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INSTITUTE OF ACTUARIES.

No. 223.—JANUARY 1906.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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1906.



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CONTENTS OF VOL. XL.

	PAGE
On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables. By George King, F.I.A., F.F.A., one of the Vice-Presidents of the Institute of Actuaries	1
On a Property of the O ^[M] Select Tables, and its Application to the Valuation of Whole-Life Policies. By O. F. Diver, M.A., F.I.A., of the Clerical, Medical and General Life Assurance Society.....	15
Notes on an approximate Method of Valuation of Whole-Life Assurances, with Allowance for Selection. By Thomas G. Ackland, one of the Vice-Presidents of the Institute of Actuaries; Honorary Fellow of the Faculty of Actuaries; with Appendix	42
Abstract of the Discussion on the above three Papers	84
Report of the Departmental Committee on Bond Investment Companies.....	99
Canadian Vital Statistics; with particular reference to the Province of Ontario. By M. D. Grant, B.A., F.I.A., Assistant Actuary, Government Insurance Department, Ottawa, Canada	125
Abstract of the Discussion on the preceding.....	143
The Variations in Masculinity under different conditions. By John Norman Lewis, F.I.A., F.F.A., and Charles James Lewis, D.Sc., M.D., F.R.C.P. (Edinburgh)	154
Abstract of the Discussion on the preceding	181
On some Special Features of Widows' and Orphans' Funds. By Ernest Charles Thomas, F.I.A., of the Gresham Life Assurance Society	188
On the Calculation of the Contributions to be made to an Annuity Fund for Widows and Children, by the members of a Society, all of whom, whether married or single, are obliged to contribute. By Oscar Schjoll, formerly Manager (Kontorechef) of the "Idun" Life Insurance Company, Christiania; Manager of the Royal Office for Workmen's Assurances (Rigsforsikringsanstalten), Christiania	200

	PAGE
Historical Memorandum on Friendly Societies in connection with Actuarial Certificates. (Extracted from the Report of the Chief Registrar of Friendly Societies for the year 1904)	212
On a form of Spurious Selection which may arise when Mortality Tables are amalgamated. By W. Palin Elderton, F.I.A., of the Guardian Assurance Company	221
Abstract of the Discussion on the preceding	234
Some Aspects of Registration of Title to Land. By James Robert Hart, F.I.A., of the Pelican and British Empire Life Office	246
Abstract of the Discussion on the preceding	273
The Life Assurance Companies of the United Kingdom. (Extracted from the Parliamentary Returns for 1905.)	313
Reversionary Securities as Investments. By C. R. V. Coutts, F.I.A., Assistant Actuary of the National Mutual Life Assurance Society.....	317
Abstract of the Discussion on the preceding	332
French Assurance Law, 1905	343
Fifth International Congress of Actuaries	416
ACTUARIAL NOTES :	
On an alternative type of formula for Approximate Summation. By W. Palin Elderton, F.I.A.....	116
Interpolation by Finite Differences (Two Independent Variables). By Herbert H. Edwards	289
Some Practical Hints on Two-Variable Interpolation. By John Spencer, F.I.A.	293
On the Law of Uniform Seniority	302
CORRESPONDENCE :	
Letter from Mr. Stenart E. Macnaghten on the Determination of Average Ages by Methods of Weighting	120
„ „ Mr. Duncan C. Fraser on the use of $O^{(M)}$ Select Premiums for Valuation Purposes	122
„ „ Mr. James Bacon on "Spurious Selection"	304
„ „ Mr. W. Palin Elderton on the same subject	309

THE INSTITUTE OF ACTUARIES :

Notice of Lectures on the Actuarial subjects of Parts III and IV of the Examinations	124
Alteration in Rules of Examination	312
Examination Papers—Associate (Part I), April 1906	375
" " " (Part II), April 1906	378
" " Fellow (Part III), April 1906	381
" " " (Part IV), April 1906	385
Proceedings of the Institute—Session 1905-1906	389
Report, 1905-1906	390
Revenue Account and Balance Sheet, for the Year ending 31 March 1906.....	392-393
Results of the Examinations, 1906	395
Proceedings at the Annual General Meeting, 1906.....	397
Additions to the Library.....	403

ORIGINAL TABLES :

Continuous Temporary Annuities, $O^M 2\frac{3}{4}\%$	368
--	-----

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CONTENTS OF NO. 223.

	PAGE
List of Members, as on 1 January 1906	i
On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables. By George King, F.I.A., F.F.A., one of the Vice-Presidents of the Institute of Actuaries	1
On a Property of the O ^[M] Select Tables, and its Application to the Valuation of Whole-Life Policies. By O. F. Diver, M.A., F.I.A., of the Clerical, Medical and General Life Assurance Society.....	15
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ACTUARIAL NOTE:	
On an alternative type of formula for Approximate Summation. By W. Palin Elderton, F.I.A.....	116
CORRESPONDENCE :	
Letter from Mr. Steuart E. Macnaghten on the Determination of Average Ages by Methods of Weighting	120
,, ,, Mr. Duncan C. Fraser on the use of O ^[M] Select Premiums for Valuation Purposes	122
THE INSTITUTE OF ACTUARIES :	
Notice of Lectures on the Actuarial subjects of Parts III and IV of the Examinations	124

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1897	†Kentish, Owen, <i>Economic Life Assurance Soc.</i> , 6 New Bridge-street, E.C.	1885	Mackenzie, Alexander George, F.F.A., 47 York-terrace, Regent's-park, N.W.
1874	†King, George, F.F.A., F.A.S. (VICE-PRESIDENT and LECTURER), 15 Walbrook, E.C.	1900	†Macnaghten, Stuart Edye, A.C.A., <i>Equity & Law Life Assur. Soc.</i> , 18 Lincoln's-inn-fields, W.C.
1887	†Kyd, Thomas, F.F.A., <i>Northern Assurance Company</i> , 1 Union-terrace, Aberdeen.	1901	†Macphail, Donald, F.F.A., <i>Yorkshire Insurance Company</i> , Cape Town, South Africa.
1882	Lancaster, William John, J.P., South Lynn, Putney-hill, s.w.	1870	†Manly, Henry William, F.A.S. (PAST PRESIDENT, 1898-1900), Glenthorne, 157, Highbury New-park, N.
1894	†Laughton, Alexander Millar, F.F.A., <i>National Mutual Life Assoc. of Australasia, Limited</i> , Corner of Collins and Queen-streets, Melbourne, Australia.	1890	†Marks, Geoffrey (LIBRARIAN), <i>National Mutual Life Assur. Soc.</i> , 39 King-street, Cheapside, E.C.
1887	†Lemon, William Kent, Barrister-at-Law, 1 Vanbrugh-terrace, Blackheath, S.E.	1900	†Marr, Vyvyan, F.F.A., <i>Edinburgh Life Assurance Co.</i> , 22 George-street, Edinburgh.
1896	†Levine, Abraham, M.A., <i>Alliance Assurance Co., Ltd.</i> , Bartholomew-lane, E.C.	1902	†May, Basil, <i>Royal Exchange Assur. Corp.</i> , Royal Exchange, E.C.
1896	†Lewis, John Norman, F.F.A., <i>London Assurance Corporation</i> , 7 Royal Exchange, E.C.	1897	†May, George Ernest, <i>Prudential Assurance Company</i> , Holborn-bars, E.C.
1892	†Lidstone, George James, <i>Equitable Life Assurance Soc.</i> , Mansion-house-street, E.C.	1897	†Miller, Neville, <i>London Assurance Corporation</i> , 7 Royal Exchange, E.C.
1901	†Little, James Fulton, <i>Mutual Life Association of Australasia</i> , Perth, West Australia.		

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1905	† Milligan, Charles Livingstone, <i>Provident Life Office, 50 Regent-street, w.</i>	1901	† Norton, William Ernest, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1893	† Milner, John William, <i>North British & Mercantile Insur. Co., 61 Threadneedle-street, E.C.</i>	1905	† Oakley, Henry John Percy, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>
1892	† Milton, Henry, M.A., <i>Law Debenture Corporation, Ltd., 41 Threadneedle-street, E.C.</i>	1899	† Parker, Robert Peter, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1899	† Moir, Henry, F.F.A., F.A.S., <i>Provident Savings Life Assur. Soc., 346 Broadway, New York, U.S.A.</i>	1864	Pearson, Arthur, <i>Betchworth-house, The Bank, Highgate, N.</i>
1890	† Molyneux, Arthur Ernest, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>	1905	† Penman, William, Jr., <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1901	† Moorhouse, Alfred, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>	1891	† Phelps, William Peyton, M.A., <i>Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>
1897	† Moors, Elphinstone MacMahon, M.A., <i>University of Sydney, Australia.</i>	Under the Charter.	Priestley, John George, <i>44 St. German's-road, Forest-hill, S.E.</i>
1896	† Moran, Joseph Flack, <i>Reversionary Interest Society, 30 Coleman-street, E.C.</i>	1891	† Pulley, William Pritchard, <i>Norwich Union Life Insur. Soc., 71 & 72 King William-st., E.C.</i>
1900	† Morgan, Benjamin Charles, M.A., <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1903	† Rae, Joseph, <i>Finance Department, Town-hall, Upper-street, N.</i>
1895	† Muter, Percy, <i>New Zealand Government Life Insurance Department, Wellington, New Zealand.</i>	1899	† Raisin, Arthur Herbert, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>
1888	† Nash, Willie Oscar, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), W.C.</i>	1897	† Rees, Martin, <i>Law Reversionary Interest Soc., Limited, Thanet-house, 231 & 232 Strand (opposite the Law Courts), W.C.</i>
1883	Neison, Francis G. P., F.S.S., <i>19 Abingdon-st., Westminster, S.W.</i>	1901	† Reeve, Charles Ernest, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1888	† Newman, Philip Lewin, B.A., <i>Yorkshire Insurance Co., York.</i>	1902	† Richmond, George William, <i>Scottish Widows' Fund and Life Assur. Society, 28 Cornhill, E.C.</i>
1865	Newton, Algernon, M.A., <i>c/o London & Westminster Bank, 94 & 96 High-st., Kensington, W.</i>	1904	† Rietschel, Hermann Julius, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1887	† Nightingale, Harry Ethelston, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>	1898	† Robinson, George Frederick, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1903	† Norris, Charles Arthur, <i>National Mutual Life Association of Australasia, Limited, Melbourne, Australia.</i>	1905	† Robinson, Hugh Thomas Kay, <i>Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary, S.W.</i>
		1888	† Rusher, Edward Arthur, F.S.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1882	†Ryan, Gerald Hemmington, F.A.S., (TREASURER), <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>	Under the Charter	Stevens, Charles, <i>Aberdeen Ho., Preston, Brighton.</i>
1898	†Salmon, Richard George, F.S.S., <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1888	Stewart, John, F.F.A., <i>City of Glasgow Life Assur. Co., 30 Renfield-street, Glasgow.</i>
1883	Saunders, Harris Charter Lindon, F.R.A.S., "Marquise," Twickenham.	1898	Stirling, Robert, F.F.A., <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>
1886	†Schooling, Frederick, F.A.S. (VICE-PRESIDENT), <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1892	†Straker, Edward Robert, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>
1901	†Searle, George Morley, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1878	†Straker, Frank Arthur, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1901	†Sharman, William Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	†Strong, William Richard, <i>London Guarantee & Accident Co., 61 Moorgate-street, E.C.</i>
1905	†Sherriff, Francis Henry, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>	1884	†Stuart, John Moody, F.F.A., <i>Leeds Permanent Benefit Building Society, Victoria-buildings, Park-lane, Leeds.</i>
1896	†Sim, William Abernethy, F.F.A., <i>Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.</i>	1900	†Sutherland, John, M.A., <i>Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.</i>
1875	†Smith, Arthur, <i>Inces, Scaynes-hill, Hayward's-heath.</i>	1889	†Tarn, Arthur Wyndham, <i>Westminster and General Life Assurance Association, 28 King-street, Covent-garden, W.C.</i>
1881	†Somerville, William Finlay, <i>Liverpool and London and Globe Insurance Co., 1 Dale-street, Liverpool.</i>	1887	Teece, Richard, F.F.A., F.A.S., <i>F.S.S., Australian Mutual Provident Society, Sydney, Australia.</i>
1877	†Sorley, James, F.S.S., F.R.S.E., 82 Onslow-gardens, S.W.	1872	Templeton, Col. John M., C.M.G., <i>National Mutual Life Association of Australasia, Melbourne, Australia.</i>
1898	†Spencer, John (SUB-EDITOR OF JOURNAL), <i>English and Scottish Law Life Assurance Assoc., 12 Waterloo-place, S.W.</i>	1864	†Terry, James, <i>Hernlee, Lyme Regis, Dorset.</i>
1894	†Sprague, Alfred Ernest, D.Sc., M.A., F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>	1889	†Thiselton, Herbert Cecil, F.F.A., F.A.S., <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1857	Sprague, Thomas Bond, M.A., LL.D., Hon. F.F.A., F.S.S., F.R.S.E. (PAST PRESIDENT, 1882-86), 29 Buckingham-ter., Edinburgh.	1901	†Thodey, Robert, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1896	†Stahlschmidt, Louis, <i>St. John's College, Agra, India.</i>	1893	†Thomas, Ernest Charles, <i>Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1899	† Thomas, Robert Arthur Caradoc, <i>Pelican and British Empire Life Office, 12 Dalhousie-sq., Calcutta.</i>	1904	† Weatherill, Henry, <i>National Debt Office, E.C.</i>
1905	† Thompson, Thomas Percy, B.A., <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>	1880	† Whittall, Wm. Joseph Hutchings, F.A.S., <i>Clerical, Medical & General Life Assur. Soc., 15 St. James's-sq., S.W.</i>
1895	† Thomson, Herbert Archer, B.A., <i>Umberlade, Boscobel-road, St. Leonard's-on-Sea.</i>	1905	† Wilson, John Sydney, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1893	† Thorne, Alfred Charles, <i>Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>	1864	Wilson, Robert, <i>44 Talfourd-rd., Camberwell, S.E.</i>
1891	† Tilt, Robert Ruthven, <i>General Reversionary & Investment Co., Ltd., 26 Pall-mall, S.W.</i>	1888	† Wilson, Robert, Jr., <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1902	† Tinner, Thomas, <i>Comptroller's Depart., London County Council, Spring-gardens, S.W.</i>	Under the Charter.	Winser, Thomas Boorman, F.R.G.S., F.R.N.S., <i>81 Shooter's-hill-road, Blackheath, S.E.</i>
1881	† Todd, George, M.A. (HON. SEC.), <i>Economic Life Assurance Society, 6 New Bridge-street, E.C.</i>	1899	† Winter, Arthur Thomas, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>
1894	† Todhunter, Ralph, M.A., <i>University Life Assur. Soc., 25 Pall-mall, S.W.</i>	1897	† Wintle, Lancelot Andrewes, <i>Economic Life Assurance Soc., 6 New Bridge-street, E.C.</i>
1899	† Trouncer, Harold Moltke, M.A. <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1904	† Wood, Arthur Barton, B.A., F.A.S., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1878	† Turnbull, Andrew Hugh, F.F.A., F.R.S.E., <i>18 Whitehouse-loan, Edinburgh.</i>	1884	† Woods, Ernest, F.A.S. (VICE-PRESIDENT), <i>Westminster and General Life Assurance Association, 28 King-street, Covent-garden, W.C.</i>
1889	Wallace, Thomas, F.F.A., <i>North British & Mercantile Insurance Co., 64, Princes-street, Edinburgh.</i>	1902	† Woolmer, Alfred Henry, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>
1905	† Wandless, John Robert, <i>Canada Life Assurance Co., 14 King William-street, E.C.</i>	1902	† Workman, William Arthur, <i>Equitable Life Assur. Society, Mansion-house-street, E.C.</i>
1888	† Warner, Samuel George (HON. SEC.), <i>Law Union & Crown Insur. Co., 126 Chancery-lane, W.C.</i>	1902	† Worthington, William, <i>Royal Insurance Co., Liverpool.</i>
1893	† Watson, Alfred William, <i>Manchester Unity Friendly Soc., Nottingham.</i>	1875	† Wyatt, Frank Bertrand, F.A.S., <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, S.W.</i>
1895	† Watson, James Douglas, F.A.S., <i>English & Scottish Law Life Assr. Assoc., 12 Waterloo-place, S.W.</i>	1874	Young, Thomas Emley, B.A., F.R.A.S., F.A.S. (PAST-PRES., 1896-8), <i>108 Evering-road, Stoke Newington, N.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1900	² Adams, Cecil Francis, <i>New Zealand Accident Insurance Company, Wanganui, New Zealand.</i>	1894	³ Barton, Robert Whitechurch, <i>Clerical, Medical & General Life Assurance Society, 26 Mosley-street, Newcastle-on-Tyne.</i>
1905	(²) Adamson, William, F.F.A., <i>Scottish Accident Life & Fidelity Insurance Co., Ltd., 115 George-street, Edinburgh.</i>	1903	² Baxter, Edwin Herbert, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1869	² Adey, Theodore Henry, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>	1901	² Benjamin, Stanley O., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1899	³ Adlard, Stanley, A.K.C., <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1881	Birks, Edmund Alfred, <i>Yorkshire Insurance Co., York.</i>
1899	² Ansell, George Frederic, <i>National Debt Office, E.C.</i>	1898	(²) Blount, Edward Thos. J., F.F.A., F.S.S., <i>Standard Life Assurance Co., Shanghai, China.</i>
1904	² Ashley, Charles Henry, <i>British Widows' Assurance Co., 1 Old-street, E.C.</i>	1873	² Boon, Gerald Inglis, <i>United Legal Indemnity Insur. Soc., Limited, 222 Strand, W.C.</i>
1883	² Ashley, John Geo., M.A., <i>War Office, S.W.</i>	1889	(²) Bremner, Thomas William, F.F.A., <i>Mutual Life Insurance Co. of New York, Sydney, Australia.</i>
1901	² Ashton, William Richard, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1905	(²) Brodie, Robert Raynal, F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-sq., Edinburgh.</i>
1904	² Atkins, Leonard George, <i>Law Union & Crown Insurance Co., 126 Chancery-lane, W.C.</i>	1896	(²) Brown, George Andrew (AUDITOR), <i>Clerical, Medical & General Life Assurance Society, 1 King William-street, E.C.</i>
1881	² Ayling, Charles Stephen, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1899	² Brown, Harold, <i>Scottish Union and National Insurance Co., 3 King William-street, E.C.</i>
1905	² Bain, William Algernon, <i>Manufacturers' Life Insurance Co., Toronto, Canada.</i>	1886	Buckley, Thomas John Wesley, <i>9 St. Andrew-street, Holborn-circus, E.C.</i>
1903	² Ball, Sidney Robertson, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>	1882	Burke, David, F.S.S., <i>Royal Victoria Life Insur. Co., Montreal, Canada.</i>
1905	² Barford, Frederick William, M.A., <i>The High School, Perth, West Australia.</i>	1900	² Burnley, Isaac, <i>Australian Mutual Prov. Society, Sydney, Australia.</i>
1904	² Barrett, William Goodsman, <i>United Kingdom Temperance and General Provident Institution, 1 Adelaide-place, London-bridge, E.C.</i>	1895	³ Butterfield, William Thos., A.C.A., <i>9 Market-street, Bradford, Yorkshire.</i>
1885	Barton, Arthur, <i>Royal Insurance Company, Maidstone.</i>	1905	(²) Cameron, Finlay James, F.F.A., <i>Friends' Provident Institution, Bradford, Yorkshire.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of
becoming an
Associate.

Date of
becoming an
Associate.

- 1876 Carter, Eric Mackay,
33 Waterloo-street, Birmingham.
- 1899 ² Catchlove, Chas. Hamilton Leyland
*Australian Mutual Provident
Society, c/o H. S. King & Co.,
65 Cornhill, E.C.*
- 1904 ⁽²⁾ Cathles, Lawrence MacLagan,
F.F.A.,
*Franklin Life Insurance Co.,
Springfield, Ill., U.S.A.*
- 1905 ² Chubb, William,
*Sun Life Assurance Company of
Canada, Montreal, Canada.*
- 1898 ² Coates, Thomas Linnaeus,
*North British and Mercantile
Insurance Co., 61 Threadneedle-
street, E.C.*
- 1904 ² Collier, Charles Aubrey,
30 Crockerton-road, Tooting, s.w.
- 1871 Cook, Arthur James, M.J.I.,
*Victoria Mutual Assur. Society,
Farringdon-street, E.C.*
- 1899 ³ Cook, William Playfair,
*Guardian Assurance Company,
11 Lombard-street, E.C.*
- 1897 ² Coop, Charles Rowland,
*United Kingdom Temperance and
General Provident Institution,
28 High-street, Birmingham.*
- 1905 ² Cooper, John James,
*Sun Life Assurance Co. of
Canada, Montreal, Canada.*
- 1891 ² Coote, Ernest Charles,
*Alliance Assurance Co., Ltd.,
Bartholomew-lane, E.C.*
- 1900 ² Corbett, Edwin Somerville,
*Australasian Temperance and
General Mutual Life Assurance
Society, Sydney, Australia.*
- 1871 Countts, Edwin Arthur,
*North British and Mercantile
Insurance Company, 12 Low-
pavement, Nottingham.*
- 1900 ² Covington, Oliver Henry,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1884 Craig, Robert Alexander,
*Abstainers' and General Assur.
Co., City Buildings, Birmingham.*

- 1901 ³ Culley, Alfred Benjamin,
*Star Life Assurance Society, 32
Moorgate-street, E.C.*
- 1900 ³ Curtis, William Allen,
*Clerical, Medical & General
Life Assurance Society, 15 St.
James's-square, s.w.*
- 1904 ² Daman, Gerard William, B.A.,
*Commercial Union Assur. Co.
26 New-bridge-street, E.C.*
- 1902 ³ Denmead, John Charles, M.A.,
F.F.A.,
*Estate Duty Office, Somerset-
house, w.c.*
- 1901 ² Diamond, George Frederick,
*City Mutual Life Assurance
Society, Hunter-street, Sydney,
Australia.*
- 1855 Dix, James,
Hurstdale, Wood-la., Highgate, n.
- 1901 ⁽²⁾ Donald, Alexander Graham, M.A.,
F.F.A.,
*Scottish Provident Institution,
6 St. Andrew-square, Edinburgh.*
- 1881 Donaldson, John,
*Australian Widows' Fund Life
Assurance Society, Collins-street-
west, Melbourne, Australia.*
- 1899 ² Dougharty, Harold, F.S.S.,
F.C.I.S.,
*London and Lancashire Life
Assurance Company, 66 & 67
Cornhill, E.C.*
- 1902 ² Doust-Smith, Ernest Charles,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1881 Dovey, William Roadly, F.F.A.,
F.A.S.,
26 Drylands-rd., Crouch End, n.
- 1905 ² Downes, Edward George,
*c/o T. G. Ackland, Esq.,
5 & 6 Clement's-inn, w.c.*
- 1870 Dowson, John,
Royal Insur. Company, Liverpool.
- 1898 ² Doyle, Arthur James,
54 Bourke-st., Sydney, Australia.
- 1901 ² Earle, Arthur Percival,
*Reliance Life Insurance Co.,
Farmers' Bank-buildings, Pitts-
burgh, Pa., U.S.A.*

ASSOCIATES.

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Date of becoming an Associate.		Date of becoming an Associate.	
1868	Eaton, Henry William, <i>Liverpool & London & Globe Insurance Company, William-street, New York, U.S.A.</i>	1894	² Fraser, Thomas John, <i>Australian Alliance Assurance Company, Melbourne, Australia.</i>
1904	² Ebihara, Kaitaro, 151 <i>Denmark-hill, S.E.</i>	1901	(2) Gaff, William Robertson, C.A., F.F.A., 53 <i>New Broad-street, E.C.</i>
1904	² Ecroyd, Cuthbert W., <i>Friends' Provident Institution, Ocean Chambers, 44 Waterloo-street, Birmingham.</i>	1873	² Gage, Uriah Woodard, <i>North British and Mercantile Insurance Company, 61 Thread-needle-street, E.C.</i>
1905	² Elderton, Robert Lapidge, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1895	² Galwey, Charles Edmund, <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>
1905	² Ellis, Reginald George Gregson, 12 <i>Manson-pl., Queen's-gate, S.W.</i>	1893	² Gardiner, Robert Edward, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1872	² Evans, William, F.F.A., F.R.S.E., 38 <i>Morningside-park, Edinburgh.</i>	1885	² Gayford, Herbert Stannard, <i>Northern Assurance Co., 1 Moor-gate-street, E.C.</i>
1905	² Falk, Oswald Toynbee, B.A., <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>	1899	³ Gibb, James Burnett, F.F.A., <i>Penn Mutual Life Insce. Co. of Philadelphia, 923 Chestnut-st., Philadelphia, U.S.A.</i>
1905	² Farmer, Ernest Chattock, <i>London, Edinburgh & Glasgow Assurance Company, Limited, Insurance-bldgs., Farringdon-street, E.C.</i>	1871	² Glennie, William Gordon, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>
1896	² Featherstonehaugh, William Irwin, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	1897	² Goggs, Frank Sidney, <i>Scottish Metropolitan Life Assur. Co., Ltd., 25 St. Andrew-sq., Edinburgh.</i>
1903	² Ferguson, Colin C., B.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>	1882	Goldman, Leopold, F.S.S., <i>North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.</i>
1905	² File, Lorne K., B.A., <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1904	² Goodman, Gilbert, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1897	² Findlay, Alexander Wynaud, LL.D., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	² Goodwyn, John, Jr., <i>Norwich & London Accident Insurance Association, 53 Queen-street, Melbourne, Australia.</i>
1902	² FitzGerald, Charles R., <i>State Mutual Life Assur. Co., Worcester, Mass., U.S.A.</i>	1905	² Gould, W. H., M.A., <i>Sovereign Life Assurance Co., Toronto, Canada.</i>
1901	² FitzGerald, William George, B.A., 82 <i>Dupont-street, Toronto, Canada.</i>	1902	² Gray, Robert Alexander, B.A., 324 <i>Markham-street, Toronto, Canada.</i>
1890	(2) Fox, Charles Edward, F.F.A., <i>Standard Life Assurance Co., 53 King William-street, E.C.</i>		
1886	Fox, Morris, F.A.S., <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>		

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Date of becoming an Associate.		Date of becoming an Associate.	
1868	Greig, John Andrew, <i>Sun Life Assurance Society</i> , 60 <i>Charing-cross</i> , S.W.	1884	Higham, William Samuel, <i>Equitable Life Assurance Soc.</i> , <i>Mansion-house-street</i> , E.C.
1869	Griffith, E. Clifton, 4 <i>Carlton-chambers</i> , S.W.	1905	² Hitchins, William Richmond, B.A., F.A.S., 336, <i>Shaw-st.</i> , Toronto, Canada.
1903	² Hall, John Bertram, <i>Imperial Life Assurance Co. of</i> <i>Canada</i> , Toronto, Canada.	1894	² Hollingworth, Albert Charles, <i>Australian Mutual Provident</i> <i>Society</i> , Melbourne, Australia.
1893	² Hall, John Francis Edmund, <i>Eagle Insurance Company</i> , 79 <i>Pall-mall</i> , S.W.	1883	Holt, Edward Hallett, <i>Law Life Assurance Society</i> , 187 <i>Fleet-street</i> , E.C.
1905	² Hallman, M. S., <i>Mutual Life Assurance Company</i> <i>of Canada</i> , Waterloo, Ontario, Canada.	1898	² Howell, Chas. Edward, B.A., LL.D., <i>Standard Life Assurance Compy.</i> , 59 <i>Dawson-street</i> , Dublin.
1905	² Hammond, Reginald, <i>British Equitable Life Assur.</i> <i>Co., Ltd.</i> , Queen-street-place, E.C.	1899	² Hudson, Alfred James, <i>Northern Assurance Company</i> , 1 <i>Moorgate-street</i> , E.C.
1869	Hann, Robert George, F.A.S., <i>Equitable Life Assur. Soc. of</i> <i>the United States</i> , 120 <i>Broadway</i> , New York.	1903	³ Humphreys, Henry Thompson, <i>Sun Life Assurance Society</i> , 63 <i>Threadneedle-street</i> , E.C.
1894	² Harcastle, Edward Edgington, M.A., F.A.S., <i>Union Central Life Office</i> , Cin- cinnati, Ohio, U.S.A.	1875	Hunt, Richard Aldington, F.S.S., <i>Wesleyan & General Assur. Soc.</i> , <i>Steelhouse-lane</i> , Birmingham.
1900	² Harding, Harry Burnard, <i>Commercial Union Assur. Co.</i> , 26 <i>New Bridge-street</i> , E.C.	1893	(²) Hunter, Arthur, F.F.A., F.A.S., F.S.S., <i>New York Life Insurance Co.</i> , 346 & 348 <i>Broadway</i> , New York, U.S.A.
1896	³ Harris, Frederick Joseph, <i>Australian Mutual Provident</i> <i>Society</i> , Sydney, Australia.	1902	² Hunter, Robertson G., F.A.S., <i>New York Life Insurance Co.</i> , <i>New York Life Building</i> , Chicago, Ill., U.S.A.
1904	² Harriss, Walter James, <i>Law Life Assurance Society</i> , 187 <i>Fleet-street</i> , E.C.	1887	² Hunter, Samuel, <i>Patriotic Assurance Company</i> , 9 <i>College-green</i> , Dublin.
1897	² Haycraft, William Melhuish, <i>Prudential Assurance Company</i> , <i>Holborn-bars</i> , E.C.	1904	(²) Imrie, John Hamilton, M.A., F.F.A., <i>Life Association of Scotland</i> , 82 <i>Princes-street</i> , Edinburgh.
1897	² Hazell, James Stanley, <i>National Provident Institution</i> , 48 <i>Gracechurch-street</i> , E.C.	1889	(²) Jacobs, Frederick Job, <i>Australian Mutual Provident</i> <i>Society</i> , Sydney, Australia.
1895	² Heness, Leonard Thomas, <i>Prudential Assurance Company</i> , <i>Holborn-bars</i> , E.C.	1876	² James, George Trevelyan, 12 <i>Waterloo-place</i> , S.W.
1878	Henry, Alfred, F.C.A., <i>Throgmorton-house</i> , <i>Copthall-</i> <i>avenue</i> , E.C.	1905	(²) Jamieson, Charles William Steele, F.F.A., <i>Scottish Amicable Life Assur.</i> <i>Society</i> , 35 <i>St. Vincent-place</i> , Glasgow.
1900	³ Hicks, Arthur Joseph, <i>Law Life Assurance Society</i> , 187 <i>Fleet-street</i> , E.C.		

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Date of becoming an Associate.		Date of becoming an Associate.	
1905	² Jefferson, John Arthur, c/o T. G. Ackland, Esq., 5 & 6 Clement's-inn, W.C.	1902	² Kitchin, Frederick Harcourt, B.A., Corehythe, Kingston-lane, Ted- dington.
1871	Jellicoe, George Rogers, Eagle Insurance Company, 79 Pall-mall, S.W.	1905	(²) Laing, James Murray, F.F.A., National Mutual Life Assocn. of Australasia, 5 Cheapside, E.C.
1883	Jerman, Richard, Commercial Union Assurance Company, Exeter.	1893	² Laing, William Claud, North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.
1896	² Jobson, Alexander, Equitable Life Assurance Society of the United States, Sydney, Australia.	1897	² Lane, Arthur Vere, B.A., City of Glasgow Life Assurance Co., 1 Princess-street, Albert- square, Manchester.
1894	² Johannessen, Nikolai Mikal, Hygea Life Assurance Company, Bergen, Norway.	1905	² Langstaff, James Miles, Imperial Life Assurance Co. of Canada, Toronto, Canada.
1894	² Johnston, Frederick H., F.A.S., Prudential Life Insurance Co. of America, Newark, N.J., U.S.A.	1905	² Latham, Bertrand, Australian Mutual Provident Society, Melbourne, Australia.
1903	² Jones, Leonard Alexander Mouat, Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.	1899	² Lawton, George Herbert, Clerical, Medical & General Life Assurance Society, 15 St. James's- square, S.W.
1903	² Jones, Wallace Mouat, General Reversionary & Invest- ment Company, Limited, 26 Pall- mall, S.W.	1885	Ledward, Archibald Prentice, B.Sc., 29 Langland-gardens, N.W.
1898	² Kaufman, Henry N., A.A.S., Phoenix Mutual Life Insurance Co., Hartford, Connecticut, U.S.A.	1905	² Leigh, Samuel George, Refuge Assurance Co., Oxford- street, Manchester.
1876	Kearry, Joseph, 44 Charwood-street, Belgrave- road, S.W.	1879	Leitch, Alexander, Scottish Provident Institution, 17 King William-street, E.C.
1899	² Kelly, John Joseph, Citizens' Life Assurance Co., Sydney, Australia.	1897	² Le Maitre, Frank William, Sun Life Assurance Society, 63 Threadneedle-street, E.C.
1897	² Kemp, Julian Ernest Sandford, Eagle Insurance Company, 79 Pall-mall, S.W.	1885	Leveaux, Arthur Michael, F.S.S., Registry of Friendly Societies, Central Office, 28 Abingdon- street, Westminster, S.W.
1902	² Kilgour, David Errett, M.A., North American Life Assurance Co., North American Life Building, 112-118 King-street- west, Toronto, Canada.	1868	Litchfield, Edward, 92 St. Vincent-street, Glasgow.
1874	King, Arthur Thomas, I.S.O., National Debt Office, E.C.	1876	² Lucey, Herbert, General Assurance Company, 103 Cannon-street, E.C.
1882	² King, William Alfred, Northern Assurance Company, 1 Moorgate-street, E.C.	1890	(²) Lugton, Hugh, F.F.A. (AUDITOR), North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.

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Date of becoming an Associate.		Date of becoming an Associate.	
1900	³ McArthur, Harry de C., <i>Box 282, Dunedin, New Zealand.</i>	1900	² Maunder, George Harvard, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1867	Macdonald, William Rae, F.F.A., <i>Scottish Metropolitan Life Assur. Co., Limited, 25 St. Andrew- square, Edinburgh.</i>	1902	(²) Maxwell, Benjamin Bell, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>
1882	³ McDougald, Alfred, <i>Pelican and British Empire Life Office, Montreal, Canada.</i>	1903	³ May, Walter Thomas, <i>Liverpool and London and Globe Insurance Co., 1 Cornhill, E.C.</i>
1905	² Macfarlane, James Allan, <i>North American Life Assurance Co., North American Life Building, 112-118 King-street- west, Toronto, Canada.</i>	1899	² Meade, Gerald Willoughby, <i>North British & Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>
1884	Mackay, Alexander, <i>Law Union & Crown Insur. Co., 126 Chancery-lane, W.C.</i>	1896	² Merfield, Percy Henry, <i>Law Life Assurance Society, 187 Fleet-street, E.C.</i>
1905	² McKechmie, James Baldwin, <i>Manufacturers' Life Insurance Company, Toronto, Canada.</i>	1874	Miller, John W., F.S.S., <i>Scottish Widows' Fund and Life Assur. Soc., 28 Cornhill, E.C.</i>
1901	³ Mackenzie, Michael Alexander, <i>c/o Messrs. John Mackay & Co., Investment Brokers, 7 & 9 King- street-east, Toronto, Canada.</i>	1905	² Monilaws, William Barrington, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>
1896	² Macmillan, John Campbell, <i>Northern Assur. Co. and Law Union and Crown Insurance Co., Apartado Postal No. 872, Mexico.</i>	1879	Monilaws, William Macgeorge, (AUDITOR), <i>Scottish Provident Institution, 17 King William-street, E.C.</i>
1905	² McPhail, Frederick Charles, <i>Colonial Mutual Life Assurance Soc., Ltd., Melbourne, Australia.</i>	1905	² Monkhouse, Charles Cosmo, B.A., <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>
1883	² Makeham, William Reed, <i>Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane, W.C.</i>	1877	Moon, James, <i>Prudential Assurance Company, 30 Dale-street, Liverpool.</i>
1905	² Makepeace, Francis Lucas, B.A., <i>29 Deronda-road, Herne-hill, S.E.</i>	1877	Moon, John, <i>Prudential Assurance Company, 76 King-street, Manchester.</i>
1883	Mannering, George Willsher, <i>London and Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>	1879	Moon, Sidney Norman Laming, <i>52-54 William-street, New York, U.S.A.</i>
1880	Manwaring, Henry, <i>National Debt Office, E.C.</i>	1903	² Moore, George Cecil, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>
1896	² Martin, Sidney George, <i>National Mutual Life Assoc. of Australasia, Ltd., 295 Queen- street, Brisbane, Australia.</i>	1905	² Moore, George Edward, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1897	² Mascal, Alfred John, <i>Standard Life Assurance Co., 3 Pall-mall East, S.W.</i>	1905	² Moore, Gerald Leslie, A.C.A., <i>1 Rosebery-gardens, Muswell- hill, N.</i>
1904	² Maudling, Reginald G., <i>London and Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>		

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1898	² Moore, Joseph Patrick, <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1884	Park, Leslie John, <i>Colonial Mutual Life Assurance Society, Melbourne, Australia.</i>
1871	² Moore, Roderick Mackenzie, <i>United Kingdom Temperance and General Provident Institution, 1 Adelaide-place, London-bridge, E.C.</i>	1905	² Paton, Albert George, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1893	² Munro, Donald Alexander, <i>Brook-house, 10 Walbrook, E.C.</i>	1898	(²) Pearce, Henry John, F.F.A., <i>Scottish Amicable Life Assurance Soc., St. Vincent's-place, Glasgow.</i>
1900	² Nash, Alfred Charles, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, S.W.</i>	1899	² Peele, Thomas, <i>Universal Insurance Company, 77 New Briggate, Leeds.</i>
1903	³ Neill, Samuel Bennett, <i>London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance- buildings, Farringdon-st., E.C.</i>	1900	² Peters, Charles Furness, <i>L'pool. Victoria Legal Friendly Society, 18 St. Andrew-street, E.C.</i>
1897	² Newling, Sidney Wallis, B.A., <i>Woodleigh, South Woodford, Essex.</i>	1895	(²) Pierson, Israel Coriell, F.A.S., <i>141 Broadway, New York, U.S.A.</i>
1905	² Newnham, Ernest Whiffin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	² Pigrome, George Davey, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	² Nicholls, Arthur William, <i>Australian Mutual Provident Society, Brisbane, Australia.</i>	1899	² Pipe, Sidney Herbert, <i>London and Lancashire Life Insurance Company, Montreal, Canada.</i>
1884	Nicoll, John, F.F.A., <i>Life Association of Scotland, 82 Princes-street, Edinburgh.</i>	1883	Pitts, Thomas, <i>Commercial Union Assurance Company, Exeter.</i>
1883	Orr, Lewis P., F.F.A., <i>Scottish Life Assur. Co., Ltd., 19 St. Andrew-sq., Edinburgh.</i>	1890	² Powell, Alfred, <i>Alliance Assurance Company, Limited, Bartholomew-lane, E.C.</i>
1886	Owen, Evan Frederick, F.S.S., <i>Actuary for Friendly Societies, Melbourne, Australia.</i>	1881	Price, William John, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>
1895	² Pagden, Lionel King, <i>Union Assurance Society, 81 Cornhill, E.C.</i>	1869	Pringle, James, C.A., F.F.A., <i>42 Drumsheugh-gardens, Edin- burgh.</i>
1864	Panton, Edward Henry, <i>50 Wood-vale, Forest Hill, S.E.</i>	1884	Pullar, James, F.F.A., <i>Colonial Mutual Life Assurance Society, Melbourne, Australia.</i>
1901	³ Papps, Percy Charles Herbert, F.A.S., <i>Manufacturers' Life Insurance Co., Toronto, Canada.</i>	1881	Purves, Thomas Peter, <i>New York Life Insurance Com- pany, Sydney, Australia.</i>
1895	² Paradice, William Henry, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1904	(²) Rankin, John Adam, F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>
1869	Park, David Francis, C.A., F.F.A., <i>Crédit Foncier of Mauritius, Limited, 39 Lombard-st., E.C.</i>	1867	Rattray, Patrick, C.A., <i>115 St. Vincent-street, Glasgow.</i>
		1874	² Ray, Charles Richard, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>

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1905 ² Raynes, Harold Ernest, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>	1884 Schooling, John Holt, <i>Fotheringay-house, Montpelier-row, Twickenham.</i>
1885 Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., <i>22 Charing-cross, S.W.</i>	1899 ² Schouten, Pieter, <i>Verzekering Maatschappij, "Arnhem," Stations-plein, 17, Arnhem, Holland.</i>
1898 ² Reid, Edward E., B.A., <i>London Life Insurance Co., London, Ontario, Canada.</i>	1873 Scott, Ernest Willem, F.A.S., <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.</i>
1901 ² Rhodes, Francis, B.A., <i>Royal Insurance Co., Liverpool.</i>	1904 ² Searle, Arthur Joseph, <i>English & Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>
1887 Richardson, Josephus Hargreaves, F.F.A., F.A.S., <i>New Zealand Government Life Insurance Department, Wellington, New Zealand.</i>	1861 ² Searle, Thomas John, <i>Mansion - house - chambers, Bucklersbury, E.C.</i>
1879 Roberts, Thomas B., <i>Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.</i>	1900 ² Searls, Edwin Richard, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1904 ³ Robertson, Frederick William, F.F.A., <i>Caledonian Insurance Company, 19 George-street, Edinburgh.</i>	1900 ² Sharpe, Edgar Cecil Engledue, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1904 ⁽²⁾ Robertson, James Leask, F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>	1894 ³ Sheppard, Herbert Norman, B.A., F.A.S., <i>Home Life Insurance Company, 256 Broadway, New York, U.S.A.</i>
1878 Robertson, William, F.F.A., <i>29 Stafford-street, Edinburgh.</i>	1897 ² Shimmell, James Edward, <i>United Provident Assurance Co., Ltd., 96 Oxford-rd., Manchester.</i>
1876 Robinson, Andrew, <i>Sunningdale-park, Sunningdale, Berks.</i>	1896 ² Shlager, Joseph, <i>Equitable Life Assurance Society of the United States, Mansion-house-chambers, Adderley-street, Cape Town, South Africa.</i>
1885 Ronald, Thomas Robert, <i>Law Guarantee and Trust Soc., Ltd., 49 Chancery-lane, W.C.</i>	1903 ² Shovelton, Sydney Taverner, B.A., <i>7 Dymham-road, West Hampstead, N.W.</i>
1904 ² Rudd, Alfred James, <i>Australian Widows' Fund Life Assurance Society, Grenfell-street, Adelaide, South Australia.</i>	1905 ² Shute, Oxenham Bent, <i>National Provincial Bank of England, 53 Baker-street, W.</i>
1897 ² Ryley, Edmund, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1864 Smith, Howard Samuel, F.F.A., F.C.A., F.S.S., <i>Bank-chambers, 11 Waterloo-street, Birmingham.</i>
1896 ² Sanderson, Frank, M.A., F.F.A., F.A.S., F.S.S., <i>Canada Life Assurance Company, Toronto, Canada.</i>	1898 ² Smith, Robert Parker, <i>Royal Insurance Company, Liverpool.</i>
1904 ² Sare, Thomas Henry, <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1884 Smithett, Edward Henry, <i>"Hillside," Fitzroy-park, Highgate, N.</i>
1905 ² Savery, Robert S. B., <i>Gresham Life Assurance Society, Giselastrasse, No. 1, Vienna.</i>	

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1905	² Somerville, Walter Harold, <i>Mutual Life Assur. Co. of Canada,</i> <i>Waterloo, Ontario, Canada.</i>	1904	(²) Tatlock, John, M.A., F.R.A.S., F.A.S., 376 West-end-avenue, New York, U.S.A.
1871	Spencer, Robert James, F.S.S., 75 King's-road, Southsea.	1893	² Taylor, Arthur, <i>Westminster and General Life</i> <i>Assurance Assoc., 28 King-street,</i> <i>Covent-garden, w.c.</i>
1868	Spens, William George, <i>Scottish Amicable Life Assur.</i> <i>Soc., 35 St. Vincent-pl., Glasgow.</i>	1875	Taylor, J. Wilford, <i>North British and Mercantile</i> <i>Insur. Co., 61 Threadneedle-st., E.C.</i>
1902	³ Spurgeon, Ernest Frank, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1904	(²) Thomson, John Walter, F.F.A., <i>Scottish Life Assur. Co., 19 St.</i> <i>Andrew-square, Edinburgh.</i>
1904	² Stamp, Horatio E., "Appuldurcombe," Park Lane, Wallington, Surrey.	1883	² Titmuss, Walter George, <i>Provident Life Office, 50 Regent-</i> <i>street, w.</i>
1860	Stark, James, <i>Reversionary Interest Society,</i> <i>30 Coleman-street, E.C.</i>	1905	² Touzel, Philip Duncan, <i>Australian Mutual Provident</i> <i>Society, Melbourne, Australia.</i>
1866	Stark, William Emery, <i>Chapel-walks, Manchester.</i>	1905	² Townley, Ebenezer William, <i>National Mutual Life Assurance</i> <i>Soc., 39 King-st., Cheapside, E.C.</i>
1878	Stevenson, Charles, 9 Albert-square, Manchester.	1902	² Traversi, Antonio Thomas, <i>New Zealand Government Life</i> <i>Insurance Department, Wellin-</i> <i>ton, New Zealand.</i>
1880	Stock, Edward James, <i>National Mutual Life Assoc. of</i> <i>Australasia, Melbourne, Aus-</i> <i>tralia.</i>	1883	Tregaskis, George Alfred, <i>Commercial Union Assur. Co.,</i> <i>26 New Bridge-street, E.C.</i>
1905	² Strong, Allan Wilmot, <i>Sun Life Assurance Co. of</i> <i>Canada, Montreal, Canada.</i>	1894	² Trenerry, Charles Farley, B.A., <i>University of London, South</i> <i>Kensington, s.w.</i>
1896	² Stuckey, Jos. James, M.A., <i>Salisbury Chambers, 49a King</i> <i>William-street, Adelaide, South</i> <i>Australia.</i>	1905	² Tully, Arthur Patrick Thomas, <i>Legal and General Life Assur.</i> <i>Society, 10 Fleet-street, E.C.</i>
1905	² Stuckey, Reginald Robert, <i>Australian Mutual Provident</i> <i>Society, Adelaide, S. Australia.</i>	1891	² Turnbull, A. D. Lindsay, C.A., F.F.A., F.C.I.S., <i>Scottish Widows Fund and Life</i> <i>Assurance Society, 9 St. Andrew-</i> <i>square, Edinburgh.</i>
1905	² Sturt, Herbert Rothsay, c/o C. H. E. Rea, Esq., 22 Charing-cross, s.w.	1884	Vian, William Collett, <i>Railway Passengers' Assurance</i> <i>Company, 64 Cornhill, E.C.</i>
1904	² Sugars, Robert Morrison, B.A., <i>Gresham Life Assurance Society,</i> <i>St. Mildred's-house, Poultry, E.C.</i>	1884	Vincent, Frederick James, F.S.S., <i>London, Edinburgh & Glasgow</i> <i>Assurance Co., Ltd., Insurance-</i> <i>buildings, Farringdon-street, E.C.</i>
1899	³ Symmons, Frank Percy, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1899	² Vokius, George Alfred, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1882	Tarn, Walter George, <i>Reversionary Interest Society,</i> <i>30 Coleman-street, E.C.</i>		

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1879	Wall, Walter George, 3 <i>Shrewsbury-road</i> , <i>Birkenhead</i> .	1904	² Williams, Frederick Alfred, F.S.S., <i>Hurstpierpoint</i> , <i>Hornchurch</i> , <i>Essex</i> .
1878	Walton, William Gandy, F.F.A., <i>Scottish Provident Institution</i> , 6 <i>St. Andrew-square</i> , <i>Edinburgh</i> .	1904	² Wilson, Arthur Benjamin, <i>Australian Mutual Provident Soc.</i> , <i>Wellington</i> , <i>New Zealand</i> .
1905	² Wansbrough, Thomas Percival, <i>English and Scottish Law Life Assurance Association</i> , 37 <i>Queen Victoria-street</i> , E.C.	1900	² Wilson, George, <i>Standard Life Assurance Company</i> , 3 <i>George-st.</i> , <i>Edinburgh</i> .
1902	³ Wares, Harold Wallace, <i>Yorkshire Insurance Company</i> , <i>York</i> .	1870	² Wilson, Henry Edward, <i>Northern Assurance Co.</i> , 1 <i>Moorgate-street</i> , E.C.
1862	Waterhouse, Edwin, M.A., F.C.A., F.S.S., 3 <i>Frederick's-place</i> , <i>Old Jewry</i> , E.C.	1873	² Windett, Charles, <i>Legal & General Life Assurance Society</i> , 10 <i>Fleet-street</i> , E.C.
1903	² Watherston, Charles F., B.A., <i>War Office</i> , s.w.	1905	² Winstanley, Charles William, <i>North British & Mercantile Insurance Co.</i> , 61 <i>Threadneedle-street</i> , E.C.
1883	² Watson, John Robertson, <i>British Law Fire Insurance Co.</i> , 105 <i>West George-st.</i> , <i>Glasgow</i> .	1903	² Wood, William Archibald Porter, B.A., <i>Canada Life Assurance Co.</i> , <i>Toronto</i> , <i>Canada</i> .
1894	² Watt, George, <i>Royal Insurance Co.</i> , <i>Liverpool</i> .	1883	Woodhouse, Lister, A.C.A., F.S.S., <i>City Comptroller</i> , <i>City-hall</i> , <i>Westminster</i> , s.w.
1900	(²) Watt, James, F.F.A., 18 <i>Moray-place</i> , <i>Edinburgh</i> .	1877	² Woods, Arthur Biddle, <i>Rock Life Assurance Company</i> , 15 <i>New Bridge-street</i> , E.C.
1883	Weall, Bertram, 16 <i>Waldegrave-park</i> , <i>Twickenham</i> .	1866	Woods, Bernard, <i>Metropolitan Life Assurance Society</i> , 13 <i>Moorgate-street</i> , E.C.
1902	² Weatherill, Charles, <i>Scottish Office</i> , s.w.	1879	Wornum, Thornton Selden, <i>Rock Life Assurance Company</i> , 15 <i>New Bridge-street</i> , E.C.
1894	(²) Weeks, Rufus Wells, F.A.S., <i>New York Life Insurance Co.</i> , 346 & 348 <i>Broadway</i> , <i>New York</i> , <i>U.S.A.</i>	1903	² Worth, Bertram Oliver, <i>Clerical, Medical & General Life Assurance Society</i> , 15 <i>St. James's-square</i> , s.w.
1898	³ Whigham, Charles Frederick, F.F.A., C.A., <i>Messrs. Moncrieff & Horsburgh</i> , 46 <i>Castle-street</i> , <i>Edinburgh</i> .	1871	Yardley, John, <i>Prudential Assurance Company</i> , <i>Holborn-bars</i> , E.C.
1884	Whyte, Alexander, <i>c/o Messrs. Lever Bros., Ltd.</i> , <i>Cambridge, Mass., U.S.A.</i>	1873	Young, Alexander Hunter, 60 <i>Market-street</i> , <i>Melbourne</i> , <i>Australia</i> .
1897	² Wiekens, Charles H., <i>Government Statistician's Office</i> , <i>Perth, West Australia</i> .	1900	³ Young, Arthur Stanley, <i>Metropolitan Life Assurance Society</i> , 13 <i>Moorgate-street</i> , E.C.
1896	² Wilkinson, Edward Berkeley, 24 <i>Maxilla-gardens</i> , <i>N. Kensington</i> , w.		
1903	² Wilkinson, William Magnay, <i>Citizens' Life Assurance Co.</i> , 24 & 25 <i>King William-street</i> , E.C.		

STUDENTS.

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Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.		Date of becoming a Student.	
1892	¹ Aaron, David Hyam, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1896	¹ Barry, David, <i>Royal Commission on the University of Melbourne, Supreme Court Library, Melbourne, Australia.</i>
1903	¹ Acum, Wilfred Harry, <i>15 Lordship-lane, Wood Green, N.</i>	1905	¹ Beamish, Edwin Cooper, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905	¹ Adam, Cyrus Cyril, <i>Northern Assurance Company, 7 Westmoreland-street, Dublin.</i>	1898	¹ Bennell, Samuel Thomas, <i>25 Meath road, Ilford.</i>
1904	¹ Addey, Leonard, <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, S.W.</i>	1903	¹ Bennett, Reginald, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1905	¹ Agutter, William John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1898	¹ Bennett, Samuel, <i>National Deposit Friendly Soc., 37 Queen-square, W.C.</i>
1905	¹ Alder, Milton Cromwell, <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1902	¹ Biden, Norman Frederick, <i>Standard Life Association, 28 Elizabeth-st., Sydney, Australia.</i>
1905	¹ Allen, John, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1895	¹ Bigby, Robert Frederick Mitchell, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1904	¹ Allison, Sinclair E., <i>Canada Life Assurance Co., Toronto, Canada.</i>	1900	¹ Bingeman, Milton H., <i>Great-West Life Assurance Co., Winnipeg, Manitoba, Canada.</i>
1904	¹ Armstrong, Charles Henry, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1903	¹ Binney, Charles Eardley-Wilmot, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1886	Arnold, Thomas, Jr., <i>British Equitable Life Assur. Co., Ltd., Queen-street-place, E.C.</i>	1891	¹ Bird, Edward William, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1902	¹ Askwith, Thomas Nowell, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1905	¹ Blackadar, E. Gordon, B.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1905	¹ Atkins, Francis Cuthbert, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	¹ Blake, Francis Seymour, <i>62 Oakhurst-grove, East Dulwich, S.E.</i>
1904	¹ Ayseongh, Ivan, <i>Equity and Law Life Assurance Soc., 18 Lincoln's-inn-fields, W.C.</i>	1902	² Blanchard, Norman, B.A., <i>Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>
1899	¹ Baber, Walter Crosbie, <i>Royal Victoria Life Insur. Co. of Canada, Montreal, Canada.</i>	1901	¹ Blehl, Ernest M., A.M., <i>Security Trust & Life Insurance Co., 26th and Broadway, New York, U.S.A.</i>
1903	¹ Baggs, Henry Ernest, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>	1887	Blossom, James, <i>186 South-view-road, Sheffield.</i>
1899	¹ Barnett, Isaac, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1904	¹ Boag, Harold, <i>14 Avondale-terrace, Gateshead.</i>
		1892	¹ Boddy, Henry Mitchell, <i>Manufacturers' Life Insurance Co., Cape Town, South Africa.</i>

STUDENTS.

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Date of
becoming
a Student.

- 1903 ¹ Bodley, Rupert Frank,
Star Life Assurance Society,
32 Moorgate-street, E.C.
- 1897 Bond, Frederic D.,
122 South 39th Street, Phila-
delphia, U.S.A.
- 1900 ¹ Borrajo, Edward Joseph William,
Prudential Assurance Company,
Holborn-bars, E.C.
- 1902 ¹ Bowerman, Judah Philip,
472 Lafayette-avenue, Brooklyn,
New York, U.S.A.
- 1897 ¹ Bowles, Francis Marsh,
Pearl Life Assurance Company,
London-bridge, E.C.
- 1891 ¹ Boyd, Henry Norris,
City of Glasgow Life Assurance
Company, 21 St. Andrew-square,
Edinburgh.
- 1903 ¹ Bradbury, Algernon Charles,
Australian Mutual Provident
Society, Melbourne, Australia.
- 1905 ¹ Bradshaw, Frank Law,
Law Guarantee and Trust Soc.,
Ltd., 49 Chancery-lane, W.C.
- 1899 ¹ Brady, John Francis,
Citizens' Life Assurance Co.,
Sydney, Australia.
- 1897 ¹ Brierley, William Ernest,
Refuge Assurance Company,
Oxford-street, Manchester.
- 1904 ¹ Broad, Laurence C.,
Prudential Assurance Company,
Holborn-bars, E.C.
- 1905 ¹ Bromby, Wilfrid,
Australasian Temperance and
General Mutual Life Assurance
Society, Melbourne, Australia.
- 1902 ¹ Brook, Frank A.,
Refuge Assurance Co., Oxford-
street, Manchester.
- 1894 ¹ Brough, Frank,
Federal Life Assurance Company,
Hamilton, Ontario, Canada.
- 1904 ¹ Brown, Arthur Ewart,
Scottish Widows' Fund and Life
Assur. Society, 28 Cornhill, E.C.
- 1905 ¹ Brown, James,
Robert-st., Marrickville, Sydney,
Australia.

Date of
becoming
a Student.

- 1891 ¹ Brown, William Heron,
Gresham Life Assur. Soc., Ltd.,
St. Mildred's-house, Poultry, E.C.
- 1905 ¹ Burrows, George Eastoe,
2 Northholme-road, Highbury-
park, N.
- 1905 ¹ Burrows, Victor Albert,
2 Northholme-road, Highbury-
park, N.
- 1901 ¹ Caldwell, Richard H.,
North British and Mercantile
Insurance Co., Birmingham.
- 1904 ¹ Canter, Harold,
National Provident Institution,
48 Gracechurch-street, E.C.
- 1903 ¹ Capon, Frank Christopher,
Prudential Assurance Company,
Holborn-bars, E.C.
- 1902 ¹ Capon, Geoffrey William,
Norwich Union Life Insurance
Society, Norwich.
- 1903 ¹ Carpenter, Thomas B. Boyd,
Clergy Mutual Assur. Society,
2 & 3 The Sanctuary, S.W.
- 1901 ¹ Carter, George Stanley,
Life Association of Scotland,
5 Lombard-street, E.C.
- 1899 ² Carter, Norman John,
Eagle Insurance Company, 79
Pall-mall, S.W.
- 1900 ¹ Chambers, John Joseph,
53 Richmond-mount, Heading-
ley, Leeds.
- 1902 ¹ Chandler, Frederick Joseph,
Eagle Insurance Co., 79 Pall-
mall, S.W.
- 1905 ¹ Cherry, John Arnold,
Chamber of London, Guildhall,
E.C. (Reinstated.)
- 1903 ¹ Cheshire, Harold Frank,
Woodside, Shornden, St.
Leonard's-on-Sea.
- 1903 ¹ Child, Robert Harold,
North British and Mercantile
Insurance Company, 61 Thread-
needle-street, E.C.
- 1905 ¹ Clarke, Herbert George,
Australian Widows' Fund Life
Assurance Society, Melbourne,
Australia.

STUDENTS.

Those marked 1, 2, or 3 have passed one, two, or three of the four Examinations of the Institute.

Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.		Date of becoming a Student.	
1905	¹ Clemens, Frederic Broadbent, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1901	¹ Coventry, Cameron H., <i>Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.</i>
1897	¹ Clinton, George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1904	¹ Cowdy, Henry Leslie, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>
1902	² Clinton, Louis Ernest, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1894	Cox, Edward William, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1902	² Coates, Frederick George, <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1894	Cox, Herbert Coplin, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1901	¹ Cockerton, John Leonard, <i>Pioneer Life Assurance Co., Ltd., Century-buildings, 31 North John-street, Liverpool.</i>	1905	¹ Cox, Stanley Nelson, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1895	¹ Cogar, William Edward, <i>New York Life Insurance Co., Trafalgar-square, W.C.</i>	1887	¹ Cross, Henry John, <i>3 Park-rd., Wandsworth-common, S.W.</i>
1899	¹ Collins, Patrick A., <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1897	² Crump, Percy C., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1902	¹ Collins, William Ernest, <i>61 Osmond-terrace, Norwood, South Australia.</i>	1904	¹ Cushing, Robertson Macaulay, <i>Sun Life Assurance Company of Canada, Montreal, Canada.</i>
1896	¹ Cook, Henry Milton, <i>Standard Life Assurance Com- pany, Dalhousie-square, Calcutta, India.</i>	1904	¹ Dalrymple, Alfred George, <i>Canada Life Assurance Company, Toronto, Canada.</i>
1900	¹ Cooper, Bernard Hugh, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	¹ Dalton, John, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1902	¹ Corbett, Archibald Gladstone, <i>Australian Mutual Provident Society, Collins-st., Melbourne, Australia.</i>	1905	¹ Dark, Thomas Arthur, <i>Excelsior Life Insurance Co., Toronto, Canada.</i>
1905	¹ Cotterill, William Ernest, <i>Mutual Life Association of Australasia, Ltd., Sydney, Australia. (Reinstated.)</i>	1889	¹ Davies, Hugh Myddleton, <i>Royal Insurance Co., Liverpool.</i>
1903	¹ Cotton, Arthur Sparkes, <i>Scottish Office, S.W.</i>	1900	¹ Davies, William Allison, <i>Borough Treasurer's Office, Town Hall, Birkenhead.</i>
1897	³ Court, Alexander George Dacus, <i>Union Assurance Society, 81 Cornhill, E.C.</i>	1905	⁽¹⁾ Davis, Mervyn, B.A., <i>Connecticut General Life Insur. Co., Hartford, Conn., U.S.A.</i>
1905	¹ Countts, Kenneth Vawdrey, <i>Clergy Mutual Assurance Soc., 2 & 3 The Sanctuary, S.W.</i>	1899	¹ Davison, Horace Williams, <i>Manufacturers' Life Ins. Co., Toronto, Canada.</i>
		1891	¹ Dawson, Frank Aubrey, <i>Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand, W.C.</i>
		1902	¹ Deck, James Gilbert, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>

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Date of
becoming
a Student.

- 1904 ¹ Defries, Frederick,
*Union Assurance Society, 81
Cornhill, E.C.*
- 1902 ¹ Denmark, Robert John,
*Norwich Union Life Insurance
Society, Norwich.*
- 1901 ¹ Dent, Ernest Edward,
*London and Lancashire Life
Assurance Company, 66 & 67
Cornhill, E.C.*
- 1905 ¹ Derrick, Victor Percival Augustine,
43 Falmouth-road, S.E.
- 1896 ¹ de Ville, Francis,
*Clergy Pensions Institution, 11
Norfolk-street, Strand, W.C.*
- 1890 ¹ Docker, Leslie,
*North British and Mercantile
Insurance Co., 61 Threadneedle-
street, E.C.*
- 1897 ¹ Dorrian, John Christopher,
*Citizens' Life Assurance Com-
pany, Sydney, Australia.*
- 1901 ² Downes, Sidney Cecil,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1904 ¹ Drake, Charles Clifford Hall,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1905 ¹ Dulley, John Francis,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1905 ¹ Eastcott, William Merrill,
*Sun Life Assur. Co. of Canada,
Montreal, Canada.*
- 1892 ¹ Edwards, Edward Samuel,
*Australian Mutual Provident
Society, Sydney, Australia.*
- 1905 ¹ Edwards, Herbert Alfred,
*British Homes Assurance Corp.,
6 Paul-street, Finsbury, E.C.*
- 1905 ¹ Edwards, Herbert Horace,
31 Haverstock-road, N.W.
- 1902 ¹ Edwards, Thomas Baker,
*Comptroller's Dept., London
County Council, Spring-gardens,
S.W.*
- 1892 ¹ Eedy, Arthur Malcolm,
*Citizens' Life Assurance Com-
pany, Sydney, Australia.*

Date of
becoming
a Student.

- 1901 ¹ Eggleton, Harold Edward,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1904 ¹ Eldridge, Ernest Edward Booth,
*Atlas Assurance Company, Ltd.,
92 Cheapside, E.C.*
- 1903 ¹ Ellis, Thomas Barnes,
*Public Works Loan Board, Old
Jewry, E.C.*
- 1893 ¹ Emery, John M.,
*Des Moines Life Insurance Co.,
Des Moines, Iowa, U.S.A.*
- 1904 ¹ Esler, John,
*Crown Life Insurance Company,
Toronto, Canada.*
- 1892 ¹ Farrell, John,
*Citizens' Life Assurance Co.,
Sydney, Australia.*
- 1902 ¹ Farrow, Alfred Ellis,
Flaxton-rectory, York.
- 1901 ¹ Fielder, William Crowhurst,
*Atlas Assurance Company, Ltd.,
92 Cheapside, E.C.*
- 1904 ¹ Fippard, Richard Clift,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1901 ¹ Fisher, John William, B.A.,
*Crown Life Insurance Co.,
Toronto, Canada.*
- 1896 ¹ Fisk, George William Victor, F.S.S.,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1904 ¹ Fletcher, Andrew W. A. C.,
*Standard Life Assurance Co.,
3 George-street, Edinburgh.*
- 1905 ¹ Flynn, Benedict Devine,
*Travellers' Insurance Company,
Hartford, Conn., U.S.A.*
- 1904 ¹ Foot, Alfred Helsdon,
*Law Accident Insurance Society,
Limited, 215 Strand, W.C.*
- 1905 ¹ Forbes, James,
*Great-West Life Assurance Co.,
Winnipeg, Manitoba, Canada.*
- 1901 ¹ Franklin, Herbert Dare,
*Australian Mutual Provident
Society, Melbourne, Australia.*

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Date of becoming a Student.		Date of becoming a Student.	
1903	¹ Fulford, William John, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1897	² Gosset, Thorold, <i>16 Durham-road, Wimbledon,</i> <i>S.W.</i>
1890	Gamman, Robert Ebenezer, <i>London Joint Stock Bank,</i> <i>Princes-street, E.C.</i>	1886	Gover, Frederick Field, F.S.S., <i>10 Lee-park, Blackheath, S.E.</i>
1886	Garcke, Emile, F.S.S., M.I.E.E., <i>Ditton-house, near Maidenhead.</i>	1886	Greening, Herbert Joseph, <i>Abstainers' & General Insur. Co.,</i> <i>City-buildings, Birmingham.</i>
1900	¹ Garner, James, <i>9 Arlington-gardens, Chiswick, W.</i>	1901	¹ Hall, Arthur F., <i>North American Life Assurance</i> <i>Co., North American Life Build-</i> <i>ing, 112-118 King-street-west,</i> <i>Toronto, Canada.</i>
1901	⁽¹⁾ Gerrish, Frank Wilfred, B.A., <i>Minerva-villa, Albert-rd.-south,</i> <i>Buckhurst-hill, Essex.</i>	1902	² Hallett, William Sebastian, B.A., <i>Equitable Life Assurance Soc.,</i> <i>Mansion-house-street, E.C.</i>
1899	¹ Giles, Hylton Lloyd, <i>Pelican & British Empire Life</i> <i>Office, 70 Lombard-street, E.C.</i>	1901	¹ Hamilton, George Powell, <i>North American Life Assurance</i> <i>Co., McLean Block, 6 Douglas-</i> <i>street, Guelph, Ontario, Canada.</i>
1895	¹ Gill, James Stewart, <i>Australian Widows' Fund Life</i> <i>Assur. Soc., Sydney, Australia.</i>	1905	¹ Hamley, Ernest Fountain, <i>Australasian Temperance and</i> <i>General Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>
1901	¹ Glassford, David Murray, <i>Mutual Life Association of Aus-</i> <i>traliasia, Sydney, Australia.</i>	1902	¹ Hammant, Francis Clive, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1893	Glasson, George Cornish, <i>Economic Life Assurance Soc.,</i> <i>4 St. Stephen's-chbrs., Baldwin-</i> <i>street, Bristol.</i>	1905	¹ Hammond, Harry Pierson, B.A., <i>A.A.S.,</i> <i>Mutual Life Insurance Co. of</i> <i>New York, New York, U.S.A.</i>
1902	¹ Gleave, Charles Sheldon, <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>	1892	Hancock, Arthur Tom, <i>Clerical, Medical & General Life</i> <i>Assurance Society, 15 St. James's-</i> <i>square, S.W.</i>
1893	¹ Gledstone, W. L., <i>Royal Exchange Assur. Corpora-</i> <i>tion, Royal Exchange, E.C.</i>	1903	² Hancock, Edwin J., <i>72 Tredegar-road, Bow, E.</i>
1902	¹ Godsill, Richard Collis, <i>Liverpool Victoria Legal</i> <i>Friendly Soc., 18 St. Andrew-</i> <i>street, E.C.</i>	1902	¹ Hardy, Reginald Herbert, <i>32 Highfield-street, Leicester.</i>
1894	¹ Golding, Arthur, <i>40 Allerton-road, Stoke New-</i> <i>ington, N.</i>	1903	¹ Harley, Brian, <i>Guardian Assurance Co., 11</i> <i>Lombard-street, E.C.</i>
1905	¹ Goodall, Ernest Victor, <i>51 Ardgowan-road, Hither-</i> <i>green, S.E.</i>	1905	¹ Harnack, Frederick William, <i>Sceptre Life Association, Ltd.,</i> <i>40 Finsbury-pavement, E.C.</i>
1888	¹ Gooding, Harold John, <i>Law Guarantee and Trust Soc.,</i> <i>Ltd., 56 Moorgate-street, E.C.</i>	1901	¹ Harper, Henry, <i>103 Waverley-road, Small Heath,</i> <i>Birmingham.</i>
1903	¹ Gopp, John Ive, <i>14 Church-hill-road, Waltham-</i> <i>stow, E.</i>	1905	¹ Harrington, Eustace Woods, <i>Northern Assurance Company,</i> <i>1 Moorgate-street, E.C.</i>
1902	¹ Gordon, Walter Hamilton, <i>45 Braydon-road, Stamford-</i> <i>hill, N.</i>		

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1903	¹ Harris, Ernest Arthur, 40 Lambert-rd., Brixton-hill, s.w.	1902	¹ Hugill, Herbert, " Briarfield," Keighley.
1889	¹ Harris, Henry, Friends' Provident Institution, 17 Gracechurch-street, E.C.	1904	¹ Humphreys, Harry Lewis, Pelican and British Empire Life Office, 70 Lombard-street, E.C.
1905	¹ Harrison, Launcelot, Citizens' Life Assurance Co., Sydney, Australia.	1902	¹ Humphreys, John A., National Mutual Life Assurance Society, 39 King-street, Cheap- side, E.C.
1896	Haskins, George Frederick, A.C.A., 18 Walbrook, E.C.	1902	¹ Humphry, Edmund William, Life Association of Scotland, 5 Lombard-street, E.C.
1894	¹ Hatten, David Leslie, Standard Life Assurance Co., 83 King William-street, E.C.	1891	Hunt, Arthur Leonard, Wesleyan and General Assur. Soc., 101 Finsbury-pavement, E.C.
1905	(1) Heron, David, M.A., 21 Upper Bedford-place, Russell- square, W.C.	1902	(1) Jackson, Charles William, M.A., c/o M. M. Dawson, Esq., 11 Broadway, New York, U.S.A.
1905	¹ Hill, Charles Dorell, Queen's-villa, High-road, Chis- wick, W.	1902	² Jackson, Herbert Moore, Australian Mutual Provident Society, Sydney, Australia.
1903	¹ Hill, Frank Wilson, Norwich Union Life Insurance Society, Norwich.	1890	² Jackson, Samuel, F.F.A., Scottish Widows' Fund and Life Assurance Society, Liverpool.
1896	² Hines, Walter Robert, Norwich Union Life Insurance Society, Norwich.	1905	¹ Jennings, Reginald Webb, Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.
1902	¹ Hodge, Cecil Wilfred, Star Life Assurance Society, 32 Moorgate-street, E.C.	1896	¹ Jepps, John Blacklee, English and Scottish Law Life Assurance Assoc., 12 Waterloo- place, s.w.
1896	¹ Hogg, Charles, 10 Whitehall-place, s.w.	1905	¹ Johns, Arthur Humphreys, Colonial Mutual Life Assurance Society, Melbourne, Australia.
1905	¹ Homan, Russell Charles, 3 The Terrace, Camden-sq., N.W.	1904	¹ Johnson, Frank Henry, Law Life Assurance Society, 187 Fleet-street, E.C.
1898	² Hooper, George Duncan, Prudential Assurance Company, Holborn-bars, E.C.	1898	¹ Johnston, Arthur Edward, 3 Cumnor-road, Sutton.
1895	² Horn, Ernest Frederick, Equity & Law Life Assur. Soc., 18 Lincoln's Inn Fields, W.C.	1902	¹ Jones, Edward Furnival, A.S.A.A., Hearts of Oak Benefit Society, 17 Charlotte-street, W.
1902	¹ Houston, Charles Cornelius, Metropolitan Asylums Board, Victoria-embankment, E.C.	1903	¹ Jones, Ernest Stephens, National Debt Office, E.C.
1901	¹ Howell, Archibald Rennie, B.A., Royal Insurance Co. of England, Montreal, Canada.	1896	¹ Jones, Richard Foxley, Refuge Assurance Co., Orford- street, Manchester.
1898	Hughes, Arthur J., China Mutual Life Insur. Co., Shanghai, China.		
1902	¹ Hughes, Charles, A.A.S., Insurance Department of the State of Connecticut, Hartford, Conn., U.S.A.		

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1905	¹ Keevil, Norman Alexander Clement, Blagdon, Park-road, Watford, Herts.	1904	¹ Lee, Frank Sidney, Ocean Accident and Guarantee Corporation, 36-44 Moorgate- street, E.C.
1893	³ Kellham, Cyril Stephen, Prudential Assurance Company, Holborn-bars, E.C.	1904	¹ Lee, Frederick, Ecclesiastical Insurance Office, 11 Norfolk-street, Strand, W.C.
1902	¹ Kemper, J. M. de Bosch, La Mutuelle Hollandaise Com- pagnie d'Assurance, 21 Avenuede l'Opera, Paris.	1894	Leonard, Maurice, 14 Sotheby-road, Highbury, N.
1905	¹ Kenchington, Frank, 6 Hamilton-road, Highbury, N.	1904	¹ Levey, Ralph, Prudential Assurance Company, Holborn-bars, E.C.
1905	¹ Kidd, Hubert D., Mutual Life Insur. Co. of New York, 16, 17 & 18 Cornhill, E.C.	1904	¹ Lewty, Francis Arthur, Equity and Law Life Ass. Soc., 18 Lincoln's-inn-fields, W.C.
1905	¹ King, Albert Edward, Provident Clerks' and General Mutual Life Assur. Association, 27 & 29 Moorgate-street, E.C.	1896	¹ Ley, James, Office of the Actuary for Friendly Societies, Melbourne, Australia.
1894	² Kingsbury, James William, Australian Mutual Provident Society, Sydney, Australia.	1889	¹ Lighton, Harold John, Law Union & Crown Insurance Co., 126 Chancery-lane, W.C.
1900	¹ Kirkham, Alfred, c/o Messrs. Thos. Mitchell & Co., Lonsdale-street, Melbourne, Australia.	1904	¹ Linzmeyer, Louis, F.A.S., Manhattan Life Insurance Co., 64-70 Broadway, New York, U.S.A.
1903	¹ Kirsopp, Frederick, Liverpool Victoria Legal Friendly Society, 18 St. Andrew-street, E.C.	1895	¹ Littell, Lewis Lloyd, Standard Life Assurance Co., 83 King William-street, E.C.
1895	¹ Knight, Alfred Murray, Bank-house, Chapel-st., Devon- port.	1904	¹ Littlefair, James Taylor, Refuge Assurance Co., Oxford- street, Manchester.
1905	¹ Lafford, Harry George, Legal and General Life Assur. Society, 10 Fleet-street, E.C.	1890	Love, Robert, Ecclesiastical Insurance Office, 11 Norfolk-street, Strand, W.C.
1905	¹ Laing, John Morrison, Mutual Life Assurance Co. of Canada, Waterloo, Ontario, Canada.	1888	¹ McConway, James Robert, Royal Insurance Company, Liver- pool.
1902	¹ Lang, Frederick John, Royal London Friendly Society, Finsbury-square, E.C.	1903	¹ McDonald, Charles Joseph Angus, Australian Mutual Provident Society, Wellington, New Zealand.
1905	¹ Langstaff, Milton Palmer, Imperial Life Assurance Co. of Canada, Toronto, Canada.	1903	¹ Macdonald, Charles Strange, M.A., Confederation Life Association, Toronto, Canada.
1904	¹ Latham, Percy James, Inland Revenue Department, Falkirk, N.B.	1904	¹ Macfarlane, Edmond Scales, Manufacturers' Life Insurance Company, 23 Water Street, Yokohama, Japan.
		1902	¹ McGee, Cyril H. Box 981, St. Thomas, Ontario, Canada.

STUDENTS.

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Those marked (1) have been exempted under the Bye-laws from the Examination in Part I.

Date of becoming a Student.		Date of becoming a Student.	
1905	¹ McKechnie, John Henry, <i>Waterford High School, Waterford, Ontario, Canada.</i>	1903	¹ Moore, Roderick John, <i>102, Caithness-road, Mitcham, Surrey.</i>
1902	¹ McKellar, John A., <i>Equitable Life Assur. Society of the United States, 120 Broadway, New York, U.S.A.</i>	1898	¹ Moore, Stanley, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	² Maltby, Charles Hugh, <i>Scottish Widows' Fund and Life Assurance Society, 28 Cornhill, E.C.</i>	1904	¹ Moran, Albert James, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1903	¹ Manly, George William, B.A., <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>	1902	¹ Morton, Francis, <i>Commercial Union Assurance 24, 25 & 26 Cornhill, E.C.</i>
1904	¹ Marlin, James Harold, <i>Ocean Accident and Guarantee Corporation, 36-44 Moorgate-street, E.C.</i>	1902	¹ Muckle, Charles Park, <i>Union Life Assurance Co., Toronto, Canada.</i>
1905	¹ Marshall, Arthur William, <i>c/o Messrs. A. Marshall & Co., 93 Queen Victoria-street, E.C.</i>	1904	¹ Muleahy, Francis Benedict, <i>Citizens' Life Assurance Co., Sydney, Australia.</i>
1905	¹ Marshall, John Edwin, <i>Prudential Assurance Company, 13 Waterhouse-street, Halifax.</i>	1902	¹ Mullin, Alexander, B.A., <i>76 Major-st., Toronto, Canada.</i>
1903	¹ Martin, Frederick Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	¹ Myers, Harry Duxbury, A.S.A.A., <i>64 Devonshire-street, Keighley.</i>
1904	¹ Matheson, Donald, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>	1896	¹ Neale, Maurice Baldwin, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>
1895	¹ Mayhew, Percy Craske, <i>4 Princess-road, Selhurst, S.E.</i>	1903	¹ Neill, William Adam Hoyes, <i>Scottish Widows' Fund & Life Assur. Soc., 28 Cornhill, E.C.</i>
1890	¹ Meikle, Henry George Watson, F.F.A., <i>Oriental Government Security Life Assurance Co., Limited, Bombay, India.</i>	1905	⁽¹⁾ Nicholl, Charles Carlyon, B.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>
1901	² Melville, Henry Edward, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>	1904	¹ Norris, Isaac Taylor, <i>Collegiate Institute, Ottawa, Canada.</i>
1892	¹ Meyers, Henry Wilson, <i>National Mutual Life Association of Australasia, 5 Cheapside, E.C.</i>	1903	¹ Northeott, John Arthur, <i>St. Andrew's College, Toronto, Canada.</i>
1899	² Minus, Ernest Edwin, <i>Norwich Union Life Insurance Society, Norwich.</i>	1901	¹ Nugent, James, <i>Cornwall, Ontario, Canada.</i>
1902	¹ Moore, Hubert Fred, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1903	¹ Oates, Percy Tuckfield, <i>30 High-street, Wimbledon, S.W.</i>
		1902	⁽¹⁾ O'Connor, William, M.A., M.D., <i>Mutual Life Insurance Co. of New York, Toronto, Canada.</i>
		1892	¹ O'Reilly, Anthony James, <i>Government Insurance Department, Ottawa, Canada.</i>
		1897	¹ Osborn, Nathaniel Banner Francis, <i>11 Bruce-grove, Tottenham, N.</i>

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Date of
becoming
a Student.

- 1905 ¹ Osborne, William Arthur,
*Ocean Accident & Guar. Corp.,
Ltd., 36-44 Moorgate-street, E.C.*
- 1905 ⁽¹⁾ Owen, David John, B.A.,
62 Union-road, Rotherhithe, S.E.
- 1893 ¹ Owen, Edgar Theodore, F.S.S.,
*Under Secretary for Labour,
Registrar of Friendly Societies
and Government Actuary, Perth,
West Australia.*
- 1901 ¹ Papworth, Frederick William,
*A.S.A.A.,
St. David's, Longley-road,
Tooting, S.W.*
- 1904 ¹ Parker, John G.,
*University of Toronto, Toronto,
Canada.*
- 1904 ¹ Parker, Walter Montgomery,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1895 ¹ Pascoe, William Yeoman Bennett,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1897 ¹ Paton, Harry Arthur,
*Royal Exchange Assurance Cor-
poration, Royal Exchange, E.C.*
- 1905 ¹ Patrick, James,
*Audit Office, Town-hall, Birken-
head. (Reinstated.)*
- 1896 ² Penny, Charles Augustus,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1905 ¹ Perry, Sidney James,
*Northern Assurance Company,
1 Moorgate-street, E.C.*
- 1901 ¹ Petter, Herbert,
*Britannic Assurance Co., Ltd.,
Broad-st.-corner, Birmingham.*
- 1905 ¹ Phillips, Thomas Ashley,
*117 W. 16th-street, New York,
U.S.A.*
- 1904 ¹ Phillips, Walter, A.C.I.S., A.S.A.A.,
*8 Riverdale-terrace, Richmond,
Surrey.*
- 1905 ¹ Pickup, John Richardson,
*National Provident Institution,
48 Gracechurch-street, E.C.*
- 1905 ¹ Pollard, Edward Cecil,
The Castle, Rochester.

Date of
becoming
a Student.

- 1898 Poort, Willem Anthonie, Phil. Nat.
Doct.,
*Algemeene Friesche Levens-
verzekerings Maatschappij Leeu-
warden, Leeuwarden, Holland.*
- 1904 ¹ Portch, Albert Garfield,
*Canada Life Assurance Company,
Toronto, Canada.*
- 1903 ⁽¹⁾ Porter, Frank, M.A.,
Mansfield House, Canning Town, E.
- 1893 ¹ Pownall, Herbert Wilfred,
*Australian Mutual Provident
Society, Adelaide, Australia.*
- 1901 ¹ Ramsay, Cecil Byron,
*Mutual Life Insur. Co. of New
York, 16, 17 & 18 Cornhill, E.C.*
- 1905 ¹ Reeve, Gilfrid Montier,
*Newlands, Prospect-hill, Wal-
thamstow, E.*
- 1898 ¹ Reynell, Guy Courtenay,
*National Mutual Life Assurance
Society, 39 King-st., Cheapside,
E.C.*
- 1904 ¹ Reyner, Harry Fane,
*Refuge Assurance Company,
Oxford-street, Manchester.*
- 1903 ¹ Reynolds, William Daniel,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1904 ¹ Rice, George Ritchie,
*58 Croxted-road, West Dulwich,
S.E.*
- 1894 ¹ Richards, Gilbert P. A.,
*Oak Cottage, Bulwer-road, New
Barnet.*
- 1904 ¹ Ridgway, Wulfric,
*Sun Life Assurance Society,
63 Threadneedle-street, E.C.*
- 1902 ¹ Robertson, Aubrey Charles,
*London Assurance Corporation,
7 Royal Exchange, E.C.*
- 1903 ¹ Robertson, Bernard,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1903 ¹ Robinson, Ernest William,
*Standard Life Association, Ltd.,
28 Elizabeth-street, Sydney,
Australia.*
- 1896 ¹ Robinson, Frederick Charles,
*Royal Exchange Assur. Corpora-
tion, Royal Exchange, E.C.*

STUDENTS.

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Date of becoming a Student.		Date of becoming a Student.	
1893	¹ Roll, Frederick James, <i>Pearl Life Assurance Company, London-bridge, E.C.</i>	1904	¹ Sinclair, William Alexander, <i>Canada Life Assurance Company, Toronto, Canada.</i>
1893	¹ Roodenburch, Bartholomeus Adrianus, <i>Verzekeeringsbank Victoria, 126 Keizersgracht, Amsterdam.</i>	1891	¹ Sindall, Alfred John, <i>London and Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>
1895	¹ Ross, Christopher Watson, <i>c/o Messrs. M. Moss & Co., Flinders-lane, Melbourne, Aus- tralia.</i>	1888	² Slinon, William James, F.F.A., <i>10 Mayfield-terrace, Edinburgh.</i>
1901	¹ Rountree, Arthur FitzGerald, <i>The Rectory, Stretford, near Manchester.</i>	1905	¹ Sloan, Joseph James Eastwood, <i>Corporation Audit Department, Liverpool.</i>
1905	¹ Rowland, Stanley Jackson, <i>Clerical, Medical and General Life Assurance Society, 15 St. James's-square, s.w.</i>	1902	¹ Smith, Septimus Wontner, <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>
1895	Rowley, James Edward, A.C.A., <i>7 Waterloo-street, Birmingham.</i>	1903	¹ Smith, William, <i>Standard Life Association, Ltd., 28 Elizabeth-street, Sydney, Australia.</i>
1899	¹ Rutter, Edward Valentine, <i>Pelican & British Empire Life Office, 70 Lombard-street, E.C.</i>	1902	¹ Smither, Herbert Buxton, <i>University Life Assurance Soc., 25 Pall-mall, s.w.</i>
1904	¹ Sadler, Percy, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	² Sneddon, Andrew William, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1894	Salter, George Ferry, F.A.S., <i>123 N. 16th-street, E. Orange, N.J., U.S.A.</i>	1904	¹ Spring, Stanley Harold, <i>London Guarantee and Accident Company, 61 Moorgate-st., E.C.</i>
1905	¹ Schooling, Terence Holt, <i>40B Lavender Sweep, s.w.</i>	1904	¹ Sprules, Alfred M., <i>"Trelawne," Sutton, Surrey.</i>
1897	¹ Scott, Alexander Lewis, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>	1903	¹ Stanford, Harold William, <i>43 Lower Stockbridge-road, Winchester.</i>
1888	Sewell, Richard, F.F.A., C.A., <i>British Widows' Assurance Co., 1 Old-street, E.C.</i>	1901	¹ Steffensen, Johan F., <i>Forsikringsraadet, 1 Christians- gade, Copenhagen.</i>
1905	¹ Sharp, Harold Gregory, <i>Friends' Provident Institution, 17 Gracechurch-street, E.C.</i>	1898	³ Stewart, Lionel William, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>
1902	¹ Shrubsole, Stanley Smith, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1886	² Stirling, James, F.F.A., <i>Law Union and Crown Insur. Co., 126 Chancery-lane, w.c.</i>
1892	¹ Simpson, William Murray, <i>North British and Mercantile Insurance Company, 61 Thread- needle-street, E.C.</i>	1905	¹ Stone, Mark, <i>Merton College, Oxford.</i>
1905	¹ Sinclair, Coll Claude, <i>Great-West Life Assurance Co., Vancouver, B.C., Canada.</i>	1903	² Story, Cyril Lionel William Steane, <i>Norwich Union Life Insurance Society, 71 & 72 King William- street, E.C.</i>
		1888	¹ Stott, Walter, <i>Royal Insurance Co., Liverpool.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1893	¹ Streeter, Theodore Edward, <i>P.O. Box 1415, Winnipeg, Manitoba, Canada.</i>	1897	¹ Townshend, Edward Villiers, <i>Scottish Widows' Fund and Life Assurance Society, 28 Baldwin- street, Bristol.</i>
1904	¹ Strong, Gordon Gilbert, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1901	¹ Tregaskis, George, <i>Sun Insur. Office, 40 Chancery- lane, W.C.</i>
1902	¹ Strong, William Boughton, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1905	¹ Tutill, Hubert Linzee, <i>English & Scottish Law Life Assur. Association, 12 Waterloo- place, S.W.</i>
1904	¹ Stuart, Arthur William, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1891	Tyler, Edgar Alfred, F.S.S., <i>9 Old Jewry-chambers, Bank, E.C.</i>
1905	¹ Stuart, C. J. S., <i>Canada Life Assurance Co., Toronto, Canada.</i>	1904	¹ Underwood, Reginald, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1904	¹ Sturt, Arthur James, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>	1905	¹ Walker, Dwight A., <i>Equitable Life Assurance Soc. of the United States, 120 Broad- way, New York, U.S.A.</i>
1903	¹ Suddaby, William A., <i>Liverpool Victoria Legal Friendly Society, 18 St. Andrew- street, E.C.</i>	1904	¹ Warnock-Fielden, Francis Hugh, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1904	¹ Tamkin, Walter Ellis, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1904	¹ Warren, Cyril Ferdinand, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905	¹ Thompson, John Henry Reginald, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	¹ Watson, Alexander R. D., <i>Devonport, Auckland, New Zealand.</i>
1905	¹ Thompson, John Spencer, <i>Mutual Life Insurance Co. of New York, New York, U.S.A.</i>	1900	¹ Watt, Arthur W., <i>Sun Life Assur. Co. of Canada, Montreal, Canada.</i>
1905	¹ Thompson, Joseph William, <i>Norwich Union Life Insurance Society, Norwich.</i>	1898	¹ Webb, Lloyd, <i>Commercial Union Assur. Co., 26 New-bridge-street, E.C.</i>
1904	¹ Thompson, William George, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>	1905	¹ Wellington, Frank, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1900	¹ Thomson, Frederick Robert T., <i>Kent-house, Church-end, Finch- ley, S.</i>	1902	¹ Wellisch, Frederick, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1902	¹ Thwaites, Frederick George, <i>Norwich Union Life Insurance Society, Norwich.</i>	1893	¹ Welman, Arthur Joseph, <i>Legal & General Life Assurance Soc., 15 Tithebarn-st., Liverpool.</i>
1897	¹ Tipping, Oswald, <i>Trustees, Executors and Agency Co., Limited, 412 Collins-street, Melbourne, Australia.</i>	1905	¹ Welsh, Willis, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1902	¹ Tope, Maurice William, <i>National Mutual Life Assurance Society, 39 King-street, Cheap- side, E.C.</i>		

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1904	¹ Wenn, Albert Edward <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	² Wolfenden, Edgar Sydney, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1905	¹ White, Osborn Denyer, <i>Commercial Union Assurance Co., 24, 25 & 26, Cornhill, E.C.</i>	1895	¹ Wood, David James, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1905	¹ White, Robert Campbell <i>Shearburn, 250 King's-road, Kingston-hill, Surrey.</i>	1901	¹ Wood, Roland Stuart, <i>Liverpool & London & Globe Insurance Co., 1 Cornhill, E.C.</i>
1902	¹ White, Wilfred Clare, <i>Manufacturers' Life Insurance Co., Toronto, Canada.</i>	1902	¹ Woodhouse, David Alfred, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1886	¹ Williams, David, <i>181 Queen Victoria-street, E.C.</i>	1905	¹ Woodward, James Howard, <i>137 Gillott-road, Birmingham.</i>
1905	¹ Williams, Hugh Corden, <i>Australian Mutual Provident Society, Hobart, Tasmania, Australia.</i>	1900	¹ Woolston, Paul Livingston, B.S., <i>50 Maine avenue, Ocean-grove New Jersey, U.S.A.</i>
1895	¹ Williams, Henry Samuel Walter, <i>Liverpool & London & Globe Insurance Company, Melbourne, Australia.</i>	1894	¹ Wyatt, George Matthew, <i>Central Insurance Company, 12 & 13 Nicholas-lane, E.C.</i>
1900	(1) Williams, Lewis, B.A., <i>Commercial Union Assur. Co., 26 New Bridge-street, E.C.</i>	1886	Yeatman, Alexander Alfred, <i>2 Coleman-street, E.C.</i>
1901	² Wilton, Herbert George, <i>Norwich Union Life Insurance Society, Norwich.</i>	1895	¹ Yeldham, William James, <i>Prudential Assurance Company Holborn-bars, E.C.</i>
1894	¹ Windett, Sydney V., <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>	1903	¹ Young, Henry J., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1905	¹ Wisdom, Sidney Herbert, <i>Estate Duty Office, Somersct-house, W.C.</i>	1897	¹ Younger, R. Hugh, <i>Liverpool & London & Globe Insurance Co., 1 Dale-street, Liverpool.</i>
		1904	¹ Zumstein, Herbert Christian, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>

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Astronomical Observatory.

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M. Albert Quiquet, F.A.S.,
Actuaire, La Nationale Compagnie d'Assurances sur la vie. Membre Correspondant de l'Association des Actuaires Belges. Secrétaire de l'Institut des Actuaires Français; 17 Rue Laffitte (IX^e).

M. Alfred Thomereau,
8 Rue le Peletier.

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GOTHA.

Dr. Johannes Karup,
Actuary of the Gotha Life Assurance Company.

M. Karl Samwer, Dr. Jur.,
Manager of the Gotha Life Assurance Company, Ohrdrufferstrasse, 4.

Hungary.

BUDAPEST.

M. Julius Altenburger,
Consulting Actuary. Corresponding Member of the "Institut des Actuaires Français," and of the "Association des Actuaires Belges." Member of the "Versicherungs-Wissenschaftliche Vereinigung of Vienna"; vii. Hajtsár ut, 20.

Italy.

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M. Guido Toja,
Actuary of "La Fondiaria" Life Assurance Company.

Russia.

ST. PETERSBURG.

M. Serge de Savitch,
Professor of Mathematics in the Electro-Technical Institute and in the University of St. Petersburg. Manager of the Tariffs' Service of the United Russian Insurance Companies; Nikolaevskaia, 35.

Spain.

MADRID.

Dr. José Maluquer y Salvador,
Insurance Counsellor (Ex-Actuary at the Home Office). Member of the Institute of Social Reforms; calle de Campomanes, 10.

Sweden.

STOCKHOLM.

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Switzerland.

ZURICH.

Herr Dr. Gottfried Schaertlin,
Manager of the "Schweizerischen Lebensversicherungs-und Rentenanstalt."

United States.

NEW YORK.

Mr. David Parks Fackler, F.A.S.,
Ex-President of the Actuarial Society of America (1891-93). Consulting Actuary; 35 Nassau-street.

* * * It is requested that any inaccuracy in the foregoing list may be pointed out to the ASSISTANT SECRETARY.

RULES

FOR THE

REGULATION OF THE LIBRARY.

1. The Library is open daily, from Ten to Five, from 1st of May to 30th September, and from Ten to Six from 1st of October to 30th April, except on Saturdays, when it is open from Ten to Two.

2. Members of the Institute are permitted to take out Two Books on making application in person, or by letter addressed to the Assistant Secretary; but no Member may keep any work longer than a Fortnight. If a Book be retained beyond that period, the borrower shall pay a fine of One Shilling per volume for each week, or part of a week, during which it is so retained, and shall not be permitted to obtain another from the Library until the missing book has been returned and the fine paid. When a Book is returned by a Member, it can be borrowed by him again, provided it has not been bespoken in the meantime by another Member.

3. Scientific Journals and Periodicals are not circulated until the volumes are completed and bound.

4. Cyclopædias and works of reference and certain other volumes are not circulated.

5. For the convenience of Members, a card Index to the contents of the Library, including all additions, is available for reference; also a card Index of the contents of the *Journal* of the Institute, from the commencement, up to the last published number.

6. Any Member damaging, or losing, a work must pay an amount to be fixed by the Council as the equivalent of the damage or loss sustained.

7. Works taken from the shelves for reference are not to be replaced, but must be laid on the Library table.

8. A list of defaulters shall be submitted monthly at each meeting of the Council.

By Order of the Council.

December, 1905.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables. By GEORGE KING, F.I.A., F.F.A. :
one of the Vice-Presidents of the Institute of Actuaries.

[Read before the Institute, 27 November 1905.]

1. ON 29 December 1902 I had the honour of reading a paper before the Institute on the "Comparative Reserves of Life Assurance Companies according to various Tables of Mortality at various Rates of Interest." That paper was intended to complete up to date the investigations which had been made in 1876 and 1877 on the same subject, by bringing in the New British Offices Experience Tables, O^M and $O^{M(5)}$. When that paper was read, the Select Tables, based on the British Offices data, were not available, and in the comparisons then instituted they could not be included. The case is now different, and a valuation has been made of the Model Office, on the basis of the $O^{[M]}$ Select Tables, at 3 per-cent interest, and in Table I of the Appendix the results are given, and compared with the results of the valuation by the O^M Aggregate Table, at the same rate of interest. The valuations by the O^M and $O^{M(5)}$ Tables in their various combinations are also repeated, so that reference to the previous paper may not be necessary. It will, however, be convenient to note that Table I in the Appendix is a continuation of Table 12 in the paper above

mentioned, (see *J.I.A.*, xxxvii, pp. 476 and 477), and also of Table V in the paper of March 1877, (*J.I.A.*, xxi, pp. 268—272).

2. In order to facilitate comparison between the Select reserves and the Aggregate reserves, from various points of view, in Tables II and III of the Appendix the reserves are shown of the Model Office for existing policies effected at each central age at entry; and also the liability of the Model Office, under policies existing in each quinquennium of insurance, which were effected at each central age at entry. These two tables are continuations of Tables 13 to 18, (*J.I.A.*, xxxvii, pp. 478 to 489), and Table 19, (*J.I.A.*, xxxvii, pp. 490 and 491) respectively.

3. It has not been deemed necessary on the present occasion to make a Select valuation of the Model Office at more than one rate of interest. The $O^{[M]}$ Select reserve, at 3 per-cent interest, is compared with the O^M Aggregate reserve, also at 3 per-cent; and it may safely be assumed that, if valuations were to be made at any other rates of interest, the Select reserves and the Aggregate reserves would vary in almost identical proportions. Therefore, for all practical purposes, the valuation by the Select Table at one rate of interest, will prove sufficient, and it is not necessary to cumber the *Journal* with more extended monetary tables.

4. An examination of Tables I to III of the Appendix will show that the estimate of liability, when the $O^{[M]}$ Select Table is employed, is greater for a company that has been established any considerable time, than by any table, or combination of tables, hitherto used by actuaries. This is an important fact, because it is generally believed, and I think justly, that a Select table supplies the most accurate measure attainable, both of premiums and of reserves. In saying this, I do not suggest that other bases of valuation, which result in a lower estimate of liability, are unsound; because the margins, both of premium and interest, are always so large, that any practical differences in net premium valuation estimates are not of great consequence. We naturally, however, wish to know as nearly as may be what is the truth, and the opinion is general that the fundamental truth is best brought out by Select tables.

5. It was shown in the paper in volume xxxvii of the *Journal*, that, while the O^M Table gives a higher estimate of liability, taken as a whole, with an average company, than the H^M Table, yet, at the older ages at entry, the reserves by the O^M Table are actually

smaller than those by the H^M . That appears from Tables 13 and 15, (*J.I.A.*, xxxvii, pp. 478 and 482), where, for central ages at entry 20 to 35 inclusive, in a company of fifty years' standing, the O^M reserves are the larger, while, for central ages at entry 40 to 65, they are the smaller; the excess, however, at the younger entry ages more than counterbalancing the defect at the older. Yet it might happen that, with a company which, for any special reason, secured an unusually large proportion of old entrants, the O^M Table might give an actually smaller total reserve than the H^M .

6. Looking now at Table II of the Appendix to this paper, in connection with Table 13 of the previous paper, it will be seen that, when the $O^{[M]}$ Select Mortality Table is used, the reserves are throughout greater for every age at entry, than those produced by an H^M valuation. This weighty consideration renders it all the more desirable to value on a Select basis where practicable.

7. I am not aware that a Select valuation has ever hitherto been applied in practice. The reasons are not far to seek. Although the Institute Experience was taken out in the first instance in Select form, only Aggregate tables were graduated, and monetary values were published originally on the Aggregate basis alone, the Select tables being viewed much as if they were merely a scientific curiosity. Moreover, the Select tables themselves were Select only in a sense. They did not accurately follow policy years, and their deviation, although probably not wide, was an uncertain quantity. In confirmation of this statement, we need only recall the controversy which was at one time carried on briskly as to year 0 of assurance. Also, in my first paper on the Comparative Reserves of Life Offices, (*J.I.A.*, xx, p. 233), it was shown that a valuation by the Combined H^M and $H^{M(5)}$ Tables gave results practically identical with a Select valuation; and, the Combined Tables being thought easier in use than the Select, there was no immediate object in pursuing the subject further.

8. In a letter addressed to the *Insurance Record* on 7 January 1881, I showed how a Select valuation could be easily and accurately made on the principles here illustrated, but the matter was then one rather of academic interest than of practical use, and it lay dormant. Now, however, we have Select tables, constructed strictly by policy years, on an extended basis which is beyond suspicion. These show that selection affects the mortality for more than the five years previously assumed, and, therefore, the Select mortality has

not been merged into the Aggregate until ten years have elapsed. This extension of the Select period renders, it is generally imagined, valuations on a Select basis more troublesome than when the Select period was thought to be limited to five years, and it also causes a combination of tables, such as that of the H^M and $H^{M(5)}$, to be less practicable. The importance, therefore, becomes greater of devising some plan by which a Select valuation can be made easily, and without involving too great clerical work.

9. If British offices could follow the American system of valuing the policies individually, so as to display only the reserves, without separating the values of the sums assured and of the future premiums, there would be no complication. Tables of policy-values are as easily prepared on the Select basis as on the Aggregate, and as easily used. But, in the United Kingdom, the Board of Trade requirements virtually necessitate the displaying of the policy-value in its component parts, the value of the sum assured, less the value of the premiums; therefore, a valuation of the individual policies becomes impracticable and we must resort to some method of grouping.

10. For a valuation to be acceptable in the United Kingdom, we must have,

- (a) The values of sums assured and of premiums, shown separately.
- (b) Due allowance for fractional durations of policies; or, what is the same thing, adjustment for the incidence of the premium income of the company.
- (c) The proper adjustment for half-yearly and quarterly cases.

11. In the method of valuation now submitted, we first find the total of the policy-values, which we afterwards dissect into the total value of the sums assured, and the total value of the future premiums.

12. We have

$${}_tV_x = 1 - \frac{1 + a_{x+t}}{1 + a_x}$$

$$(1 + a_{x+t}) = (1 + a_r)(1 - {}_tV_x)$$

$${}_tV_x = A_{x+t} - P_x(1 + a_{x+t})$$

$$= A_{x+t} - P_x(1 + a_x)(1 - {}_tV_x)$$

$$= A_{x+t} - A_x(1 - {}_tV_x)$$

$$A_{x+t} = {}_tV_x(1 - A_r) + A_x.$$

If there be any number of policies, no matter of what durations, all effected at age x , with total sums assured S , and if $\Sigma(SV)$ be the sum of their values, and $\Sigma(SA)$ the total value of the sums assured, and $\Sigma(Pa)$ the total value of the net premiums, then

$$\Sigma(SA) = (1 - A_x) \times \Sigma(SV) + S \times A_x$$

$$\Sigma(Pa) = \Sigma(SA) - \Sigma(SV).$$

This is the formula to be used. It was devised in 1878 for the transformation of Model Office valuations, and demonstrated and illustrated, (*J.I.A.*, xx, pp. 257-262.) It has since been incorporated in the *Text-Book*, Part II, chap. xviii, pp. 325-6.

13. To use the formula, we must have a prepared table of ${}_tV_{[x]}$, and also the values of $A_{[x]}$ and $P_{[x]}$ at date of assurance only.

14. We group the policies, first, by age at entry, preferably the nearest age, and, secondly, by nearest duration. For present purposes we assume the valuation to be made on 31 December, but the method, with obvious slight modifications, will equally suit any other date.

15. It is convenient to use cards, filled in with the necessary data; and the particulars from the cards, when completed, will be entered in suitable schedules. As an illustration, a complete valuation of a group of policies, effected at nearest age 30, is appended, and, in the following explanations, reference is made to the headings of the columns of the schedule.

Valuation in Groups of Whole-Life Policies by Select Tables.

O^[M] Interest 3 per-cent. Age at Entry, 30.

Nearest Duration <i>t</i>	Number of Policies	Office Yearly Premiums	Fractional Premium Correction	SUMS ASSURED			$V_{[r]}$	Value of Policies	Nearest Duration <i>t</i>
				α	β	Total			
				$t = c + 1$	$t = c$				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	11	£ 98 7 11	350	...	4,000	4,000	·00000	0	0
1	18	201 7 7	750	3,000	5,200	8,200	·01532	125	6
2	24	320 3 7	2,875	3,800	9,200	13,000	·02927	380	5
3	18	229 18 5	1,050	3,850	5,500	9,350	·04299	402	0
4	17	305 7 11	2,750	5,650	6,700	12,350	·05670	700	2
5	16	204 3 9	850	2,300	6,000	8,300	·07047	584	9
6	17	195 11 11	1,238	2,600	5,350	7,950	·08433	670	4
7	14	223 6 8	1,375	5,800	3,200	9,000	·09819	883	7
8	16	249 16 2	375	3,800	6,400	10,200	·11214	1,143	8
9	14	195 2 10	525	4,450	3,500	7,950	·12619	1,003	0
10	14	237 16 4	1,725	1,950	7,700	9,650	·14028	1,353	7
1	14	176 16 4	225	2,450	4,750	7,200	·15456	1,112	8
2	13	169 8 4	350	3,500	3,400	6,900	·16908	1,166	7
3	7	99 0 8	1,000	1,600	2,400	4,000	·18388	735	5
4	6	123 14 2	875	2,400	2,600	5,000	·19891	994	6
15	5	83 0 4	1,500	1,200	2,050	3,250	·21423	696	2
6	4	98 2 3	375	500	3,500	4,000	·22978	919	1
7	6	75 17 8	250	1,500	1,600	3,100	·24552	761	1
8	7	93 6 0	175	1,200	2,600	3,800	·26149	993	7
9	6	106 8 2	625	2,300	2,000	4,300	·27770	1,194	1
20	7	88 10 11	500	800	2,800	3,600	·29405	1,058	6
1	6	46 8 9	100	1,000	900	1,900	·31063	590	2
2	9	96 0 5	450	800	3,100	3,900	·32731	1,276	5
3	6	90 17 3	50	1,600	2,100	3,700	·34417	1,273	4
4	3	38 5 6	100	1,400	150	1,550	·36113	559	8
25	3	12 5 0	50	100	400	500	·37824	189	1
6	5	56 4 11	150	800	1,500	2,300	·39543	909	5
7	5	44 3 8	...	1,000	800	1,800	·41263	742	7
8	2	24 7 6	...	400	600	1,000	·42991	429	9
9	8	72 13 0	475	1,250	1,700	2,950	·44720	1,319	2
30	5	56 1 3	...	1,200	1,100	2,300	·46449	1,063	3
1	4	25 16 1	150	700	300	1,000	·48173	481	7
2	3	24 12 5	...	900	100	1,000	·49893	498	9
3	2	10 19 6	...	250	200	450	·51608	232	2
4	3	18 9 0	...	150	600	750	·53308	399	8
35	2	29 5 0	...	200	1,000	1,200	·54995	659	9
6	2	19 5 1	375	250	500	750	·56667	425	0
7	2	4 18 9	50	...	200	200	·58321	116	6
8	4	47 13 3	...	400	1,550	1,950	·59956	1,169	1
9	3	15 18 3	50	300	350	650	·61567	400	2
40	3	19 10 0	...	500	300	800	·63151	505	2
1	3	44 1 2	150	1,500	300	1,800	·64706	1,164	7
2	2	17 3 9	100	...	700	700	·66232	463	6
3	4	75 2 4	1,250	700	2,250	2,950	·67722	1,997	8
4	4	63 13 8	250	2,000	600	2,600	·69183	1,798	8
45	1	9 15 0	400	400	·70602	282	4
6	·71988
7	1	9 15 0	...	400	...	400	·73331	293	3
8	2	42 13 3	...	750	1,000	1,750	·74632	1,306	1
9	2	27 5 2	525	...	1,050	1,050	·75891	796	9
Totals	353	£4,618 11 10	24,013	73,200	114,200	187,400	...	38,231	0
									Totals

16. If c be the curtate, and t the nearest, duration, then, the valuation being on 31 December,

$$c = (\text{Valuation Year}) - (\text{Entry Year}),$$

and $t = (1 + c)$, for policies effected in the first half of the year, which we shall call α policies,

$t = c$, for policies effected in the second half of the year, which we shall call β policies.

17. We therefore group policies effected in the first half of any calendar year with those effected in the second half of the preceding year. To facilitate this process, we insert, for permanent use, in the cards, the (Valuation Entry Year), that is, the year of which 31 December is nearest to the day of entry; and against the (Valuation Entry Year) on the cards, we mark α or β , according as the policy was effected between 1 January and 30 June inclusive, or between 1 July and 31 December inclusive. The other usual particulars are written on the cards, but, for valuation by Select tables, the net premium is not required. We must, however, have the Fractional Premium Correction, explained in paragraph 20.

18. The cards are sorted into groups according to age at entry, and into sub-groups according to (Valuation Entry Year), and are then ready to be recorded in the schedule. Columns 2 and 3 of the schedule do not require explanation, except that it must be noted that in half-yearly cases double the half-yearly premium, and in quarterly cases four times the quarterly premium, is taken.

19. The sums assured under α policies are entered in column 5, and those under β policies in column 6, the total of the two categories being entered in column 7.

20. Fractional premiums arise in half-yearly and quarterly cases, because, where n is integral and κ fractional, the formula

$${}_{n+\kappa}V_x = {}_{n+0}V_x - \kappa({}_{n+0}V_x - {}_{n+1}V_x)$$

assumes that a full year's premium was paid at the beginning of the policy year, and makes a full reserve of premium for the unexpired term.

In half-yearly cases,

$$\frac{1}{2}P_x \left\{ \begin{array}{l} \text{will be outstanding under} \\ \text{policies which entered in the} \end{array} \right\} \text{second half of the year.}$$

and in quarterly cases,

$$\frac{1}{4}P_x \left\{ \begin{array}{l} \text{will be outstanding under} \\ \text{policies which entered in the} \end{array} \right\} \text{second quarter of the year.}$$

$$\begin{array}{lll} \frac{1}{2}P_x & \text{do.} & \text{third} \quad \text{,,} \quad \text{,,} \\ \frac{3}{4}P_x & \text{do.} & \text{fourth} \quad \text{,,} \quad \text{,,} \end{array}$$

For permanent use, we enter on the cards the proportion of the sum assured corresponding to these fractional corrections; and, in column 4 of the schedule, these fractional corrections find place.

21. The total sums assured in column 7 are multiplied by the policy-value, ${}_tV_{[x]}$, in column 8, and the result is entered in column 9. The sum of column 9 gives the total liability, $\Sigma(SV)$, but without adjustment for the unexpired terms of the policy years, and without adjustment for the outstanding premiums in half-yearly and quarterly cases. We have, in fact, multiplied $P_{[x]}$ by $(1 + a_{[x]+t})$ throughout.

22. Applying the formula of dissection of paragraph 12 to $\Sigma(SV)$, we find $\Sigma(SA)$, the value of the sums assured, and $\Sigma(Pa)$, the unadjusted value of the net premiums.

23. Then we multiply the sum of column 6 by $P_{[x]}$, and deduct the result from $\Sigma(Pa)$. This supplies the correction for the unexpired terms of the policy years, and makes a complete and accurate automatic adjustment for the incidence of the premium income of the company. We have, in fact, now multiplied by $(1 + a_{[x]+t})$ the premiums due in less than six months, and by $a_{[x]+t}$ those due in more than six months. This is in accordance with *Text-Book*, Part II, chap. xviii, art. 104, where endowment assurances are treated of. The method is very convenient, and is applicable to policies of every kind. It is probably more accurate than if we were to value the premiums by $\kappa + a_{x+t}$, where κ has to be determined by an examination into the incidence of the premium income. We could, however, follow the latter plan, and, in so doing, dispense with columns 5 and 6 (α and β). If, for instance, the premium income is so distributed that, on the average, the next renewal falls due four months hence, we have merely to deduct from the unadjusted value of the net premiums two-thirds of the total of the net premiums, instead of the whole of the net premiums on the β assurances. In the illustrative example the adjustment under these conditions would be 2230.1, instead of the more accurate adjustment, 2038.5, given in paragraph 26. Also we could, with a little more trouble, make a valuation by exact durations, if it were thought worth while to take that course.

24. To adjust for outstanding premiums in half-yearly and quarterly cases, we multiply the sum of column 4 by $P_{[x]}$, and add the result to the value of the net premiums.*

* There remains the small adjustment for such cases, which is referred to in the *Text-Book*, Part II, ch. xviii, §§ 87 to 91, but which is usually of very small importance.

25. To obtain the value of the office premiums, we take the proportion to the net premiums, after adjustment for fractional durations, but before correction for the outstanding premiums in half-yearly and quarterly cases; because the office premiums already include that adjustment, through their having been taken at the half-yearly and quarterly rates. Here I seem to differ from Mr. Lidstone (*J.I.A.*, xxxiv, p. 75), who applies the correction, I venture to think unnecessarily, to the office premiums.*

26. The following are the calculations for the illustrative example :

$$P_{[30]} = \cdot 01785. \quad A_{[30]} = \cdot 38002. \quad (1 - A_{[30]}) = \cdot 61998.$$

	Sums Assured	Net Premiums
Assurances α	£73,200	£1,306·620
Assurances β	114,200	2,038·470
Total assurances	£187,400	£3,345·090
Fractional correction	<u>£24,013</u>	<u>£428·632</u>
$(1 - A) \times$ policy-values		£23,702·5
$A \times$ total sums assured		71,215·7
Value of sums assured, $\Sigma(SA)$		£94,918·2
Policy-values, $\Sigma(SV)$		38,231·0
Value of premiums unadjusted, $\Sigma(Pa)$		£56,687·2
Premiums on β assurances		2,038·5
Value of net premiums adjusted for durations		<u>£54,648·7</u>
Fractional premiums		428·6
Value of net premiums, fully adjusted		<u>£55,077·3</u>
Value of sums assured		£94,918·2
Value of net premiums		55,077·3
Net liability		<u>£39,840·9</u>
Value of office premiums $= 54,648·7 \times \frac{4618·6}{3345·1} =$		<u>£75,453·8</u>

* See, however, Mr. Lidstone's explanation on p. 85.

27. For the sixth schedule of the Board of Trade Returns, the policies have to be re-grouped according to ages attained, but that does not necessitate a re-sorting of the cards. We have merely to summarize the valuation schedules, and this does not involve much labour. The policies effected at age x , and of t years' duration, have to be combined with those effected at $x+1$, with $t-1$ years' duration, with those effected at age $x+2$, with $t-2$ years' duration, and so on. That being so, it is well to include in the valuation schedules columns for reversionary bonuses, and for bonus reductions of premium. These benefits can then be valued by the ultimate table, according to ages attained, in the usual way. This course would produce a slight overstatement of liability for reversionary bonuses, and understatement for bonus reductions of premiums, in the few cases under ten years' duration that already have vested bonuses. The amount vested in such cases must always be very small.

28. The reversionary bonuses can, however, be valued, like the sums assured themselves, by the Select table. The total bonuses for the two categories α and β must be entered in one column, separate columns not being required. The valuation is effected just as in the case of the sums assured. The liability is dissected, and the sum assured liability taken. This is the net liability for the bonuses, because there are no premiums payable in respect of these.

29. It may also be noted, more, perhaps, as a matter of curiosity, than of practical use, that the bonus reductions of premium may likewise be valued by the formula of paragraph 12. They must be entered in α , β , and total, columns. The total column must be valued by ${}_tV_{[x]}$ and the result dissected. We now take the value of the net premiums and correct it for fractional durations. We thus have the value of $P_{[x]}$ per annum for each £1 of bonus reduction. If now we divide by $P_{[x]}$, we have the value of the bonus reductions themselves.

30. These methods of valuing reversionary bonuses and bonus reductions of premium, display the extraordinary power of the valuation formula of paragraph 12.

31. In the illustration the policies have been valued throughout their whole duration by the Select method, but that course is not necessary. Those only of less than ten years' duration need be scheduled according to ages at entry. The others may be scheduled in the usual way, according to ages attained, and valued by the ultimate table. The net premiums

for the latter would, however, have to be entered on the cards and scheduled.

32. A valuation of this kind is scarcely more troublesome than one by, for instance, the Combined H^M and $H^{M(5)}$ Tables, and it involves no approximations, but is exact according to nearest ages at entry and nearest durations. It is Mr. D. C. Fraser's method No. 12, (*J.I.A.*, xxxviii, p. 417), and he shows that it gives results practically identical with those of a valuation by exact ages at entry, and exact durations. According to him, there is a deviation of 1 in 1,000 in the case of policies of less than five years' standing; while, in the case of a company thirty years old or more, the deviation is only 1 in 10,000. That is, on a liability of £1,000,000, there is a deviation of £100.

33. The method suits companies, whatever may be their valuation epochs. One, valuing every three years, may keep the ages at entry separate for the business of the last three trienniums; one, valuing every five years, for that of two quinquenniums; and one, valuing every seven years, for that of two septenniums. There is, therefore, much greater elasticity in this respect than when the Combined H^M and $H^{M(5)}$ Tables are employed. Personally, however, I should value all the business by the Select method, as in the example.

34. The illustrative valuation is founded on fact, and I have to thank Mr. Whittall for kindly supplying the set of cards which has been used. Some of the particulars on the cards had, however, to be modified a little, in order adequately to bring out the various points which had to be dealt with.

35. I desire also to record my cordial thanks to Mr. George W. Richmond, F.I.A., who chivalrously volunteered to prepare the table of ${}_tV_{[30]}$ which was necessary, and to modify the cards, and make the valuation itself. The labour was considerable, and his knowledge and skill invaluable.

36. In conclusion, I would express the hope that in the subsidiary volume, now in course of preparation, of monetary tables, based on the British Offices Experience, a complete set of Select Policy-Values may find place. For many purposes it would be convenient if a page were devoted to one age at entry, all the rates of interest being kept together in columns side by side. Also, in the heading of each column, the values of $a_{[x]}$, $A_{[x]}$, and $P_{[x]}$ might be inserted.

APPENDIX.

TABLE I.

Showing the Reserves of the Model Office by the O^M Select Table, and making comparison with the Reserves by the O^M and the $O^{M(5)}$ Aggregate Tables, and the H^{MJ} Select Table. Interest 3 per-cent.

AGE OF OFFICE	5 YEARS		10 YEARS		15 YEARS		20 YEARS		25 YEARS	
	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000
O^{MJ} Select	28,430	11,410	87,756	10,932	167,361	10,633	259,725	10,454	357,582	10,314
O^M Aggregate	24,918	10,000	80,278	10,000	157,397	10,000	248,435	10,000	345,693	10,000
$O^{M(5)}$	23,954	9,613	77,648	9,672	152,977	9,719	242,350	9,755	338,188	9,783
O^M and $O^{M(5)}$ combined throughout	29,248	11,738	86,801	10,813	165,147	10,492	256,886	10,340	354,545	10,256
O^M and $O^{M(5)}$ combined after 5 years	24,918	10,000	82,471	10,273	160,817	10,217	252,556	10,166	350,215	10,131
H^{MJ} Select	28,978	11,629	87,408	10,888	165,989	10,546	256,716	10,333	352,803	10,206
AGE OF OFFICE =	30 YEARS		35 YEARS		40 YEARS		45 YEARS		50 YEARS	
	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000	Actual Reserve	Comparative Reserve O^M 3% =10,000
O^{MJ} Select	452,991	10,273	538,894	10,226	610,043	10,196	664,048	10,177	700,420	10,166
O^M Aggregate	440,973	10,000	526,969	10,000	598,306	10,000	652,496	10,000	688,999	10,000
$O^{M(5)}$	432,315	9,804	517,442	9,819	588,160	9,830	641,947	9,838	678,222	9,844
O^M and $O^{M(5)}$ combined throughout	450,034	10,206	536,131	10,174	607,504	10,154	661,704	10,141	698,214	10,134
O^M and $O^{M(5)}$ combined after 5 years	445,704	10,107	531,801	10,092	603,174	10,081	657,374	10,075	693,884	10,071
H^{MJ} Select	446,519	10,126	530,674	10,070	600,575	10,038	653,849	10,021	689,931	10,014

N.B.—This Table is supplementary to Table 12, *J.I.A.*, xxxvii, pp. 476 and 477, and see also Table V, *J.I.A.*, xx, pp. 268-272.

TABLE II.

Showing the Reserves of the Model Office for Existing Policies effected at each Central Age at Entry.

O^[M] Select Mortality Table. Interest 3 per-cent.

Central Age at Entry	AGE OF OFFICE					Central Age at Entry
	5 Years	10 Years	15 Years	20 Years	25 Years	
20	1,005	2,946	5,590	8,871	12,741	20
25	3,318	10,313	20,281	32,889	47,750	25
30	4,784	15,173	29,991	48,546	69,874	30
35	4,979	15,847	31,214	49,943	70,855	35
40	4,419	13,968	27,057	42,371	58,557	40
45	3,587	11,133	21,062	32,007	42,641	45
50	2,770	8,365	15,389	22,642	28,957	50
55	1,819	5,305	9,304	13,007	15,667	55
60	1,164	3,201	5,192	6,689	7,553	60
65	585	1,505	2,281	2,760	2,987	65
Total	28,430	87,756	167,361	259,725	357,582	Total
	30 Years	35 Years	40 Years	45 Years	50 Years	
20	17,142	22,047	27,343	32,772	37,776	20
25	64,414	81,806	98,966	114,873	128,059	25
30	92,652	115,134	135,580	152,577	164,141	30
35	92,246	111,963	128,015	138,462	143,513	35
40	73,883	86,787	95,455	99,647	101,024	40
45	51,411	57,159	59,912	60,821	61,009	45
50	33,254	35,410	36,059	36,181	36,183	50
55	17,084	17,623	17,747	17,749	17,749	55
60	7,868	7,927	7,928	7,928	7,928	60
65	3,037	3,038	3,038	3,038	3,038	65
Total	452,991	538,894	610,043	664,048	700,420	Total

N.B.—This Table is supplementary to Tables 13 to 18, *J.I.A.*, xxxvii, pp. 478 to 489.

TABLE III.

Showing the Liability of the Model Office under Policies existing in each Quinquennium of Insurance, and effected at each Central Age at Entry.

O^[M] Select Mortality Table. Interest 3 per-cent.

Central Age at Entry	QUINQUENNium OF INSURANCE					Central Age at Entry
	First	Second	Third	Fourth	Fifth	
20	1,005	1,941	2,644	3,281	3,870	20
25	3,318	6,995	9,968	12,608	14,861	25
30	4,784	10,389	14,818	18,555	21,328	30
35	4,979	10,868	15,367	18,729	20,912	35
40	4,419	9,549	13,089	15,314	16,186	40
45	3,587	7,546	9,929	10,945	10,634	45
50	2,770	5,595	7,024	7,253	6,315	50
55	1,819	3,486	3,999	3,703	2,660	55
60	1,164	2,037	1,991	1,497	864	60
65	585	920	776	479	227	65
Total	28,430	59,326	79,605	92,364	97,857	Total
	Sixth	Seventh	Eighth	Ninth	Tenth	
20	4,401	4,905	5,296	5,429	5,004	20
25	16,664	17,392	17,160	15,907	13,186	25
30	22,778	22,482	20,446	16,997	11,564	30
35	21,391	19,717	16,052	10,447	5,051	35
40	15,326	12,904	8,668	4,192	1,377	40
45	8,770	5,748	2,753	909	188	45
50	4,297	2,156	649	122	2	50
55	1,417	539	124	2	...	55
60	315	59	1	60
65	50	1	65
Total	95,409	85,903	71,149	54,005	36,372	Total

N.B.—This Table is supplementary to Table 19, *J.I.A.*, xxxvii, pp. 478 and 479.

On a Property of the O^[M] Select Tables, and its Application to the Valuation of Whole-Life Policies. By O. F. DIVER, M.A., F.I.A., of the Clerical, Medical and General Life Assurance Society.

[Read before the Institute, 27 November 1905.]

THE subject dealt with in this paper has already been introduced by a letter to the *Insurance Record*, of 24 February 1905, but I am now able to put forward certain further developments of the theory and method there set out.

The property referred to in the title is to be found in the O^[M] Select Annuity Tables, and holds for the three rates of interest that I have tested below. The function which displays this property may be described as measuring the effect of selection upon the annuity-value, being the difference between the values of two annuities, one on a select life after the lapse of any number of years, less than 10, since selection, and the other on a life of the same age, which has passed the selection limit of 10 years. It is convenient to give this function a distinctive* symbol and to write

$$\phi_{[x]+t} = a_{[x]+t} - a_{x+t}.$$

Specimen values at 3 per-cent are given in Table I, while others at 2½ per-cent will be found in Table V.

TABLE I.

Specimen Values of $\phi_{[x]+t}$ ($=a_{[x]+t} - a_{x+t}$).

O^[M] 3 PER-CENT.O^[M] 3 PER-CENT.

Age at Entry [x]	$\phi_{[x]}$	$\phi_{[x]+1}$	$\phi_{[x]+2}$	$\phi_{[x]+3}$	$\phi_{[x]+4}$	$\phi_{[x]+5}$	$\phi_{[x]+6}$	$\phi_{[x]+7}$	$\phi_{[x]+8}$	$\phi_{[x]+9}$	Age at Entry [x]
20	·272	·190	·143	·108	·079	·054	·036	·020	·009	·002	20
25	·275	·193	·145	·110	·080	·056	·036	·021	·008	·003	25
30	·283	·200	·152	·114	·084	·058	·037	·021	·010	·003	30
35	·298	·215	·162	·123	·091	·062	·040	·022	·010	·002	35
40	·323	·236	·180	·136	·099	·069	·044	·025	·011	·003	40
45	·361	·268	·204	·154	·113	·078	·050	·027	·012	·002	45
50	·414	·312	·239	·180	·131	·090	·058	·032	·014	·004	50
55	·483	·368	·282	·213	·154	·106	·067	·037	·017	·004	55
60	·567	·435	·334	·251	·182	·124	·078	·043	·019	·005	60
65	·659	·508	·389	·291	·210	·143	·090	·051	·022	·006	65
70	·750	·578	·441	·328	·236	·160	·101	·056	·025	·006	70
75	·821	·631	·478	·354	·253	·172	·108	·060	·027	·007	75

If these are plotted out in a diagram the similarity of the curves for different values of t is at once evident, and suggests a simple relation between the functions.

* The symbol $\delta_{[x]+t}$ originally used has been altered since this paper was read, in order to bring the notation into conformity with Mr. Ackland's.

If the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]}}$ be tabulated for different values of t , it is found that on the whole the terms increase with x . If, on the other hand, the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ be investigated, we notice a remarkable constancy in the terms corresponding to each value of t greater than 1. Specimen values of this ratio are given in Tables II, III and IV, for the rates of interest $2\frac{1}{2}$, 3 and $3\frac{1}{2}$ per-cent, from which it may be remarked that the approximate constancy extends also to the different rates of interest.

TABLE II.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$. $0^{[M]} 2\frac{1}{2}$ PER-CENT. $0^{[M]} 2\frac{1}{2}$ PER-CENT.

Age at Entry [x]	DURATION t									Age at Entry [x]
	0	2	3	4	5	6	7	8	9	
20	1.45	.755	.572	.418	.289	.183	.106	.048	.010	20
25	1.44	.754	.569	.412	.289	.180	.104	.043	.009	25
30	1.42	.757	.569	.417	.289	.184	.106	.050	.009	30
35	1.40	.759	.573	.418	.289	.181	.103	.043	.009	35
40	1.38	.763	.573	.419	.289	.186	.103	.043	.012	40
45	1.35	.766	.573	.420	.287	.183	.101	.045	.010	45
50	1.33	.767	.576	.418	.288	.182	.103	.045	.012	50
55	1.32	.768	.577	.418	.286	.180	.100	.044	.013	55
60	1.31	.768	.575	.415	.283	.178	.099	.044	.011	60
65	1.30	.766	.573	.410	.280	.176	.098	.043	.009	65
70	1.30	.761	.566	.404	.276	.174	.095	.042	.010	70
75	1.30	.759	.558	.398	.275	.172	.097	.042	.009	75

TABLE III.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$. $0^{[M]} 3$ PER-CENT. $0^{[M]} 3$ PER-CENT.

Age at Entry [x]	DURATION t									Age at Entry [x]
	0	2	3	4	5	6	7	8	9	
20	1.43	.753	.568	.416	.284	.190	.105	.047	.011	20
25	1.43	.751	.570	.415	.290	.187	.109	.041	.016	25
30	1.42	.760	.570	.420	.290	.185	.105	.050	.015	30
35	1.39	.753	.572	.423	.288	.186	.102	.047	.009	35
40	1.37	.763	.576	.420	.292	.187	.106	.047	.013	40
45	1.35	.761	.575	.422	.291	.187	.101	.045	.007	45
50	1.33	.766	.577	.420	.288	.186	.103	.045	.013	50
55	1.31	.766	.579	.418	.288	.182	.100	.046	.011	55
60	1.30	.768	.577	.418	.285	.179	.099	.044	.012	60
65	1.30	.766	.573	.413	.281	.177	.100	.043	.012	65
70	1.30	.763	.568	.408	.277	.175	.097	.043	.010	70
75	1.30	.758	.561	.401	.273	.171	.095	.043	.011	75

TABLE IV.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$.

 $0^{[M]} 3\frac{1}{2}$ PER-CENT.

 $0^{[M]} 3\frac{1}{2}$ PER-CENT.

Age at Entry [<i>x</i>]	DURATION <i>t</i>									Age at Entry [<i>x</i>]
	0	2	3	4	5	6	7	8	9	
20	1.44	.750	.570	.419	.297	.186	.107	.047	.018	20
30	1.41	.757	.573	.416	.292	.189	.108	.043	.011	30
40	1.37	.763	.580	.425	.292	.187	.105	.046	.009	40
50	1.33	.768	.580	.423	.290	.188	.102	.044	.014	50
60	1.30	.769	.579	.421	.286	.180	.101	.043	.012	60
70	1.30	.767	.571	.411	.280	.178	.099	.043	.011	70

We may thus approximately express the relation between the functions by an equation of the form

$$\phi_{[x]+t} = U_t \phi_{[x]+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

where U_t is a function of t only.

This property does not, however, belong in the same degree to the series of terms for which $t=0$. Here the ratio decreases noticeably as x increases, and this fact suggests that we might more closely express the relation between the functions by an equation of the form

$$\phi_{[x]+t} = X_t \phi_{[x]+1} + Y_t \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

where X_t and Y_t are independent of x .

Two equations are required to determine the coefficients which will give the closest results in practice, and one of these would naturally be obtained by taking the sums of the terms of all equations of the form $\phi_{[x]+t} = X_t \phi_{[x]+1} + Y_t$, where t remains constant and x takes all possible values. Assuming, for the sake of example, that the facts for ages 20 to 70 are used, the first equation will be

$$\sum_{x=20}^{70} \phi_{[x]+t} = X_t \sum_{x=20}^{70} \phi_{[x]+1} + 51 Y_t.$$

This makes the sum of the errors zero; and the sum of the accumulated errors might also be made zero by a double

summation of the series $\phi_{[x]+t}$ and $\phi_{[x]+1}$ commencing at the youngest age. Under these circumstances, the second equation might be written

$$\sum_{x=20}^{70} (71-x)\phi_{[x]+t} = X_t \sum_{x=20}^{70} (71-x)\phi_{[x]+1} + \frac{52 \times 51}{2} Y_t.$$

In this equation, however, greatest weight is evidently given to the youngest ages.* It will be seen hereafter that it is more practically useful to give the greatest weight to an age somewhere between 30 and 40, and to let the weights diminish from this point towards both ends of the series.

This may be effected by summing the series twice from the top downwards, taking the term in the second derived series corresponding to the age next before that chosen for the maximum weight (which is assumed to be always nearer the younger end of the series), and deducting twice this term from the last term. This gives us a weighted sum of the series, such that the weights decrease successively by unity from the maximum towards both ends of the series, reaching zero at the highest age used.

In Table V a comparison is given between the actual values of $\phi_{[x]+t}$ at $2\frac{1}{2}$ per-cent and those obtained from the formula $X_t\phi_{[x]+1} + Y_t$, the coefficients being determined, in a manner similar to that described, from the quinquennial values only of the functions from age 20 to age 65, and age 30 being chosen for the maximum weight. It will be noticed that the error is always less than .002 except where $t=0$. Here the error is always less than .005.

We can now proceed to the practical application of this property to the valuation of whole-life policies by the $O^{[M]}$ Select Tables.

Waiving all practical details of fractional durations, and assuming that for this purpose the duration of a policy is the difference between the valuation age attained and the valuation age at entry, however obtained, the value of a whole-life policy by select tables may be expressed as follows—

$$\begin{aligned} {}_tV_{[x]} &= 1 - (P_{[x]} + d)(1 + a_{[x]+t}) \\ &= 1 - (P_{[x]} + d)(1 + a_{x+t}) - (P_{[x]} + d)(a_{[x]+t} - a_{x+t}) \\ &= A_{x+t} - P_{[x]}(1 + a_{x+t}) - (P_{[x]} + d)\phi_{[x]+t}. \end{aligned}$$

* As Mr. Lidstone points out, when the second equation is used in combination with the first, this statement is not true. In the following, the word "weight" must be understood to refer to the coefficients of $\phi_{[x]+t}$ and $X_t\phi_{[x]+1}$ in the second equation only, taken by itself.

TABLE V.

Comparison between the actual values of $\phi_{[x]+t}$ and those deduced from the formula $X_t\phi_{[x]+1} + Y_t$.

$0^{[M]} 2\frac{1}{2}$ PER-CENT.

$0^{[M]} 2\frac{1}{2}$ PER-CENT.

Age at Entry [x]	DURATION 0			DURATION 2			DURATION 3			Age at Entry [x]
	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	
20	·301	·2965	-·0045	·157	·1575	+·0005	·119	·1188	-·0002	20
25	·303	·3001	-·0029	·159	·1600	+·0010	·120	·1206	+·0006	25
30	·310	·3086	-·0014	·165	·1655	+·0005	·124	·1246	+·0006	30
35	·324	·3255	+·0015	·176	·1762	+·0002	·133	·1327	-·0003	35
40	·349	·3508	+·0018	·193	·1924	-·0006	·145	·1448	-·0002	40
45	·387	·3907	+·0037	·219	·2180	-·0010	·164	·1639	-·0001	45
50	·440	·4438	+·0038	·253	·2521	-·0009	·190	·1893	-·0007	50
55	·511	·5139	+·0029	·298	·2971	-·0009	·224	·2228	-·0012	55
60	·596	·5960	...	·350	·3498	-·0002	·262	·2621	+·0001	60
65	·689	·6842	-·0048	·405	·4064	+·0014	·303	·3043	+·0013	65
Age at Entry [x]	DURATION 4			DURATION 5			DURATION 6			Age at Entry [x]
	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	
20	·087	·0871	+·0001	·060	·0604	+·0004	·038	·0383	+·0003	20
25	·087	·0884	+·0014	·061	·0613	+·0003	·038	·0388	+·0008	25
30	·091	·0913	+·0003	·063	·0632	+·0002	·040	·0400	...	30
35	·097	·0970	...	·067	·0671	+·0001	·042	·0424	+·0004	35
40	·106	·1056	-·0004	·073	·0729	-·0001	·047	·0461	-·0009	40
45	·120	·1191	-·0009	·082	·0820	...	·052	·0518	-·0002	45
50	·138	·1371	-·0009	·095	·0941	-·0009	·060	·0594	-·0006	50
55	·162	·1609	-·0011	·111	·1101	-·0009	·070	·0694	-·0006	55
60	·189	·1888	-·0002	·129	·1288	-·0002	·081	·0811	+·0001	60
65	·217	·2187	+·0017	·148	·1490	+·0010	·093	·0938	+·0008	65
Age at Entry [x]	DURATION 7			DURATION 8			DURATION 9			Age at Entry [x]
	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	Actual	Deducted	Error D-A	
20	·022	·0219	-·0001	·010	·0096	-·0004	·002	·0021	+·0001	20
25	·022	·0222	+·0002	·009	·0097	+·0007	·002	·0021	+·0001	25
30	·023	·0229	-·0001	·011	·0100	-·0010	·002	·0022	+·0002	30
35	·024	·0241	+·0001	·010	·0106	+·0006	·002	·0024	+·0004	35
40	·026	·0261	+·0001	·011	·0115	+·0005	·003	·0026	-·0004	40
45	·029	·0292	+·0002	·013	·0129	-·0001	·003	·0030	...	45
50	·034	·0334	-·0006	·015	·0147	-·0003	·004	·0035	-·0005	50
55	·039	·0388	-·0002	·017	·0171	+·0001	·005	·0042	-·0008	55
60	·045	·0452	+·0002	·020	·0199	-·0001	·005	·0050	...	60
65	·052	·0521	+·0001	·023	·0230	...	·005	·0059	+·0009	65

The first part of this expression is simply the result of valuing the policy with a select net premium by the ultimate

table, an operation which may be performed for all the policies in groups according to ages attained, precisely as if a single table were being used. The second part of the expression may be looked upon as a correction, to be deducted from the value just named, in order to give effect to selection in the valuation factors as well as in the net premiums.

It remains to be shown how the calculation of this correction may be much simplified by means of the property already described.

METHOD I.

Consider all the policies of a single duration t , and let S represent the sum assured under any policy. Dropping the negative sign the total correction for the group

$$= \Sigma S(P_{[x]} + d)\phi_{[x]+t}$$

where the summation extends to all values of x . This

$$= \Sigma S(P_{[x]} + d)\{X_t\phi_{[x]+1} + Y_t\} \quad \text{by equation (B)}$$

$$= X_t \Sigma S(P_{[x]} + d)\phi_{[x]+1} + Y_t \Sigma S(P_{[x]} + d).$$

Now $(P_{[x]} + d)\phi_{[x]+1}$, which may for shortness be called $G_{[x]}$, remains constant throughout the existence of the policy. We may therefore allocate to each policy at its commencement the quantity $SG_{[x]}$, and at each valuation introduce it upon the valuation-sheets in such a way that the total for each duration up to 9 years, as well as those of the sums assured and net premiums, can be easily obtained.

The corrections may now be obtained by a few simple multiplications from the formula

$$X_t \Sigma SG_{[x]} + Y_t \{\Sigma SP + d \Sigma S\},$$

and, summing the results for each duration up to 9, we have the total correction to be deducted from the reserve value based on the select net premiums and ultimate valuation factors.

The advantage of the particular weighting used in forming the second equation for the determination of X_t and Y_t will now be apparent. Seeing that, as I presume is the case in the experience of most offices, the sum of the quantities $S(P_{[x]} + d)$ increases in

general with the age to a maximum and then decreases again till it reaches zero, the two expressions

$$\sum_x S(P_{[x]} + d) \phi_{[x]+t}$$

and

$$\sum_x S(P_{[x]} + d) \{X_t \phi_{[x]+1} + Y_t\}$$

will most closely approximate if X_t and Y_t be chosen so that

$$\sum_x F(x) \phi_{[x]+t} = \sum_x F(x) \{X_t \phi_{[x]+1} + Y_t\}$$

where $F(x)$ is a function which increases and decreases in a somewhat similar manner to the sum of the quantities $S(P_{[x]} + d)$. This condition is roughly satisfied by the weights used in the second equation for determining X_t and Y_t .

So far as I have been able to test them, values of X_t and Y_t obtained in this manner give very close results in practice.

For practical use it would be well to have calculated a set of values of X_t and Y_t corresponding to a maximum total of business at each of the ages 30, 35 and 40. With the help of these I think it should be possible to choose a set of factors suited to the particular distribution of business in hand without any difficulty.

An important feature of this method is that we can fix narrow limits between which the error in the approximation must lie. The difference between the approximate and actual corrections is

$$\sum S(P_{[x]} + d) [X_t \phi_{[x]+1} + Y_t - \phi_{[x]+t}].$$

Now, if M_t is the maximum positive value of the expression in the square bracket, and M'_t is the maximum negative value, the error in the correction must lie between

$$+ M_t \sum S(P_{[x]} + d) \text{ and } - M'_t \sum S(P_{[x]} + d).$$

The values of M_t and M'_t corresponding to any set of values of X_t and Y_t can of course be determined, and it would be convenient to have them appended to the Tables of X_t and Y_t .

METHOD II.

Another method, based on an average age, may also be used. This method has already been described by me in the *Insurance Record*, but to render the subject complete so far as it goes, I repeat it here, although I do not consider it so satisfactory as Method I.

By equation (A) we may transform the correction as follows—

$$\begin{aligned}\Sigma S(P_{[x]} + d)\phi_{[x]+t} &= U_t \Sigma S(P_{[x]} + d)\phi_{[x]+1} \\ &= U_t \Sigma SG_{[x]}.\end{aligned}$$

In order to obtain the value of the correction from $\Sigma SG_{[x]}$, we must fix in some way upon the best value of U_t for the purpose, since although this is approximately constant it is not quite. The best value must lie somewhere between the limits of the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$, and should in some way give effect to the age distribution of the business in hand. These conditions are generally fairly well satisfied by the following method—

In the table of $\phi_{[x]+1}$ find the age ξ , so that

$$\frac{\Sigma SG_{[x]}}{\Sigma S(P_{[x]} + d)} = \phi_{[\xi]+1}.$$

The value of the correction in respect of duration t is then to be taken as

$$\Sigma S(P_{[x]} + d)\phi_{[\xi]+t}.$$

The value of U_t chosen is thus evidently the value of the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ when $x=\xi$, and unless the ratio is varying very irregularly at this point, it will generally give good results.

With this method also it is possible to assign limits to the error. If ω_t and ω'_t are the greatest and least values respectively of $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ then the error must lie between

$$(\omega_t - U_t)\Sigma SG_{[x]} \text{ and } -(U_t - \omega'_t)\Sigma SG_{[x]},$$

π_t being the particular value fixed as above.

It is hardly necessary to point out that, if extreme accuracy is not looked for, a perfectly safe valuation may be made by simply taking the correction as $\omega'_t \Sigma SG_{[x]}$. We know then that the total reserve cannot be too small, while the error must be less than $(\omega_t - \omega'_t) \Sigma SG_{[x]}$.

Or, again, we might even choose an average value of U_t which would fairly well fit the facts by mere inspection of a table of $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ in conjunction with the particulars of the business to be valued.

Either of these last mentioned methods would be very simple to apply, for they merely involve summing the functions $SG_{[x]}$ for each duration. The labour is thus, I think, reduced to a minimum.

TABLE VI.

*Results of Valuation of Model Office No. II by**O^[M] 2½ per-cent Tables.**Correction for Reserve.*

Duration <i>t</i>	Actual Correction for Reserve	APPROXIMATE METHOD NO. I				APPROXIMATE METHOD NO. II					Duration <i>t</i>
		Correc- tion	Error	Posi- tive Limit	Nega- tive Limit	Correc- tion	Average Age	Error	Posi- tive Limit	Nega- tive Limit	
0	7,930	7,934	+ 4	86	109	7,881	39·60	-49	452	389	0
1	5,544	5,544	5,544	1
2	3,906	3,906	...	29	20	3,913	39·70	+ 7	41	31	2
3	2,781	2,782	+ 1	25	23	2,788	39·73	+ 7	24	15	3
4	1,938	1,939	+ 1	31	20	1,939	39·70	+ 1	33	14	4
5	1,286	1,289	+ 3	20	16	1,202	39·70	+ 6	40	...	5
6	786	785	- 1	14	15	799	39·63	+13	39	4	6
7	431	431	...	3	10	432	39·58	+ 1	21	13	7
8	183	183	...	11	16	177	39·50	- 6	4	24	8
9	40	41	+ 1	14	14	47	39·43	+ 7	12	4	9
Total	24,825	24,834	+ 9	233	243	24,812	...	-13	666	494	...

TABLE VII.

*Results of Valuation of Actual Example by O^[M] 3 per-cent Tables.**Correction for Reserve.*

Duration <i>t</i>	Actual Correction for Reserve	APPROXIMATE METHOD NO. I				APPROXIMATE METHOD NO. II					Duration <i>t</i>
		Correc- tion	Error	Posi- tive Limit	Nega- tive Limit	Correc- tion	Average Age	Error	Posi- tive Limit	Nega- tive Limit	
0	3,391	3,388	- 3	44	59	3,379	43·87	-12	145	192	0
1	5,667	5,667	5,667	44·78	1
2	5,553	5,544	- 9	39	44	5,584	48·43	+31	124	...	2
3	4,082	4,074	- 8	47	62	4,107	45·90	+25	78	...	3
4	1,653	1,654	+ 1	32	37	1,663	45·00	+10	55	4	4
5	1,166	1,166	...	36	27	1,177	44·71	+11	57	4	5
6	980	979	- 1	22	24	979	44·19	- 1	53	26	6
7	410	410	...	11	14	414	44·00	+ 4	28	20	7
8	199	199	...	19	11	194	45·34	- 5	13	26	8
9	60	61	+ 1	21	16	54	45·77	- 6	16	27	9
Total	23,161	23,142	-19	271	294	23,218	...	+ 57	569	299	...

In Tables VI and VII are given the results of the two methods applied respectively to Mr. King's Model Office No. 2, and to a group of actual policies to which I have had access. In the Model Office, if we assume a policy of 1 on every life, the average age is probably rather low, seeing that no effect is given to any increase in the average sum assured as the age increases. In the actual example I have reason to believe that the average age is an unusually high one. The Model Office figures are of course grouped into sets of five ages, and in order to avoid prohibitive labour I have adopted the same grouping for the actual example. But I do not apprehend that this at all vitiates the test.

The valuation of the actual example is at 3 per-cent, and the values of X_t and Y_t used were computed from the quinquennial values of $\phi_{[x]+t}$ and $\phi_{[x]+1}$ from age 20 to age 70, on the assumption of a maximum business at age 40. In the case of the Model Office the valuation is at $2\frac{1}{2}$ per-cent, and the same values of X_t and Y_t were used as in Table V.

In the case of the Model Office the average age in the second method is almost constant for different durations, a result which is, of course, due to the large number of facts and consequent smoothness of the figures.

In the actual example the figures show the irregularity of a small experience, and the average age varies considerably. The average ages for durations 2 and 3 unfortunately fall at points where the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ is progressing rather irregularly, and this causes an exceptionally large error.

In addition to this, there happen to be two very large policies on a life aged about 50 at entry, occurring in durations 2 and 3 respectively, which severely test the approximate methods. The effect in the case of Method I is an error of -16, and in the case of Method II one of +16. Thus, in the former case the total error is almost entirely due to these large policies. It may be remarked, however, that errors due to large policies at ages very far from the average are not of much consequence in practice. For the greater part of such policies will be re-assured, and, if the same approximate methods be used for valuing the re-assurances, the errors will be almost cancelled.

In Table VIII are shown the total sums assured for all durations less than 10, according to the ages of the lives at entry, from which the reason may be seen for the choice of ages

TABLE VIII.

Total Sums Assured for durations less than 10, according to Central Ages at Entry.

Central Age at Entry	Model Office No. II	Actual Example
20	270,074	113,775
25	785,789	256,150
30	913,731	445,400
35	732,556	506,550
40	522,269	694,315
45	337,117	409,013
50	201,765	463,772
55	101,359	170,100
60	45,844	129,450
65	14,506	41,200
70	...	12,100

30 and 40 respectively as maxima for the computation of X_t and Y_t .

Strictly, of course, we should make use of a table of $\Sigma S(P_{[x]} + d)$, but this purpose is sufficiently well served by a table of the sums assured.

I may remark that, in order to test the possibility of choosing the appropriate maximum age from a mere inspection of the particulars of the business, I selected age 40 in this way in the case of the actual example given, and applied it without further trial. The results, which are not adjusted in any way, are seen to be satisfactory. In the case of the Model Office, I have tried other ages as maxima, and find that age 30 gives the best results.

I give in Tables VI and VII the positive and negative limits of the error. It is to be understood by these terms that the amounts by which the estimated correction can exceed or fall short of the true value are respectively less than these limits.

To give an idea of the relative scale of the errors and their limits, I may add that the total reserve of the Model Office by the $O^{[M]} 2\frac{1}{2}$ per-cent Tables would appear to exceed 3 millions; and that of the group of policies used, if valued by the $O^{[M]} 3$ per-cent Table, would be in the neighbourhood of $2\frac{1}{2}$ millions.

So far the separation of the correction into its two parts, applicable respectively to the value of the sums assured and net premiums, has not been considered. The correction for the sums assured is negative and that for the net premiums positive, the numerical sum of the two joining to form the negative correction for the reserve.

It may appear to some unnecessary to effect this separation at

all, even for the Board of Trade returns, and at any rate great accuracy in the two separate corrections is comparatively unimportant if we can secure it in the correction for the reserve.

It therefore seems advisable to adopt as correction for the reserve that found by one of the above-described methods, and, after obtaining in some way a sufficiently good correction, either for the sums assured or net premiums, to deduce that for the other by subtraction from the correction for the reserve.

We could obtain a very accurate correction for the sums assured by weighting S with $\phi_{[x]+1}$ instead of $S(P_{[x]}+d)$, but this would involve dispensing with the accuracy of the correction for the reserve, for it would be far too much trouble to use both weighted functions.

The methods here put forward of obtaining a correction for the sums assured are not very scientific and not very trustworthy, but I think would suit the purpose sufficiently well.

For use with Method I, the following somewhat similar plan gives fair results. I find that $\phi_{[x]+t}$ may also be expressed fairly well in terms of $P_{[x]}$ or $P_{[x]}+d$ by an equation of the form

$$\phi_{[x]+t} = X'_t(P_{[x]}+d) - Y'_t$$

where X'_t and Y'_t are independent of x . Obtaining values of X'_t and Y'_t in precisely the same way as X_t and Y_t , we are able to obtain the correction for the sums assured by the following approximate equation :

$$\Sigma dS\phi_{[x]+t} = dX'_t\Sigma S(P_{[x]}+d) - dY'_t\Sigma S.$$

$\Sigma S(P_{[x]}+d)$ and ΣS having already been obtained in the course of the application of Method I, the extra work necessary is small.

For use with Method II, I suggested in my letter to the *Insurance Record* that the same average ages should be used for the sums assured as for the reserve. This is, perhaps, rather too rough an approximation. The average ages applicable to the sums assured must always be less than those found for the reserve. And in both the examples I have been able to test I find that a constant deduction of 1.85 years from the average ages found by Method II will give a good correction for the sums assured.

Of course, however, many further trials would be necessary fully to establish this last device, if, indeed, it could be established.

Tables IX and X show the results of using these methods.

TABLE IX.

Results of Valuation of Model Office No. II by
O^[M] 2½ per-cent Tables.
Correction for Sums Assured.

Duration <i>t</i>	Actual Cor- rection for Sums Assured	Correction by Method I	Error	Correction by Method II	Error	Duration <i>t</i>
0	3,966	3,977	+ 11	3,931	- 35	0
1	2,758	2,768	+ 10	2,759	+ 1	1
2	1,938	1,947	+ 9	1,932	- 6	2
3	1,379	1,387	+ 8	1,377	- 2	3
4	962	968	+ 6	966	+ 4	4
5	638	643	+ 5	639	+ 1	5
6	390	393	+ 3	396	+ 6	6
7	215	216	+ 1	213	- 2	7
8	91	92	+ 1	91	...	8
9	20	21	+ 1	24	+ 4	9
Total ...	12,357	12,412	+ 55	12,328	- 29	...

TABLE X.

Results of Valuation of Actual Example by O^[M] 3 per-cent Tables.
Correction for Sums Assured.

Duration <i>t</i>	Actual Cor- rection for Sums Assured	Correction by Method I	Error	Correction by Method II	Error	Duration <i>t</i>
0	1,737	1,742	+ 5	1,738	+ 1	0
1	2,814	2,848	+ 34	2,796	- 18	1
2	2,551	2,548	- 3	2,572	+ 21	2
3	1,997	2,001	+ 4	1,969	- 28	3
4	814	825	+ 11	830	+ 16	4
5	582	586	+ 4	584	+ 2	5
6	497	501	+ 4	494	- 3	6
7	209	211	+ 2	214	+ 5	7
8	99	99	...	102	+ 3	8
9	30	30	...	30	...	9
Total ...	11,330	11,391	+ 61	11,329	- 1	...

I can touch but briefly upon a few most obvious details of the application of these methods, as I have had no opportunity of putting any of them into actual practice. It has been seen that for Methods I and II we require for each duration less than ten years the total sums assured, net premiums, and *G* functions. If we are dealing with valuation sheets on which the details of every policy are set out, we must arrange the policies in order of duration, and leave a space after each duration so that the totals may be cast separately. The complete totals for each duration must then be taken out on separate sheets.

If we are dealing with a valuation based on classification registers we must have a separate list of policies arranged in durations. The discontinuants must be deducted or ruled off, and the new entrants added each year. There is, of course, no reason why this plan should not be used for any sort of valuation.

I assume that reversionary bonuses and reductions of premiums would be treated by the ultimate table only, seeing that in the case of quinquennial valuations at any rate they would only appear in the less important durations.

I do not profess to be competent to deal with the question whether, other things equal, a valuation by select tables is advisable; but, if this be granted, I do not see why we should hesitate to adopt one or other of the methods sketched out in this paper of reducing the labour involved on the ground of untrustworthiness. The possible error has been shown to lie within very small limits, while the actual errors which appear to be likely to occur in practice are of an insignificant character. For example, by Method I they are comparable with those which might be introduced by cutting off the last figures of the valuation factors, a course, the adoption of which I do not suppose would shake anybody's confidence in the results of a valuation. Moreover, we could provide against any possibility of an under-estimation of the reserves, by giving the correction the lowest limit of its possible value.

The question now arises, how much work would actually be saved by these approximate methods. Evidently the fundamental valuation by the ultimate table occupies exactly as much time as a valuation by a single table, while the correction represents the extra work involved over and above an ordinary single valuation.

By far the greater portion of this extra work is taken up by the initial entering of the *G* functions on the cards, and re-copying of them on the valuation sheets. They would naturally be entered on the cards at the same time as the net premiums—most conveniently from a table of parallel columns—so that this work would at the most be equivalent to that of using a second set of net premiums. The remainder of the work is comparatively slight whatever method is used. The subsidiary totals on the valuation sheets hardly involve anything extra, as they could easily be added together to give the grand total required for other purposes. We should then have to copy the subsidiary totals on to 10 sheets of 3 columns and cast them.

The completion of the evaluation of the correction would then be a matter of about an hour's work.

Looking back on the process, I think the whole valuation would require less work than a combined H^M and $H^{M(5)}$ (H^M for first 5 years) valuation, and of course greatly less than an exact valuation by select tables.

The question naturally arises whether we can find any theoretical basis for this property, and the further question, whether we could—*a priori*—graduate a life table, so as to reproduce the property in the annuity tables.

The $O^{[M]}$ Life Table is constructed according to a definite mathematical law, and the annuity tables are formed from this by a definite process. It is therefore theoretically possible to express the function $\phi_{[x]+t}$ in terms of x , t , and the constants on which the tables are based. It will be seen that we can obtain an approximate series in ascending powers of e^x , the coefficients being functions of t . The empirical law suggests that the coefficients of the more important terms will all be approximately proportional to the function of t which we have called U_t , and this will be found to be the case. The form of the coefficients may then be expected to give some explanation of the property, or some indication of the nature of the relation between the constants of the table which causes it to appear.

It is not claimed that the explanation here given is a very satisfactory one. I think it well to set out, however, the results I have so far obtained, partly in the hope that, with these before the Institute, some more convincing explanation may be supplied, and partly with a view to giving a provisional answer to the further question.

Referring to Mr. G. F. Hardy's explanation in the *Principles and Methods*, we have

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t$$

where

$$f_t = m(10-t)^2 + m'(c')^t$$

and

$$\psi_t = n(10-t)^2$$

both expressions being taken as zero when $t > 10$.

Then $\log_{10} l_{[x]+t+r} = \log_{10} l_{x+t+r} - f_{t+r} - \beta c^x \psi_{t+r}$

and $\log_{10} p_{[x]+t} = \log_{10} p_{x+t} + f_t + \beta c^x \psi_t - f_{t+r} - \beta c^x \psi_{t+r},$

so that $r p_{[x]+t} = r p_{x+t} \times 10^{f_t + \beta c^x \psi_t} 10^{-f_{t+r} - \beta c^x \psi_{t+r}}.$

Multiplying by v^r and integrating, we have

$$\begin{aligned} \bar{a}_{[x]+t} &= 10^{f_t + \beta c^x \psi_t} \int_0^\infty v^r 10^{-f_{t+r} - \beta c^x \psi_{t+r}} r p_{x+t} dr \\ &= 10^{f_t + \beta c^x \psi_t} \left\{ \int_0^\infty v^r r p_{x+t} dr + \int_0^\infty v^r (10^{-f_{t+r} - \beta c^x \psi_{t+r}} - 1) r p_{x+t} dr \right\} \\ &= 10^{f_t + \beta c^x \psi_t} \left\{ t \bar{a}_{x+t} + \int_0^{10-t} v^r (10^{-f_{t+r} - \beta c^x \psi_{t+r}} - 1) r p_{x+t} dr \right\} \end{aligned}$$

for f_{t+r} and ψ_{t+r} vanish when $r > 10 - t$.

Also since the second term of f_t —namely: $m'(c')^t$ —rapidly becomes insignificant as t increases, and since we are summing for all values of r between the limits, we may, with very slight inaccuracy, take $f_{t+r} = m(10 - t - r)^2$, and therefore

$$f_{t+r} + \beta c^x \psi_{t+r} = (m + n\beta c^x)(10 - t - r)^2.$$

Now putting t' for $10 - t$ and subtracting \bar{a}_{x+t} from both sides of the equation, we have

$$\begin{aligned} \bar{\phi}_{[x]+t} &= (10^{f_t + \beta c^x \psi_t} - 1) \bar{a}_{x+t} \\ &\quad + 10^{f_t + \beta c^x \psi_t} \int_0^{t'} v^r (10^{-(m+n\beta c^x)(t'-r)^2} - 1) r p_{x+t} dr \dots (1) \end{aligned}$$

where $\bar{\phi}_{[x]+t} = \bar{a}_{[x]+t} - \bar{a}_{x+t}.$

This integral may be simplified as follows.

We have $\int_0^{t'} r v^r r p_{x+t} dr = v \frac{d}{dv} \int_0^{t'} v^r r p_{x+t} dr = v \frac{d}{dv} \bar{a}_{x+t; \overline{t'}}$

and, similarly,

$$\int_0^{t'} r^2 v^r r p_{x+t} dr = \left(v \frac{d}{dv} \right)^2 \bar{a}_{x+t; \overline{t'}}$$

and $\int_0^{t'} (t' - r)^2 v^r r p_{x+t} dr = \left(t' - v \frac{d}{dv} \right)^2 \bar{a}_{x+t; \overline{t'}}$

We may also make a similar transformation when any function of $(t'-r)^2$ is substituted. So

$$\begin{aligned} & \int_0^{t'} v^r (10^{-(m+n\beta c^x)(t'-r)^2} - 1) {}_r p_{x+t} dr \\ &= \left(10^{-(m+n\beta c^x) \left(t' - v \frac{d}{dv} \right)^2} - 1 \right) \bar{a}_{x+t:\overline{t'}} \\ &= - \left[\frac{\left(m+n\beta c^x \right) \left(t' - v \frac{d}{dv} \right)^2}{M} - \frac{\left(m+n\beta c^x \right)^2 \left(t' - v \frac{d}{dv} \right)^4}{2M^2} + \dots \right] \bar{a}_{x+t:\overline{t'}} \end{aligned}$$

where M is the modulus $\log_{10} e$.

In order to expand this last expression, and also the remaining term on the right-hand side of equation (1), I have made use of Mr. Emory McClintock's formula for the value of an annuity according to Makeham's law (*J.I.A.*, vol. xviii, p. 242). As I have found it necessary to alter the notation, partly to suit modern usage, and partly to avoid confusion with symbols used here and elsewhere with other meanings, I think it well to reproduce his demonstration, especially as, for the temporary annuity, it is necessary to start from one of his intermediate steps.

For a table following Makeham's law—

$$\bar{a}_x = \frac{1}{v^x s^x g^{c^x}} \int_x^\infty v^x s^x g^{c^x} dx.$$

Now let $z = -(\log_e g) c^x$

and $\kappa = 1 - \frac{A + \delta}{\log_e c}$

where A , as usual, $= -\log_e s$ and δ is the force of interest. Then

$$\begin{aligned} v^x s^x &= e^{x(\log_e v + \log_e s)} = e^{-x(A + \delta)} \\ &= e^{x(\kappa - 1)\log_e c} = c^{x(\kappa - 1)} \\ &= \left(-\frac{z}{\log_e g} \right)^{\kappa - 1} \end{aligned}$$

Also $g^{c^x} = e^{-z}$

and $\frac{dz}{dx} = z \log_e c.$

Therefore, transforming the integral for \bar{a}_x , we have

$$\begin{aligned}\bar{a}_x &= \frac{1}{\left(-\frac{z}{\log_e g}\right)^{\kappa-1}} e^{-z} \int_z^\infty \left(-\frac{z}{\log_e g}\right)^{\kappa-1} e^{-z} \frac{dz}{z \log_e c} \\ &= \frac{1}{\log_e c \cdot e^{-z} z^{\kappa-1}} \int_z^\infty e^{-z} z^{\kappa-2} dz.\end{aligned}$$

Now, by expansion and integration,

$$\int_z^\infty e^{-z} z^{\kappa-2} dz = C - \frac{z^{\kappa-1}}{\kappa-1} + \frac{z^\kappa}{\kappa} - \frac{z^{\kappa+1}}{2(\kappa+1)} + \dots$$

where C is a constant to be determined,

$$= C - z^{\kappa-1} \left[\frac{1}{\kappa-1} - \frac{z}{\kappa} + \frac{z^2}{2(\kappa+1)} - \frac{z^3}{3(\kappa+2)} + \dots \right]$$

Then since

$$u_0 - u_1 z + \frac{u_2 z^2}{2} - \dots = e^{-z} \left[u_0 - z \Delta u_0 + \frac{z^2 \Delta^2 u_0}{2} - \dots \right]$$

and since

$$\Delta^n \left(\frac{1}{\kappa-1} \right) = (-1)^n \frac{|n|}{(\kappa-1)\kappa(\kappa+1) \dots (\kappa+n-1)}$$

the integral

$$= C - e^{-z} z^{\kappa-1} \left[\frac{1}{\kappa-1} + \frac{z}{(\kappa-1)\kappa} + \frac{z^2}{(\kappa-1)\kappa(\kappa+1)} + \dots \right]$$

Now put $z=0$, when all on the right hand side vanishes except C, and we have

$$C = \int_0^\infty e^{-z} z^{\kappa-2} dz = \Gamma(\kappa-1)$$

which

$$= \frac{\Gamma(1+\kappa)}{\kappa(\kappa-1)}$$

by the well-known property of gamma functions.

Now returning to the expression for \bar{a}_x , and remembering that $(1-\kappa)\log_e c = A + \delta$, we have

$$\begin{aligned}\bar{a}_x &= \frac{1}{(\kappa-1)\log_e c} \left[\frac{\Gamma(1+\kappa)}{\kappa} e^{z \log_e c} - 1 - \frac{z}{\kappa} - \frac{z^2}{\kappa(1+\kappa)} - \dots \right] \\ &= \frac{1}{A+\delta} \left[-\frac{\Gamma(1+\kappa)}{\kappa} e^{z \log_e c} + 1 + \frac{z}{\kappa} + \frac{z^2}{\kappa(1+\kappa)} + \dots \right]\end{aligned}$$

It may here be remarked that κ is positive and less than 1 for the $O^{[M]}$ Table for all practical rates of interest.

Now since $zc^t = -(\log_e g)c^{x+t}$, we have, similarly,

$$\bar{a}_{x+t} = \frac{1}{A+\delta} \left[-\frac{\Gamma(1+\kappa)}{\kappa} e^{zc^t} (zc^t)^{1-\kappa} + 1 + \frac{zc^t}{\kappa} + \frac{z^2 c^{2t}}{\kappa(1+\kappa)} + \dots \right]$$

For a temporary annuity we have

$$\bar{a}_{x:\bar{n}} = \frac{1}{\log_e c \cdot e^{-z} z^{\kappa-1}} \int_z^{z^n} e^{-z} z^{\kappa-2} dz,$$

and, expanding and integrating in a similar way, we have

$$\begin{aligned} \bar{a}_{x:\bar{n}} &= \frac{e^z}{\log_e c \cdot z^{\kappa-1}} \left[\frac{z^{\kappa-1}}{\kappa-1} (c^{n(\kappa-1)} - 1) - \frac{z^\kappa}{\kappa} (c^{n\kappa} - 1) \right. \\ &\quad \left. + \frac{z^{\kappa+1}}{2(\kappa+1)} (c^{n(\kappa+1)} - 1) - \dots \right] \\ &= \frac{e^z}{\log_e c} \left[\frac{c^{n(\kappa-1)} - 1}{\kappa-1} - z \frac{c^{n\kappa} - 1}{\kappa} + \frac{z^2 c^{n(\kappa+1)} - 1}{2(\kappa+1)} - \dots \right] \end{aligned}$$

and, similarly,

$$\bar{a}_{x+t:\bar{t}'} = \frac{e^{zc^t}}{\log_e c} \left[\frac{c^{t'(\kappa-1)} - 1}{\kappa-1} - z c^t \frac{c^{t'\kappa} - 1}{\kappa} + \frac{z^2 c^{2t}}{2} \cdot \frac{c^{t'(\kappa+1)} - 1}{\kappa+1} - \dots \right]$$

Now since $v = e^{-\delta}$, $v \frac{d}{dv} = -\frac{d}{d\delta}$,

and since $\kappa = 1 - \frac{A+\delta}{\log_e c}$,

$$-\frac{d}{d\delta} = \frac{1}{\log_e c} \frac{d}{d\kappa}.$$

Therefore we may put $\left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa}\right)$ for $\left(t' - v \frac{d}{dv}\right)$.

In order to complete the integration we must expand

$$\left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa}\right)^p \frac{c^{t'(\kappa+r)} - 1}{(\kappa+r) \log_e c}$$

where r is any integer and p any positive integer.

We have

$$\frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} = t' \left[1 + \frac{t'(\kappa+r)\log_e c}{2} + \frac{\{t'(\kappa+r)\log_e c\}^2}{3} + \dots \right]$$

and

$$\left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa} \right) \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} = t'^2 \left(1 - \frac{1}{t' \log_e c} \frac{d}{d\kappa} \right) \left[1 + \frac{t'(\kappa+r)\log_e c}{2} + \dots \right]$$

Now, where ρ and σ are positive integers,

$$\begin{aligned} & \frac{\{t'(\kappa+r)\log_e c\}^{\rho-1}}{\rho+\sigma-1} - \frac{1}{t' \log_e c} \frac{d}{d\kappa} \frac{\{t'(\kappa+r)\log_e c\}^\rho}{\rho+\sigma} \\ &= \frac{\{t'(\kappa+r)\log_e c\}^{\rho-1}}{\rho+\sigma-1} \left(1 - \frac{\rho}{\rho+\sigma} \right) = \sigma \frac{\{t'(\kappa+r)\log_e c\}^{\rho-1}}{\rho+\sigma}. \end{aligned}$$

Applying this rule, we have—

$$\begin{aligned} \left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa} \right) \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} &= t'^2 \left[\frac{1}{2} + \frac{t'(\kappa+r)\log_e c}{3} \right. \\ &\quad \left. + \frac{\{t'(\kappa+r)\log_e c\}^2}{4} + \dots \right] \end{aligned}$$

$$\left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa} \right)^2 \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} = 2t'^3 \left[\frac{1}{3!} + \frac{t'(\kappa+r)\log_e c}{4} + \dots \right],$$

and

$$\begin{aligned} \left[t' - \frac{1}{\log_e c} \frac{d}{d\kappa} \right]^p \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} &= p! t'^{p+1} \left[\frac{1}{p+1} \right. \\ &\quad \left. + \frac{t'(\kappa+r)\log_e c}{p+2} + \frac{\{t'(\kappa+r)\log_e c\}^2}{p+3} + \dots \right] \\ &= \frac{t'^{p+1}}{p+1} \left[1 + \frac{t'(\kappa+r)\log_e c}{p+2} \right. \\ &\quad \left. + \frac{\{t'(\kappa+r)\log_e c\}^2}{(p+2)(p+3)} + \dots \right] \end{aligned}$$

If, then, we put

$${}_r b_{t'}^{(p)} = 1 + \frac{t'(\kappa+r)\log_e c}{p+1} + \frac{\{t'(\kappa+r)\log_e c\}^2}{(p+1)(p+2)} + \dots$$

then

$$t' {}_r b_{t'}^{(1)} = \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c}$$

and

$$\left(t' - \frac{1}{\log_e c} \frac{d}{d\kappa} \right)^p \frac{c^{t'(\kappa+r)}-1}{(\kappa+r)\log_e c} = \frac{t'^{p+1}}{p+1} {}_r b_{t'}^{(p+1)}.$$

Returning now to the expansion of $a_{x+t:\overline{t}|}$, we have

$$\bar{a}_{x+t:\overline{t}|} = t' e^{zc^t} \left[-{}_1b_t^{(1)} - zc'_0 b_t^{(1)} + \frac{z^2 c^{2t}}{2} {}_1b_t^{(1)} - \dots \right]$$

and

$$\left(t - \frac{1}{\log_e c} \frac{d}{d\kappa} \right)^p \bar{a}_{x+t:\overline{t}|} = \frac{t'^{p+1}}{p+1} e^{zc^t} \left[-{}_1b_t^{(p+1)} - zc'_0 b_t^{(p+1)} + \frac{z^2 c^{2t}}{2} {}_1b_t^{(p+1)} - \dots \right].$$

We may now substitute the expansions we have obtained in equation (1), which becomes

$$\begin{aligned} \Phi_{[x]+t} &= \frac{10^{f_t + \beta c^x \psi_t} - 1}{A + \delta} \left[1 + \frac{zc^t}{\kappa} + \frac{z^2 c^{2t}}{\kappa(1+\kappa)} + \dots \right. \\ &\quad \left. - \frac{\Gamma(1+\kappa)}{\kappa} e^{zc^t} (zc^t)^{1-\kappa} \right] \\ &- 10^{f_t + \beta c^x \psi_t} e^{zc^t} \left[\frac{m+n\beta c^x}{M} \cdot \frac{t'^3}{3} \cdot \left\{ -{}_1b_t^{(3)} - zc'_0 b_t^{(3)} + \frac{z^2 c^{2t}}{2} {}_1b_t^{(3)} - \dots \right\} \right. \\ &\quad \left. - \frac{(m+n\beta c^x)^2}{2M^2} \cdot \frac{t'^5}{5} \left\{ -{}_1b_t^{(5)} - zc'_0 b_t^{(5)} + \dots \right\} + \dots \right] \dots \quad (2) \end{aligned}$$

It will be shown that for most practical purposes very little error is introduced by discarding the terms containing $(m+n\beta c^x)^2$ and higher powers, and that, unless x is large and t is small concurrently, we can obtain a sufficiently accurate expression by putting

$$10^{f_t + \beta c^x \psi_t} = 1$$

and

$$10^{f_t + \beta c^x \psi_t} - 1 = \frac{f_t + \beta c^x \psi_t}{M}$$

for $f_t + \beta c^x \psi_t$ is in general small.

Introducing these approximations, and making use again of the Finite Difference transformation, we have

$$\begin{aligned} \Phi_{[x]+t} &= \frac{f_t + \beta c^x \psi_t}{M(A + \delta)} \left[1 + \frac{zc^t}{\kappa} + \frac{z^2 c^{2t}}{\kappa(1+\kappa)} + \dots - \frac{\Gamma(1+\kappa)}{\kappa} e^{zc^t} (zc^t)^{1-\kappa} \right] \\ &- \frac{m+n\beta c^x}{M} \cdot \frac{t'^3}{3} \left[b_t - zc^t \Delta b_t + \frac{z^2 c^{2t}}{2} \Delta^2 b_t - \dots \right] \dots \quad (3) \end{aligned}$$

where

$$b_t \text{ stands for } -{}_1b_t^{(3)}$$

$$\Delta b_t \text{ for } {}_0b_t^{(3)} - {}_{-1}b_t^{(3)}$$

$$\Delta^2 b_t \text{ for } {}_1b_t^{(3)} - 2{}_0b_t^{(3)} + {}_{-1}b_t^{(3)}$$

and so on.

$$\text{Now put } \eta_t = \frac{\beta \psi_t}{(-\log_e g) f_t} \text{ and } \eta = \frac{\beta n}{(-\log_e g) m}.$$

Then since $z = -(\log_e g) e^x$, equation (3) may be written

$$\begin{aligned} \bar{\phi}_{[x]+t} = & \frac{f_t}{M(A+\delta)} (1 + \eta_t z) \left[1 + \frac{z c^t}{\kappa} + \frac{z^2 c^{2t}}{\kappa(1+\kappa)} + \dots \right. \\ & \left. - \frac{\Gamma(1+\kappa)}{\kappa} c^{t(1-\kappa)} z^{1-\kappa} e^{z c^t} \right] \\ & - \frac{m t'^3}{3M} (1 + \eta z) \left[b_t - z c^t \Delta b_t + \frac{z^2 c^{2t}}{2} \Delta^2 b_t - \dots \right] \end{aligned}$$

And this may be set out in ascending powers of z as follows—

$$\begin{aligned} \bar{\phi}_{[x]+t} = & \left\{ \frac{f_t}{M(A+\delta)} - \frac{m t'^3}{3M} b_t \right\} - \frac{\Gamma(1+\kappa)}{\kappa} \frac{f_t}{M(A+\delta)} c^{t(1-\kappa)} z^{1-\kappa} \\ & + \left[\frac{f_t}{M(A+\delta)} \left(\eta_t + \frac{c^t}{\kappa} \right) - \frac{m t'^3}{3M} (\eta b_t - c^t \Delta b_t) \right] z \\ & - \frac{\Gamma(1+\kappa)}{\kappa} \frac{f_t}{M(A+\delta)} c^{t(1-\kappa)} (\eta_t + c^t) z^{2-\kappa} \\ & + \left[\frac{f_t}{M(A+\delta)} \left(\eta_t + \frac{c^t}{1+\kappa} \right) \frac{c^t}{\kappa} + \frac{m t'^3}{3M} (\eta \Delta b_t - \frac{c^t}{2} \Delta^2 b_t) c^t \right] z^2 \\ & - \frac{\Gamma(1+\kappa)}{\kappa} \frac{f_t}{M(A+\delta)} c^{t(2-\kappa)} \left(\eta_t + \frac{c^t}{2} \right) z^{3-\kappa} + \dots \quad (4) \end{aligned}$$

To verify the approximate correctness of the formulæ obtained, some values of $\bar{\phi}_{[x]+t}$ have been calculated at $2\frac{1}{2}$ per-cent from formula (2) neglecting terms containing $(m+n\beta e^x)^2$ and higher powers, and also from formula (3). These are compared in Table XI with the actual values obtained from the tables.

TABLE XI.

Comparison of Values of $\bar{\phi}_{[x]+t}$ obtained from Formulæ (2) and (3) with the Actual Values.

O^[M] $2\frac{1}{2}$ PER-CENT.

O^[M] $2\frac{1}{2}$ PER-CENT.

Duration t	AGE 20		AGE 40			AGE 60			Duration t
	Formula (3)	Actual	Formula (2)	Formula (3)	Actual	Formula (2)	Formula (3)	Actual	
0	·3007	·301	·3494	·3463	·349	·5934	·5801	·596	0
1	·2078	·208	·2539	·2523	·253	·4544	·4151	·456	1
3	·1180	·119	·1456	·1451	·145	·2613	·2583	·262	3
6	·0375	·038	·0466	·0465	·047	·0811	·0808	·081	6
9	·0024	·002	·0029	·0029	·003	·0048	·0048	·005	9

We see that formula (3), though not a very good approximation at higher ages and shorter durations, is sufficiently near for the purpose of investigating the forms of the coefficients of powers of z , without our being obliged to resort to the very cumbrous formula (2).

The coefficients of the first six terms of formula (4), which is only a modified form of No. (3), are given in Table XII for all values of t . The powers of z for a few ages are also given, and the actual terms themselves for duration 3 are shown below, in order to test the relative importance of the various terms.

TABLE XII.

Showing the formation of the terms in Formula (4).

$0^{[M]} 2\frac{1}{2}$ PER-CENT.

$0^{[M]} 2\frac{1}{2}$ PER-CENT.

x	z^0	z^1	z^2	$z^{1-\kappa}$	$z^{2-\kappa}$	$z^{3-\kappa}$	x
20	...	·00689	·00005	·18228	·00126	·00001	20
40	...	·04150	·00172	·33687	·01398	·00058	40
60	...	·25003	·06252	·62253	·15565	·03891	60
t	Corresponding coefficients of the above powers of z						t
0	·3620	7·2098	11·630	·5357	10·536	10·268	0
1	·2475	5·8573	10·252	·3798	8·769	9·365	1
2	·1863	4·6825	8·848	·2933	7·157	8·355	2
3	·1414	3·6384	7·410	·2278	5·671	7·230	3
4	·1041	2·7168	5·961	·1717	4·316	6·006	4
5	·0729	1·9191	4·535	·1227	3·107	4·718	5
6	·0472	1·2502	3·182	·0810	2·063	3·416	6
7	·0268	·7163	1·964	·0469	1·204	2·175	7
8	·0120	·3245	·958	·0215	·556	1·094	8
9	·0030	·0827	·263	·0056	·144	·310	9
x	Products of powers and coefficients where $t=3$						x
20	·1414	·0251	·0004	·0415	·0071	·0001	20
40	·1414	·1510	·0127	·0767	·0793	·0042	40
60	·1414	·9097	·4633	·1418	·8827	·2813	60

In Table XIII are shown the values of the ratio u_t/u_1 for various functions of t , in which may be noticed the general correspondence between the earlier coefficients and the values of $\phi_{[x]+t}$. The ratio is also given for the function f_t for purposes of comparison.

TABLE XIII.

Showing the ratios of various functions of t to the particular value of the function when $t=1$.

O^[M] 2 $\frac{1}{2}$ PER-CENT.O^[M] 2 $\frac{1}{2}$ PER-CENT.

Duration t	COEFFICIENTS OF POWERS OF z						ACTUAL VALUES OF $\bar{\phi}_{[x]+t} \div \bar{\phi}_{[x]+1}$				Duration t
	z^0	z^1	z^2	$z^{1-\kappa}$	$z^{2-\kappa}$	$z^{3-\kappa}$	$f_t \div f_1$	Age 20	Age 40	Age 60	
0	1.46	1.23	1.13	1.41	1.20	1.10	1.45	1.45	1.38	1.31	0
2	.753	.799	.863	.772	.816	.892	.748	.755	.763	.768	2
3	.572	.621	.723	.599	.647	.772	.563	.572	.573	.575	3
4	.420	.464	.582	.452	.492	.642	.412	.418	.419	.416	4
5	.295	.328	.442	.323	.354	.504	.285	.289	.289	.284	5
6	.190	.213	.310	.213	.235	.365	.183	.183	.186	.177	6
7	.108	.122	.192	.124	.137	.233	.103	.106	.103	.098	7
8	.049	.055	.094	.057	.063	.117	.046	.048	.043	.044	8
9	.012	.014	.026	.015	.017	.033	.011	.011	.012	.011	9

We may now examine the forms of the coefficients of formula (4), and for this purpose some of the subsidiary functions which appear are tabulated in Table XIV.

TABLE XIV.

Showing the Values of certain functions occurring in Formula (4).

O^[M] 2 $\frac{1}{2}$ PER-CENT.O^[M] 2 $\frac{1}{2}$ PER-CENT.

t	η_t	c^t	$\frac{c^t}{\kappa}$	$c^{t(1-\kappa)}$	b_t	Δb_t	t
0	18.667	1.000	1.520	1.000	.928	.239	0
1	21.991	1.094	1.662	1.031	.935	.214	1
2	23.202	1.197	1.819	1.063	.941	.189	2
3	23.590	1.309	1.989	1.097	.949	.164	3
4	23.713	1.432	2.176	1.131	.956	.140	4
5	23.751	1.567	2.381	1.166	.963	.116	5
6	23.765	1.714	2.605	1.202	.970	.092	6
7	23.770	1.875	2.849	1.240	.977	.069	7
8	23.771	2.051	3.117	1.278	.985	.045	8
9	23.771	2.244	3.410	1.318	.992	.023	9
...	$\eta = 23.772$

Turning attention to the term independent of x , we see that b_t is always less than 1, but differs little from it. Since $f_t = mt'^2 + m'(c')^t$, and the latter term is positive and small,

$$\frac{f_t}{M(A+\delta)} \text{ is slightly } > \frac{mt'^2}{M(A+\delta)}.$$

Also

$$\frac{mt'^3}{3M} b_t \text{ is slightly } < \frac{mt'^3}{3M}.$$

Therefore

$$\begin{aligned} \frac{f_t}{M(A+\delta)} : \frac{mt'^3}{3M} b_t \text{ is slightly } &> \frac{mt'^2}{M(A+\delta)} : \frac{mt'^3}{3M} \\ &> \frac{1}{A+\delta} : \frac{t'}{3}. \end{aligned}$$

Now $A = .0060$ for the $O^{[M]}$ Table, and thus

$$A + \delta = .0307 \text{ at } 2\frac{1}{2} \%,$$

and the greatest value of t' is 10.

Hence $\frac{f_t}{M(A+\delta)}$ is at least about 10 times as great as $\frac{mt'^3}{3M} b_t$, and the value of the term independent of x thus depends mainly on f_t , though, since t' decreases as t increases, it will decrease slightly less rapidly than f_t .

The coefficient of $z^{1-\kappa}$ is proportional to $f_t c^{t(1-\kappa)}$, and since $c^{1-\kappa} = 1.031$ at $2\frac{1}{2}$ per-cent this coefficient also decreases somewhat less rapidly than f_t .

The coefficient of z is of somewhat similar form to the term independent of z . One of the terms of the latter is multiplied by $\eta_t + \frac{c^t}{\kappa}$ and the other by $\eta b_t - c^t \frac{\Delta b_t}{b_t}$. If now η_t were constant and thus equal to η its limit value, and if η were large compared with $\frac{c^t}{\kappa}$ or $c^t \frac{\Delta b_t}{b_t}$, the two coefficients would be practically proportional. Now from Table XIV, η_t is seen to vary very slightly when $t > 1$, and ranges from about 13 times to about 7 times the magnitude of $\frac{c^t}{\kappa}$, while $c^t \frac{\Delta b_t}{b_t}$ is considerably smaller than $\frac{c^t}{\kappa}$.

The coefficient of $z^{2-\kappa}$ is simply that of $z^{1-\kappa}$ multiplied by $\eta_t + c^t$. As we have already seen, η_t is approximately constant, and it is always more than 10 times the magnitude of c^t .

This approximate constancy and considerable magnitude of η_t may be seen to depend on the forms of f_t and ψ_t and the relative magnitudes of m and n .

$$\text{For } \eta_t = \frac{\beta \psi_t}{(-\log_e g) f_t}$$

$$f_t = m(10-t)^2 + m'(c')^t$$

$$\text{and } \psi_t = n(10-t)^2.$$

Now $m'(c')^t$ rapidly becomes insignificant, and $\frac{n}{m} = 583$ about.

The coefficients of higher powers appear to diverge more and more from the type, as higher powers of c' enter as factors, but the first four terms are much the most important, the others having little influence till the age exceeds 50.

Speaking generally, we might have expected, on summing the series, to find the law in question fairly regularly satisfied for ages up to 50 and durations greater than 1. Certain divergencies would naturally appear in the case of ages over 50 because of the increased influence of terms of the series later than the fourth. Divergencies would also be expected in the case of duration 0 for two reasons, (1) because the large value of t' has here considerable influence on the term independent of x , and (2) because η_t does not commence to be approximately constant until $t=2$.

I have not been able to undertake the lengthy calculations that would be necessary to enquire more closely into the causes of the law under discussion, nor do I think it very necessary; for although perhaps the first question with which I set out on page 29 is not very satisfactorily answered, we seem to have sufficient data to give a conditionally affirmative answer to the further question, so far as at this stage it is possible so to do.

It is true that the mathematical analysis is based on the particular forms of f_t and ψ_t selected by Mr. G. F. Hardy for the graduation of the $O^{[M]}$ Select Tables. It may, I think, be assumed, however, that it would often be possible to obtain an approximate series for $\phi_{[x]+t}$ on somewhat similar lines, although doubtless sometimes f_t and ψ_t might have to be taken of such forms as to prohibit any mathematical expression.

TABLE XV.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$ by the $O^{[am]}$ $2\frac{1}{2}$ per-cent Table.

Age at Entry [x]	$t=0$	$t=2$	$t=3$	$t=4$
30	1.64	.514	.196	.038
40	1.64	.510	.191	.041
50	1.67	.500	.192	.038
60	1.70	.484	.176	.033
70	1.75	.482	.168	.032
80	1.78	.471	.169	.031
90	1.80	.465	.164	.031

TABLE XVI.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$ by the $O^{[af]}$ $2\frac{1}{2}$ per-cent Table.

Age at Entry [x]	$t=0$	$t=2$	$t=3$	$t=4$
30	1.68	.500	.176	.034
40	1.70	.485	.175	.035
55	1.77	.462	.159	.031
70	1.77	.470	.164	.033
80	1.78	.467	.161	.031
90	1.81	.466	.164	.034

TABLE XVII.

Specimen Values of $\phi_{[x]+t} \div \phi_{[x]+1}$ by the $O^{[NM]}$ $2\frac{1}{2}$ per-cent Table.

Age at Entry [x]	$t=0$	$t=2$	$t=3$	$t=4$
25	2.26	.368	.088	.000
30	2.16	.377	.098	.016
40	1.96	.429	.130	.013
50	1.80	.477	.165	.018
60	1.68	.494	.167	.025
70	1.61	.498	.173	.022

In this connection a similar investigation of the $O^{[NM]}$, $O^{[am]}$ and $O^{[af]}$ Tables would be interesting. This I have not been able to undertake, but Tables XV, XVI and XVII, showing some values of $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ for these tables, may be of interest. It will be seen that the $O^{[af]}$ Table approaches nearest, on the whole, to satisfying the law of constancy, although it is not so striking as in the case of the $O^{[M]}$ Table, while the $O^{[NM]}$ Table practically does not exhibit the property at all. I can do no more than throw out the following suggestion, which may or may not have any significance.

It will be remembered that the fact that $\frac{\psi_t}{f_t}$ is large and approximately constant for most values of t in the case of the $O^{[M]}$ Table appeared to be of importance in producing the property under consideration. Now the $O^{[af]}$ Table is constructed from two separate series, but for both of these

as well as for the $O^{(am)}$ Table the forms of the functions f_t and ψ_t are :

$$f_t = m[(5-t)^2 + (4-t)^2 + (1-t)^2]$$

and
$$\psi_t = n[(5-t)^2 + (4-t)^2 + (1-t)^2].$$

Thus, $\frac{\psi_t}{f_t}$ is absolutely independent of t , but its value is not so large as in the case of the $O^{[M]}$ Table, being about 159 for the Male Table and the second series of the Female Table, and 350 for the first series of the Female Table, as against about 583 for the $O^{[M]}$ Table.

In the case of the $O^{[NM]}$ Table, $\frac{\psi_t}{f_t}$ appears to be not even approximately constant. We may perhaps trace some congruity between these facts and the conclusions drawn above from Tables XV, XVI and XVII.

In conclusion, I must admit that it appears rash for a tyro to challenge comparison with the work of such veterans as Mr. Ackland and Mr. King, by submitting a paper on the same subject, and at the same meeting. I must, however, deprecate any such comparison. The foregoing notes contain, I am fully aware, much that is crude and ill thought out. But I have hopes that somewhere in the chaff may be found a seed, which under more skilled culture than mine may develop into a healthy plant. The property which forms the basis of the paper has, I believe, not been noticed before ; and I think it should be capable of being turned to account in some way. It is solely on the strength of such hopes that I venture to place these notes before the Institute.

Notes on an approximate Method of Valuation of Whole-Life Assurances, with Allowance for Selection. By THOMAS G. ACKLAND, one of the Vice-Presidents of the Institute of Actuaries ; Honorary Fellow of the Faculty of Actuaries.

[Read before the Institute, 27 November 1905.]

1. THE object of the following notes is to state somewhat more fully and formally a method (first published in the *Insurance Record* for 10 March 1905) of approximating to the reserves required for whole-life assurances, with due regard to the influence of selection ; and to illustrate the practical application of the method.

2. Let the age at entry, as usual, be x , and the duration of the assurance at the date of valuation, t , so that $(x+t)$ is the valuation age, and let $S_{[x]+t}$ represent the aggregate sums assured in respect of assurances effected at age x , the duration of which is t years; then the policy-value, by the usual non-select or "ultimate" tables, but employing a select net premium, is equal to

$$S_{[x]+t}\{\bar{A}_{x+t}-\pi_{[x]}\bar{a}_{x+t}\} \quad . \quad . \quad . \quad . \quad (1)$$

Continuous valuation factors are employed, in accordance with the conditions usually obtaining in practice. If t lies within the period during which selection is in operation (here assumed as ten years), we have, for the policy-value by select tables, the formula—

$$S_{[x]+t}\{\bar{A}_{[x]+t}-\pi_{[x]}\bar{a}_{[x]+t}\} \quad . \quad . \quad . \quad . \quad (2)$$

Expressing \bar{A}_{x+t} , in formula (1), as $1-\delta\bar{a}_{x+t}$, and

$$\bar{A}_{[x]+t}, \text{ in formula (2), as } 1-\delta\bar{a}_{[x]+t},$$

and deducting (2) from (1), we have—

$$\begin{aligned} -\Delta_{[x]+t} &= S_{[x]+t}\{\delta(\bar{a}_{[x]+t}-\bar{a}_{x+t})+\pi_{[x]}(\bar{a}_{[x]+t}-\bar{a}_{x+t})\} \\ &= S_{[x]+t}\{(\pi_{[x]}+\delta)(\bar{a}_{[x]+t}-\bar{a}_{x+t})\} \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

or, representing $(\bar{a}_{[x]+t}-\bar{a}_{x+t})$ by $\bar{\phi}_{[x]+t}$,

$$= S_{[x]+t}\{(\pi_{[x]}+\delta)(\bar{\phi}_{[x]+t})\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This formula represents the amount by which the reserve or net liability according to ultimate tables (but employing select net premiums) is reduced, when select tables are substituted; the value of the sums assured being *diminished* by

$$S_{[x]+t} \cdot \delta \cdot \bar{\phi}_{[x]+t}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which may be conveniently referred to as " Δ value of sums assured"; and the value of the net premiums *increased* by

$$S_{[x]+t} \cdot \pi_{[x]} \bar{\phi}_{[x]+t}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which may be referred to as " Δ value of net premiums." It follows that, if the whole-life assurances at attained age $[x]+t$ are valued by ultimate tables (with select net premiums), the

application to the net liability of the corrective amount brought out by formula (4), or the application, separately, to the value of the sums assured, and the value of the net premiums, of the corrective amounts shown respectively by formulas (5) and (6), will give the modified values, with allowance for the operation of selection.

3. If now, an average value w can be found, such that

$$S_{[x]+t} \cdot \phi_{[x]+t} + S_{[x+1]+t} \phi_{[x+1]+t} + S_{[x+2]+t} \cdot \phi_{[x+2]+t} + \dots \\ = (\Sigma S_t) \phi_{[w]+t} \quad \dots \quad (7)$$

$$(\text{where } \Sigma S_t = S_{[x]+t} + S_{[x+1]+t} + S_{[x+2]+t} + \dots),$$

it is evident that the corrective expression $\phi_{[w]+t}$ can be applied to the *total* sums assured in respect of a given duration t , at whatever entry ages the assurances were effected. In order to find the value of w , which shall fulfil the conditions of formula (7), it is necessary, either to deduce the mathematical law governing the values of the function $\phi_{[x]+t}$ for successive values of x and t (according to a given graduated mortality table), or to deduce an *approximate* expression for $\phi_{[x]+t}$, which shall express sufficiently closely the progression of that function, and which shall at the same time be practically applicable in a simple form to the valuation problem under investigation.

4. Taking, for example, the graduated $O^{[M]}$ Table, the mathematical formulas on the basis of which the values of $\log l_{[x]+t}$ and $\log l_{x+t}$ (which form the basis respectively of select and ultimate whole-life annuity-values) are calculated, are fully and clearly stated by Mr. G. F. Hardy, in the "Account of Principles and Methods, &c." (p. 158, formulas 21 to 23). I have made some attempt to investigate, on the basis of these formulas, the law governing the progression of the values of $\phi_{[x]+t}$ for successive values of x and t , according to the $O^{[M]}$ Table; but have not been able, in the time at my disposal, satisfactorily to solve the problem. In an appendix to these notes, I state the problem, and indicate some lines which have been experimentally followed. It is probable that other workers in this field may be more successful than I have been, and may arrive, possibly by different routes, at a satisfactory solution of the problem.

5. From the form of the expressions experimentally deduced in the course of this investigation, and also from other general considerations, it appeared probable that the expression

$\bar{\phi}_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$ might be approximately represented by a series in the form

$$\bar{\phi}_{[x]+t} = A_t + B_t c^x + C_t c^{2x} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where A_t , B_t , and C_t are constant for a given value of t , but independent of x ; and c is the Makeham constant employed in the graduation of the mortality table, equal, for the O^[M] Table, to 1.09396, so that $\log_{10} c = .039$. If (as is in practice found to be the case) the numerical value of the third term of the series in formula (8) is small compared with that of the second term, we shall have

$$\begin{aligned} \Delta_x \bar{\phi}_{[x]+t} &= (A_t + B_t c^{x+1}) - (A_t + B_t c^x) \text{ approximately,} \\ &= B_t c^x (c - 1) \text{ approximately,} \quad . \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

and the successive values of $\Delta_x \bar{\phi}_{[x]+t}$ will be approximately in geometrical progression, with a common ratio of $c = 1.09396$.

6. In order to test this relation, I had the values of $(\bar{a}_{[x]+t} - \bar{a}_{x+t})$ computed, according to the O^[M] Table, at $2\frac{1}{2}$ and 3 per-cent, for all values of x from 0 to 60, and for all values of t from 0 to 9 inclusive. The tabular values are given in Tables I and II, under the heading "True Values", with specimens of their first differences, and the ratio between every *fifth* value of those differences, where such ratio > 1 . It will be found that the tabulated ratios do not greatly differ, on the average, from the value of $c^5 = 1.56675$. The individual divergencies shown arise, no doubt, in part from the approximate nature of the formula; but also from the fact that, from the values of $N_{[x]+t}$ and $D_{[x]+t}$, as tabulated on the O^[M] basis, annuity-values cannot be deduced accurately to the third place of decimals, so that the fourth and subsequent places of decimals in annuity-values thus deduced must be quite untrustworthy. For example,

$$a_{[35]+5} = \frac{N_{[35]+5}}{D_{[35]+5}} = (\text{at 3 per-cent}) \frac{456,158.}{24,842.} = 18.36237. \quad \text{The}$$

tabular values of N and D , as published, are given to the nearest integer only, up to about attained age 60. If it be assumed that the true value of the denominator in the above fraction is 24,842.5 (the numerator remaining unchanged) the value of the annuity will be reduced to 18.36200. If, on the other hand, the true value of the denominator be assumed as 24,841.5 (the numerator remaining unchanged), the value of the annuity will be increased to 18.36274. It follows that the third and following places of decimals in the annuity-value cannot be correctly

determined from values of N and D , based on a radix of 100,000, and tabulated to the nearest integer only; although the true value of the annuity in the above example will most probably lie between 18.362 and 18.363.

7. These results not being entirely conclusive as to the presumed law of progression of the function $\bar{\phi}_{[x]+t}$, I applied the ingenious and useful method suggested by Mr. Lidstone in his investigation of temporary life annuities, in connection with the valuation of endowment assurances by approximate methods (see *J.I.A.*, vol. xxxviii, p. 8). The formula employed in the present investigation was

$$r = \left[\frac{[\bar{\phi}_{[40]+t} - \bar{\phi}_{[60]+t}]}{[\bar{\phi}_{[20]+t} - \bar{\phi}_{[40]+t}]} \right]^{\frac{1}{20}}, \quad \dots \quad (10)$$

where r is equal to the average ratio of increase of $\Delta_x \bar{\phi}_{[x]+t}$, for successive values of x , deduced from a range of values of the function between the practical limits of $x=20$ and $x=60$. Applying this formula to the calculated values of $\bar{\phi}_{[x]+t}$ for all values of t from 0 to 9, with interest at $2\frac{1}{2}$ and 3 per-cent, the following values of r were deduced:

t	VALUES OF r	
	$2\frac{1}{2}$ per-cent	3 per-cent
0	1.085	1.081
1	1.078	1.074
2	1.076	1.074
3	1.074	1.073
4	1.075	1.072
5	1.073	1.069
6	1.071	1.075
7	1.075	1.079
8	1.083	1.085
9	1.086	1.094

These average values of the ratio of increase were, it was thought, sufficiently near to the value of $c=1.094$, to justify the application of the proposed method in an approximate valuation.

8. In order further to test the closeness of the approximation given by formula (8), I proceeded to deduce values of A_t , B_t , and C_t , for each value of t from 0 to 9. The process followed may be interesting to students of the Institute, and I therefore state it here in some detail. If $\bar{\phi}_{[x]+t} = A_t + B_t c^x + C_t c^{2x}$, we can eliminate the function A by differencing, and then have

$$\Delta_x \bar{\phi}_{[x]+t} = B_t c^x (c - 1) + C_t c^{2x} (c^2 - 1). \quad \dots \quad (11)$$

where the term omitted in formula (9) is now included. If now the values of Δ be summed for all values of x , say from a to z , we have

$$\sum_{x=a}^{x=z} (\Delta_x \bar{\phi}_{[x]+t}) = B_t(c^z - c^a) + C_t(c^{2z} - c^{2a}). \quad (12)$$

and, if Δ be summed from a to x (where x has every value in succession from a to z), and these values be again summed, we have

$$\sum_{x=a}^{x=z} \left(\sum_{x=a}^{x=x} \Delta_x \bar{\phi}_{[x]+t} \right) = B_t \left((z-a)c^z - \frac{c^z - c^a}{c-1} \right) + C_t \left((z-a)c^{2z} - \frac{c^{2z} - c^{2a}}{c^2 - 1} \right) \quad (13)$$

From formulas (12) and (13) the values of the functions B_t and C_t can be deduced, for each separate value of t from 0 to 9 inclusive. In practice, a was taken as 20, and z as 60, giving a range of ages from 20 to 60. The approximate values of $\Delta_x \bar{\phi}_{[x]+t}$ were then constructed, within these limits, by the application of formula (11), inserting the deduced values of B_t and C_t ; and the value of A_t was finally determined from the formula—

$$\begin{aligned} \sum_{x=20}^{x=60} (\bar{\phi}_{[x]+t}) &= \sum_{x=20}^{x=60} (A_t + B_t c^x + C_t c^{2x}) \\ &= 41A_t + \sum_{x=20}^{x=60} (B_t c^x + C_t c^{2x}) \end{aligned}$$

whence

$$41A_t = \sum_{x=20}^{x=60} [\bar{\phi}_{[x]+t} - (B_t c^x + C_t c^{2x})] \quad (14)$$

9. This process, which was really a simple application of the general principles followed in Prof. Karl Pearson's useful method of "moments" (but applied to the differences, instead of to the values, of the function), gave resulting approximate values of $\bar{\phi}_{[x]+t}$, which are shown, together with the values of the functions A_t , B_t , and C_t for each value of t , in Tables I and II, at $2\frac{1}{2}$ and 3 per-cent respectively, under the heading "Approximate Values." Specimens of the first differences of these values, together with the ratios between every fifth value of those differences, are also included in Tables I and II. From the formulas and methods employed as above, the following conditions of relation between the true and approximate values will hold good for all values of t :—(1) the first and last values of the functions, $\bar{\phi}_{[20]+t}$ and $\bar{\phi}_{[60]+t}$, are accurately reproduced; (2) the sums of the true values, and of the approximate values, are

throughout equal; (3) the sums of the first differences of the true, and of the approximate, values, are equal; (4) the accumulated sums of these first differences are also equal.*

10. It will also be found that the approximate values, for particular values of x and t , very fairly reproduce the true values throughout. Having regard also to the fact, referred to in § (6) above, that the third and following places of decimals in the values of the function $\phi_{[x]+t}$ cannot be altogether relied upon, the approximate values here deduced may, I think, be considered as practically reproducing the true values, throughout, within the assigned limits of age. It may therefore be fairly stated that, whatever the true progression of the function $\phi_{[x]+t}$, when deduced with mathematical accuracy, it practically follows the law indicated by the formula

$$\phi_{[x]+t} = A_t + B_t c^x + C_t c^{2x}$$

for all values of x usually arising in practice.

11. I now proceed to apply the formula, thus deduced, in the practical work of valuation. Mr. Lidstone has shown (*J.I.A.*, xxxviii, 3) that the value of an ordinary temporary annuity, ${}_na_x$, can be represented approximately (where Makeham's law is in operation) by a formula practically identical with the above; and he applies this relation to the approximate valuation of Endowment Assurances, by weighting the sums assured with c^x (or its practical equivalent), where x is the attained age, and thence deducing an average value c^w , the power of which gives the average age at which the whole of the assurances (having identical unexpired terms) can be approximately valued. I have adapted the same method to the approximate valuation of whole-life assurances by select tables. Reverting to formula (7) (see § 3), it will be seen that, if the sums assured, in respect of the cases where selection is still in operation, be separately scheduled for each value of t ; that is, if the amounts of the assurances *effected*

* It would probably have been more satisfactory to apply Prof. Karl Pearson's method of moments more directly, by deducing the values of the functions A_t , B_t and C_t from three equations, based on the first, second and third "moments" of the function $\phi_{[x]+t}$, for successive values of x , at a given value of t ; in which case the sums of the true and approximate values of the function would have been identical, as well as their second and third summations. It was, however, found that the arithmetical labour involved, where 41 consecutive values of the function were employed, was very considerable; and the above method, whilst much less laborious, was thought to be quite sufficiently accurate for the practical purposes required, although not so defensible on theoretical grounds.

in each of the last ten years, and subsisting at the valuation date, be separately tabulated; and if an *average entry age* w be deduced separately for the assurances effected in each of these ten years, then the correction deduced from the average value of $\Phi_{[w]+t}$ can be applied to the *aggregate sums assured* effected in each of the ten years; and the correction on account of selection, to be applied to the net liability (or separately to the value of sums assured and of net premiums), can be thus deduced with close accuracy.

12. I proceed to illustrate the application of this method in a practical case. For this purpose I have taken the data of Mr. George King's O^[M] Model Office (*J.I.A.*, xxxvii, pp. 456, 457) as representing the relative number of cases in existence at each entry age and duration in an average life office. As the Model Office practically proceeded on the assumption that the sum assured was uniformly 1 at all entry ages and durations, it seemed desirable to give effect, as far as practicable, to the differing amounts assured on the average at each entry age. A brief investigation was therefore made into the relative amounts assured, by whole-life assurances, at different entry ages, from the combined data of two life offices which happened to be available. The relative amounts at every fifth entry age, roughly graduated, came out as follows :

Age at Entry	Relative Amounts Assured
20	3·80
25	3·90
30	4·00
35	4·30
40	4·10
45	4·30
50	4·00
55	3·90
60	3·70
65	3·40

It was thought that, *relatively*, the amounts assured at different entry ages would probably not differ much in different offices; and that, in any case, the amounts deduced as above would sufficiently serve for purposes of practical illustration. These relative amounts were therefore multiplied into the numbers, according to entry age, shown by the O^[M] Model Office in the first ten years of duration. The amounts, thus deduced, showed an aggregate

sum assured of upwards of £16,000,000, distributed over the first ten years of duration only.*

13. I then proceeded to value the assurances in this illustrative office (1) by Ultimate $O^{(M)}$ Tables; (2) by Select $O^{(M)}$ Tables; at $2\frac{1}{2}$ and at 3 per-cent; the valuation being made by formulas (1) and (2) respectively (employing net $O^{(M)}$ premiums throughout) separately for the assurances in each year of duration 0 to 9 inclusive, and effected at each quinary entry age. The differences between the aggregate values of the sums assured, of the net premiums, and of the net liability, as computed on a select and on an ultimate basis, were then ascertained, at each rate of interest, separately for each of the ten years of duration; and these differences represented the true values of the corrections to be applied to the respective values, when computed by ultimate tables, in order to arrive at the modified values on the basis of select tables. These true values of the correction to the value of sums assured, of the net premiums, and of the net liability, are given respectively in columns (7), (10) and (12) of Tables III and IV, at $2\frac{1}{2}$ and 3 per-cent interest, the relative sums assured being given in column (2) of those tables. These detailed valuations on a select and an ultimate basis were, of course, solely made for the purpose of testing the closeness of the approximate method to be employed; and in practice the detailed valuation on a select basis would not be necessary.

14. The next step was the application of the approximate method of valuation; and it was first necessary to ascertain an average entry age, w , in respect of the cases subsisting in each of the first ten years of duration. This was done, as already stated, by a method similar to that adopted by Mr. Lidstone in the valuation of endowment assurances. The values of $S_{[x]+t}$, the sums assured under assurances effected at entry age x , were "weighted" by computing in each case (or in respect of the total sums assured for given values of x and t) the quantity $c^x S_{[x]+t}$, where c is Makeham's constant, equal in the $O^{(M)}$ Table to 1.09396. In practice, again following a device adopted by Mr. Lidstone, as the value of the quantity c^x became inconveniently large at older ages, it was reduced by multiplication throughout by c^{-40} , where 40 was selected as representing a rough approximation to the average age at entry. In Table V are given the values

* Relative amounts assured were employed, rather than actual amounts, as the latter made the resulting figures of the Model Office unmanageably large.

of Sc^{x-40} for all values of x from 15 to 75, and values of $S=100, 200, \dots$ up to 1,000, the tabular results varying from 11 to 1,479 in respect of 100 assured. (If the weighting quantity were c^x , without reduction, the results would practically lie between 38 and 53,700 for 100 assured.) The values given in Table V are applicable for deducing the average entry age in respect of all durations, and whatever the rate of interest employed in the valuation. In practice, they would be inserted, once for all, on the valuation cards, as written up from the policy registers.

15. The weighted quantity $c^x S_{[x]+t}$ (or $c^{x-40} S_{[x]+t}$), being thus entered for all ages at entry and durations (up to 9 inclusive), the totals of this quantity were ascertained, in respect of the cases existing in each year of duration; and these totals, divided by the appropriate total of the unweighted sum assured, gave the value of c^w (or of c^{w-40}), from which the value of the average entry age w was determined, for the assurances in the particular year of duration. The values of w , as deduced from the illustrative $O^{[M]}$ data employed, are given in column (3) of Tables III and IV, and will be seen to run with great regularity, between the limits of 38.3 and 38.6.

16. The approximate valuation can now rapidly be completed, and the steps are clearly shown in Tables III and IV. In column (4) are inserted the values of $\phi_{[w]+t}$, taken from the true values of that quantity in Tables I and II. These form, as shown in formulas (4), (5) and (6), the basis of the corrective quantities required. The “ Δ value of the sums assured” is arrived at by multiplying $\phi_{[w]+t}$ by the sums assured at duration t , $S_{[x]+t}$, the product being entered in column (5) of Tables III and IV, and multiplying the result by δ (column 6). The “ Δ value of the net premiums” is arrived at by entering the values of $\pi_{[w]}$, taken from the $O^{[M]}$ Table at the average age w , in column (8), and multiplying these values by the amounts in column (5), the product, $S_{[w]+t} \phi_{[w]+t} \pi_{[w]}$, being entered in column (9). Finally, the “ Δ net liability” is at once deduced by summing the two corrective quantities entered in columns (6) and (9), and entering their sum in column (11). It is particularly to be observed that the net premiums to be valued are deduced by multiplying the sums assured by $\pi_{[w]}$, where w is the average age at entry; and that the true net premiums $\Sigma(S_{[x]+t} \pi_{[x]})$ are not employed in the approximate valuation.

17. It will be seen that the resulting values, as ascertained by detailed valuation, and by the approximate method here described, are as under :

O^[M] 2½ per-cent—

	True Value	Approx. Value
Δ Value of Sum Assured	— £51,294	— £51,780
Δ „ Net Premiums	+ 51,060	+ 51,757
Δ „ Net Liability	—£102,354	—£103,537
Amount of Error		+ 1,183.

O^[M] 3 per-cent—

	True Value	Approx. Value
Δ Value of Sum Assured	— £56,856	— £57,550
Δ „ Net Premiums	+ 44,934	+ 45,387
Δ „ Net Liability	—£101,790	—£102,937
Amount of Error		+ 1,147.

18. The resulting error is thus slightly in excess of one per-cent of the true values. As the aggregate amount of the net liability is upwards of £1,500,000 in respect of assurances in the first ten years of assurance, with an additional liability of upwards of £12,000,000 in respect of policies of longer durations, it will be seen that the errors of £1,183 and £1,147 are, relatively, quite insignificant in amount; and it will probably be agreed that the results brought out by the approximate method are sufficiently near for all practical purposes; whilst the amount of work involved is very small indeed, being all set out (after ascertainment of the average entry ages) in columns (4), (5), (6), (8), (9) and (11) of Tables III or IV. The scheduling of the sums assured according to duration (not exceeding 10 years) presents, it is thought, no difficulty, as most valuation class-books are so arranged as to enable the sums assured existing in respect of each year of duration to be readily extracted. In practice, where any approved method of deducing the valuation ages is adopted, the assurances could be scheduled according to duration, for the purpose of the approximate method here dealt with, by classifying the cases according to the difference between the entry age $[x]$, adopted for determination of the net premium, and the valuation age $[x] + t$. Thus, the cases where the entry age and valuation age were identical would be classified as of duration 0; the cases where these ages differed by 1 year, as of duration 1; and so on, up to and including a difference of 9 years between the entry age and valuation age. The values of

Sc^x (or Sc^{x-40}) can conveniently be written on the valuation cards, once for all, and entered, with the sums assured, in the valuation schedules; and the average age at entry is readily determined, by casting the two sets of amounts thus entered in respect of each of the first ten years of duration, and dividing the total of the weighted sums assured by the total of the sums assured; Table V being entered (inversely) with the quotient, in order to determine the average entry age for each year of duration.

19. I now proceed to discuss, from the point of view of theory, the degree of approximation of the method here suggested. Dealing, first, with the " Δ value of the sums assured," we have

$$\delta(S_{[x]+t}\bar{\phi}_{[x]+t} + S_{[x+1]+t}\bar{\phi}_{[x+1]+t} + S_{[x+2]+t}\bar{\phi}_{[x+2]+t} + \dots)$$

which is assumed to be equal to

$$\delta.\Sigma(S_t)\bar{\phi}_{[w]+t} \dots \dots \dots (15)$$

where $\bar{\phi}_{[w]+t} = A_t + B_t c^{tw} + C_t c^{2tw}$, and the value of w is deduced from the formula

$$S_{[x]+t}c^x + S_{[x+1]+t}c^{x+1} + S_{[x+2]+t}c^{x+2} + \dots = \Sigma(S_t)c^{tw} \dots \dots (16)$$

So far as the first and second terms of the approximate expression $A_t + B_t c^{tw} + C_t c^{2tw}$ are concerned, it is evident that the employment of the average age w introduces no error. As regards the third term, $C_t c^{2tw}$, however, an error is introduced, for the average value, say c^{2w_1} , deduced from the formula

$$\frac{S_{[x]+t}c^{2x} + S_{[x+1]+t}c^{2x+2} + S_{[x+2]+t}c^{2x+4} + \dots}{\Sigma(S_t)} \dots \dots (16a)$$

evidently differs from c^{2w} , where the value of w is deduced from formula (16). I have experimentally weighted the sums assured in the Model Office by multiplication by c^{2x} , where x is the entry age, and find that the resulting values of c^{2w_1} and of w_1 , the average age at entry, are as set out in columns (3) and (11) of Table VI. Comparing these average ages with those deduced from formula (16) and shown in column (10) of Table VI, it will be seen that the values of w and w_1 are each fairly constant for all values of t , and may be taken as 38.5 and 43.3 respectively, so that $(w_1 - w)$ is approximately equal to 4.8. The amount of the error in the third term is thus represented by

$$\Sigma(S_t)\delta.C_t(c^{2w} - c^{2w_1}) = -\Sigma(S_t)\delta.C_t c^{7t}(c^{9.6} - 1)$$

and, giving numerical values to c^{7t} and $c^{9.6}$, we have

$$-1,377.5.\delta.C_t\Sigma(S_t)$$

Computing the value of $-1,377.5 \cdot \delta \cdot C_t$, from the values of C_t given at foot of Tables I and II, at $2\frac{1}{2}$ and 3 per-cent, we arrive at the following results :

Error in “ Δ Value of Sums Assured,” applicable to a Sum Assured of 1. $(-1,377.5 \cdot \delta \cdot C_t)$

t	$2\frac{1}{2}$ PER-CENT	3 PER-CENT
0	+ .000053	+ .000078
1	.000067	.000089
2	.000057	.000073
3	.000044	.000056
4	.000032	.000043
5	.000023	.000028
6	.000013	.000017
7	.000006	.000009
8	+ .000002	.000004
9	— .000000	+ .000001

These factors, multiplied into the appropriate sums assured given in column (2) of Tables III and IV, bring out a total theoretical error in the approximate Δ value of the sums assured of £522, when interest is taken at $2\frac{1}{2}$ per-cent, and of £696 at 3 per-cent. The actual errors, arising in practice, as set out at foot of column (5) of Tables III and IV, of £486 and £694 are, it will be seen, just within the amounts of theoretical error, deduced as above.

20. Considering now the theoretical error in the “ Δ value of the net premiums,” the case is somewhat more complicated. We have

$$S_{[x]+t}(\pi_{[x]}\bar{\phi}_{[x]+t}) + S_{[x+1]+t}(\pi_{[x+1]}\bar{\phi}_{[x+1]+t}) + S_{[x+2]+t}(\pi_{[x+2]}\bar{\phi}_{[x+2]+t}) + \dots$$

which is assumed to be equal to

$$\Sigma(S_t)\pi_{[w]}\bar{\phi}_{[w]+t} \dots \dots \dots (17)$$

where the average age w , employed in determining both the average net premium and the average value of $\bar{\phi}_{[w]+t}$, is derived from the summation of $S_x c^x$ as in formula (16) above. Analyzing $\bar{\phi}_{[x]+t}$ into its constituent terms, and setting out the successive values of each of the three terms under one another, we have

$$\begin{array}{lll} (S_{[x]+t} \cdot \pi_{[x]}) & A_t + & (S_{[x]+t} \cdot \pi_{[x]}) B_t c^x + & (S_{[x]+t} \cdot \pi_{[x]}) C_t c^{2x} \\ (S_{[x+1]+t} \cdot \pi_{[x+1]}) & A_t + & (S_{[x+1]+t} \cdot \pi_{[x+1]}) B_t c^{x+1} + & (S_{[x+1]+t} \cdot \pi_{[x+1]}) C_t c^{2x+2} \\ (S_{[x+2]+t} \cdot \pi_{[x+2]}) & A_t + & (S_{[x+2]+t} \cdot \pi_{[x+2]}) B_t c^{x+2} + & (S_{[x+2]+t} \cdot \pi_{[x+2]}) C_t c^{2x+4} \\ \vdots & & \vdots & \vdots \end{array}$$

assumed to be equal, when summed vertically, to

$$\Sigma(S_t)\pi_{[w]}A_t + \Sigma(S_t)\pi_{[w]}B_t c^{2w} + \Sigma(S_t)\pi_{[w]}C_t c^{2w},$$

where the average age w is uniformly derived from the weighted

sum assured $S_x c^x$ by formula (16). It is evident that the true average ages will all be different from w , and representing these by w_2 , w_3 , and w_4 , we have

$$\Sigma(S_t)\pi_{[w_2]}A_t + \Sigma(S_t)\pi_{[w_3]}B_t c^{w_3} + \Sigma(S_t)\pi_{[w_4]}C_t c^{2w_4},$$

where w_2 , w_3 and w_4 are deduced respectively from summations as given below :

$$\frac{\sum_{x=20}^{x=65} (S_{[x]} + t\pi_{[x]})}{\Sigma(S_t)} = \pi_{[w_2]} \left| \frac{\sum_{x=20}^{x=65} (S_{[x]} + t\pi_{[x]}c^x)}{\Sigma(S_t)} = \pi_{[w_3]} c^{w_3} \right| \frac{\sum_{x=20}^{x=65} (S_{[x]} + t\pi_{[x]}c^{2x})}{\Sigma(S_t)} = \pi_{[w_4]} c^{2w_4}. \quad (18)$$

21. The total error in the “ Δ value of the net premiums” will thus be equal to

$$\Sigma(S_t)[A_t(\pi_{[w]} - \pi_{[w_2]}) + B_t(\pi_{[w]}c^{w_3} - \pi_{[w_3]}c^{w_3}) + C_t(\pi_{[w]}c^{2w} - \pi_{[w_4]}c^{2w_4})] \quad \dots \quad (19)$$

I have experimentally computed the average values of $\pi_{[w_2]}$, $\pi_{[w_3]}c^{w_3}$, and $\pi_{[w_4]}c^{2w_4}$ by summations according to the formulas given under (18) above, at $2\frac{1}{2}$ and 3 per-cent, in respect of the amounts in each of the first ten years of duration in the Model Office. The resulting average values, and the corresponding average ages w_2 , w_3 , and w_4 , are given in columns (4) to (9), and (12) to (17) respectively, of Table VI; and at foot of each column is inserted the *general average* value of each function, and of each age, taken over the whole of the sums assured at all durations. It will be seen that the average values are as follows :

Per-cent	w	w_2	w_3	w_4
$2\frac{1}{2}$	38.5	36.4	40.6	45.3
3	38.5	36.4	40.7	45.4

The employment throughout of the average age w thus *overstates* the value of the term involving A_t , and *understates* the value of the term involving B_t , and these corrections tend to neutralize one another; whilst, as the value of C_t never exceeds .000002, the correction to the term involving that function is inconsiderable. Inserting in formula (19) the values of π_w , $\pi_w c^{w_3}$, and $\pi_w c^{2w}$, where $w=38.5$, and also the average values of the terms

involving w_2 , w_3 , and w_4 , taken from the last line of Table VI, we have, for the amount of the error at $2\frac{1}{2}$ per-cent :

$$\begin{aligned} & \Sigma(S_t)[A_t(\cdot 02472 - \cdot 02311) + B_t(\cdot 783 - 1\cdot 019) \\ & \qquad \qquad \qquad + C_t(24\cdot 89 - 107\cdot 67)] \\ & = \Sigma(S_t)[\cdot 00161A_t - \cdot 236B_t - 82\cdot 78C_t] \quad . \quad . \quad (20) \end{aligned}$$

and at 3 per-cent :

$$\begin{aligned} & \Sigma(S_t)[A_t(\cdot 02335 - \cdot 02179) + B_t(\cdot 740 - \cdot 974) + C_t(23\cdot 51 - 104\cdot 05)] \\ & = \Sigma(S_t)[\cdot 00156A_t - \cdot 234B_t - 80\cdot 54C_t] \quad . \quad . \quad (21) \end{aligned}$$

22. The values of the quantities in square brackets in formulas (20) and (21) are given in the following Table :

Error in "Δ Value of Net Premiums" (the Values being deduced from the Net Premiums at the AVERAGE AGE) applicable to a Sum Assured of 1.

t	$2\frac{1}{2}$ PER-CENT	3 PER-CENT
	$\cdot 00161A_t - \cdot 236B_t - 82\cdot 78C_t$	$\cdot 00156A_t - \cdot 234B_t - 80\cdot 54C_t$
0	$\cdot 000188$	$\cdot 000140$
1	$\cdot 000106$	$\cdot 000070$
2	$\cdot 000077$	$\cdot 000050$
3	$\cdot 000061$	$\cdot 000040$
4	$\cdot 000048$	$\cdot 000033$
5	$\cdot 000034$	$\cdot 000024$
6	$\cdot 000023$	$\cdot 000018$
7	$\cdot 000013$	$\cdot 000010$
8	$\cdot 000005$	$\cdot 000004$
9	$\cdot 000001$	$\cdot 000001$

These values, multiplied into the appropriate sums assured in column (2) of Tables III and IV, bring out a total amount of theoretical error in the "Δ values of the net premiums" of £996 at $2\frac{1}{2}$ per-cent, and of £698 at 3 per-cent. The actual errors, as set out at foot of column (9) of Tables III and IV, amount to £697 and £453 respectively, and are thus well within the amounts of theoretical error, deduced as above.

23. The approximate method, as here suggested, deduces the Δ values of the net premiums, not from the *actual* net premiums, scheduled in respect of each of the first ten years of duration, but from the product of the net premium, *deduced at the average age* w , into the sums assured. The error involved in the employment of the latter method has been investigated in formulas (17) to (21); and it may be useful similarly to investigate

the error arising from the employment of the actual (instead of the average) net premiums, the result of which investigation will, it is thought, entirely justify the selection of the other method as the best for practical adoption.

24. If the actual net premiums $\Sigma(S_{[x]+t}\pi_x)$ be ascertained for each value of t from 0 to 9 inclusive, we shall have, as the “ Δ value” of such net premiums—

$$\Sigma[(S_{[x]+t}\pi_{[x]})\phi_{[x]+t}] = \Sigma[S_{[x]+t}\pi_{[x]}](A_t + B_t c^{w_5} + C_t c^{2w_6}) \quad (22)$$

where w_5 and w_6 are deduced respectively by summation as under—

$$\frac{\sum_{x=20}^{x=65} (S_{[x]+t}\pi_{[x]} c^{2x})}{\Sigma(S_{[x]+t}\pi_{[x]})} = c^{w_5}; \quad \frac{\sum_{x=20}^{x=65} (S_{[x]+t}\pi_{[x]} c^{2x})}{\Sigma(S_{[x]+t}\pi_{[x]})} = c^{2w_6} \quad (23)$$

The values of the quantities c^{w_5} and c^{2w_6} , and of the corresponding average ages w_5 and w_6 , as deduced by formulas (23) from the illustrative valuation data in each year of duration, at $2\frac{1}{2}$ and 3 per-cent interest, are given in Table VII; also the general average values and ages, deduced from the aggregate actual net premiums in the first ten years' durations combined. It will be seen that the average ages are as under—

Per-cent	w_5	w_6
$2\frac{1}{2}$	42.2	47.0
3	42.3	47.2

If, in formula (22), the average age w , deduced, as before, from the values of $S_{[x]+t}c^x$, be employed instead of the true average ages w_5 and w_6 , we have, as the amount of the error—

$$\Sigma(S_{[x]+t}\pi_{[x]})[B_t(c^w - c^{w_5}) + C_t(c^{2w} - c^{2w_6})] \quad (24)$$

The average ages w_5 and w_6 , as given above, are both greater than the average age $w (=38.5)$; and there are, here, no compensatory errors on both sides of the average age w employed, as in formula (19) where the other method is adopted. Inserting the average values of c^w and c^{2w} , and the average values of c^{w_5} and c^{2w_6} , taken from the last line of Table VII, we have, at $2\frac{1}{2}$ per-cent—

$$\Sigma(S_{[x]+t}\pi_{[x]})[-12.402B_t - 3,652.19C_t] \quad (25)$$

and at 3 per-cent—

$$\Sigma(S_{[x]+t}\pi_{[x]})[-13.009B_t - 3,767.53C_t] \quad (26)$$

The values of the quantities in square brackets in formulas (25) and (26) are given in the following table :

Error in "Δ Value of Net Premiums" (the Values being deduced from the ACTUAL Net Premiums) ; applicable to a Net Premium of 1.

t	2½ PER-CENT	3 PER-CENT
	$-12\cdot402B_t - 3,652\cdot19C$	$-13\cdot009B_t - 3,767\cdot53C_t$
0	—·015,929	—·016,338
1	—·012,779	—·013,288
2	—·009,879	—·010,265
3	—·007,295	—·007,636
4	—·005,173	—·005,415
5	—·003,480	—·003,673
6	—·002,179	—·002,296
7	—·001,214	—·001,279
8	—·000,544	—·000,560
9	—·000,146	—·000,141

Multiplying these values into the totals of the actual net premiums in each year of duration, the amount of aggregate error is found to be equal to —£2,416 at 2½ per-cent, and —£2,364 at 3 per-cent. Comparing these amounts with those deduced in §(22), by the average net premium method, of +£996 and +£698 respectively, it will be seen that the magnitude of the error is much increased where the actual net premiums are employed.

25. This result is confirmed and elucidated by a comparison of the average ages, as deduced (1) by the formula

$$\frac{\Sigma(S_{[x]+t}\pi_{[x]}\bar{\phi}_{[x]+t})}{\Sigma(S_{[x]+t})} = \pi_{[w_7]}\bar{\phi}_{[w_7]+t} \quad \dots \quad (27)$$

(2) by the formula

$$\frac{\Sigma(S_{[x]+t}\pi_{[x]}\bar{\phi}_{[x]+t})}{\Sigma(S_{[x]+t}\pi_{[x]})} = \bar{\phi}_{[w_8]+t} \quad \dots \quad (28)$$

the quantities summed being taken from the detailed valuation results, worked out, as explained, in §(13), at all entry ages, and for each of the first ten years of duration. The average values thus deduced, and the corresponding ages w_7 and w_8 , are given in Table VIII at 2½ and 3 per-cent interest. It will be seen that the age w_7 varies from 38·1 to 38·8 and averages 38·3, whilst the age w_8 varies from 40·8 to 42·3, and averages 41·2. As the average age w , deduced from $S_{[x]+t}e^x$, and employed both for the

value of the sums assured and of the net premiums by the approximate method, varies from 38·3 to 38·5 and averages 38·5 (see column (10) of Table VI), it is clear that w_7 closely approximates to w , whilst w_3 differs by about 2·7 years. In other words, the employment of the net premium, *as tabulated at the average age w deduced from the weighted sums assured*, gives a much more accurate result than the employment of the *actual* net premiums, as entered in the valuation class-books.

26. The differences in the amounts of the theoretical and practical errors, as stated in §(19) in respect of the Δ values of the sums assured, and in §(22) in respect of the Δ values of the net premiums, appear to arise from two causes:—(1) the theoretical investigation gives the amount of deviation arising from the employment of the *approximate* value of the function $\phi_{[w]+t}$ at the average age w , whereas in practice (see Tables III and IV, column 4) the *true* values at such average age are employed; and it happens that where $w=38$ or 39, the true values so employed are, for almost all values of t , slightly less than the approximate values; (2) the average ages employed, as given in column (3) of Tables III and IV, based upon $S_{[x]+t}c^x$, differ (fractionally) from the true average ages, as deduced from the Δ value of the sums assured, or from the Δ value of the net premiums at the average age.

27. In Table IX, the several average functions deduced in the course of this Paper are set out in summary form, for convenient comparison, with the average entry ages, as deduced from each function, and the average ratio of increase in each case*; these ratios having been computed by the application of the formula given in § 7 *supra*, inserting in the formula the values, at interval ages, of the particular functions specified in column (3) of Table IX.

* In order to illustrate a fundamental principle, referred to by Mr. Lidstone, and other speakers, in the course of the discussion on the Paper, the several functions given in Table IX, have been arranged in the order of the average ratios in column (4), and the average ages in column (5).

TABLE I.

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0[M]

 $2\frac{1}{2}$ per-cent.

Age at Entry [x]	t = 0						t = 1						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\Phi_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	
20	·3013	·3013	·0010	1·50	·2087	·0003	3·33	·2087	·0009	1·56	20
21	·3012	·3023	·2090	·2096	21
22	·3015	·3033	·2092	·2106	22
23	·3020	·3045	·2097	·2116	23
24	·3025	·3058	·2105	·2128	24
25	·3030	·0006	4·33	·3072	·0015	1·60	·2111	·0010	2·30	·2141	·0014	1·57	25
26	·3036	·3087	·2121	·2155	26
27	·3050	·3104	·2139	·2170	27
28	·3067	·3122	·2156	·2186	28
29	·3086	·3141	·2167	·2205	29
30	·3098	·0026	1·73	·3163	·0024	1·54	·2186	·0023	1·26	·2224	·0022	1·50	30
31	·3124	·3187	·2209	·2246	31
32	·3147	·3212	·2232	·2269	32
33	·3176	·3240	·2259	·2295	33
34	·3212	·3271	·2284	·2323	34
35	·3242	·0045	1·38	·3304	·0037	1·49	·2326	·0029	2·10	·2353	·0033	1·48	35
36	·3287	·3341	·2355	·2386	36
37	·3330	·3380	·2395	·2422	37
38	·3379	·3423	·2442	·2461	38
39	·3432	·3470	·2486	·2503	39
40	·3496	·0062	1·56	·3521	·0055	1·51	·2537	·0061	1·31	·2549	·0049	1·51	40
41	·3558	·3576	·2598	·2598	41
42	·3632	·3637	·2661	·2652	42
43	·3707	·3702	·2724	·2711	43
44	·3787	·3773	·2792	·2774	44
45	·3878	·0097	1·38	·3850	·0083	1·47	·2866	·0080	1·31	·2842	·0074	1·41	45
46	·3975	·3933	·2946	·2916	46
47	·4073	·4023	·3028	·2995	47
48	·4183	·4121	·3123	·3080	48
49	·4295	·4226	·3211	·3171	49
50	·4411	·0134	1·16	·4338	·0122	1·39	·3313	·0105	1·20	·3268	·0104	1·31	50
51	·4545	·4460	·3418	·3372	51
52	·4679	·4591	·3531	·3482	52
53	·4821	·4731	·3643	·3600	53
54	·4964	·4880	·3766	·3724	54
55	·5126	·0155	...	·5039	·0169	...	·3891	·0126	...	·3854	·0136	...	55
56	·5281	·5208	·4017	·3990	56
57	·5441	·5387	·4147	·4130	57
58	·5611	·5575	·4291	·4275	58
59	·5792	·5772	·4433	·4422	59
60	·5977	·5977	·4570	·4570	60
Σ	15·8013	·2964	...	15·8009	·2964	..	11·5845	·2483	...	11·5847	·2483	...	Σ
Σ^2	...	8·7044	8·7048	7·1525	7·1523	...	Σ^2
A_t	·290,848,0	·199,082,5	A_t
B_t	·001,744,005,5	·001,607,909,3	B_t
C_t	·000,001,560,58	·000,001,960,89	C_t

TABLE I (continued).

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0[M]

 $2\frac{1}{2}$ per-cent.

Age at Entry [x]	t = 2						t = 3						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·1568	·0003	4·33	·1568	·0007	1·57	·1185	·0003	1·67	·1185	·0005	1·80	20
21	·1571	·1575	·1188	·1190	21
22	·1573	·1583	·1185	·1196	22
23	·1575	·1592	·1192	·1203	23
24	·1585	·1601	·1198	·1210	24
25	·1594	·0013	1·46	·1611	·0011	1·55	·1207	·0005	2·00	·1217	·0009	1·44	25
26	·1607	·1622	·1212	·1226	26
27	·1613	·1634	·1214	·1235	27
28	·1623	·1648	·1227	·1245	28
29	·1639	·1662	·1232	·1256	29
30	·1652	·0019	1·37	·1678	·0017	1·55	·1249	·0010	2·50	·1267	·0013	1·54	30
31	·1671	·1695	·1259	·1280	31
32	·1688	·1714	·1270	·1294	32
33	·1712	·1734	·1292	·1310	33
34	·1738	·1757	·1306	·1326	34
35	·1759	·0026	1·92	·1781	·0026	1·54	·1326	·0025	1·24	·1344	·0020	1·50	35
36	·1785	·1807	·1351	·1364	36
37	·1823	·1836	·1376	·1385	37
38	·1855	·1867	·1396	·1409	38
39	·1892	·1900	·1432	·1434	39
40	·1933	·0050	1·24	·1937	·0040	1·48	·1464	·0031	1·61	·1461	·0030	1·43	40
41	·1983	·1977	·1495	·1491	41
42	·2026	·2020	·1524	·1523	42
43	·2074	·2066	·1570	·1558	43
44	·2137	·2116	·1606	·1595	44
45	·2191	·0062	1·39	·2170	·0059	1·39	·1650	·0050	1·08	·1636	·0043	1·42	45
46	·2253	·2229	·1700	·1679	46
47	·2323	·2291	·1745	·1726	47
48	·2334	·2358	·1795	·1776	48
49	·2460	·2430	·1856	·1830	49
50	·2538	·0086	1·15	·2507	·0082	1·28	·1915	·0054	1·39	·1887	·0061	1·28	50
51	·2624	·2589	·1969	·1948	51
52	·2704	·2676	·2039	·2012	52
53	·2799	·2768	·2101	·2080	53
54	·2888	·2864	·2173	·2152	54
55	·2980	·0099	...	·2965	·0105	...	·2238	·0075	...	·2226	·0078	...	55
56	·3079	·3070	·2313	·2304	56
57	·3185	·3179	·2392	·2384	57
58	·3294	·3289	·2467	·2465	58
59	·3392	·3406	·2545	·2546	59
60	·3511	·3511	·2627	·2627	60
Σ	8·8281	·1943	...	8·8277	·1943	..	6·6481	·1442	...	6·6482	·1442	..	Σ
Σ²	...	5·5670	5·5674	4·1226	4·1225	...	Σ²
A _t	·149,088,9	·112,721,8	A _t
B _t	·001,289,833,8	·000,966,696,0	B _t
C _t	-·000,001,675,04	-·000,001,285,15	C _t

TABLE I (continued).

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0[M]

 $2\frac{1}{2}$ per-cent.

Age at Entry [x]	t = 4						t = 5						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\Phi_{[x]+t}$	Δ	Δ_{+5} Δ	$\Phi_{[x]+t}$	Δ	Δ_{+5} Δ	$\Phi_{[x]+t}$	Δ	Δ_{+5} Δ	$\Phi_{[x]+t}$	Δ	Δ_{+5} Δ	
20	·0867	·0000	...	·0867	·0004	1·50	·0599	·0002	1·00	·0599	·0003	1·33	20
21	·0867	·0871	·0601	·0602	21
22	·0867	·0875	·0605	·0605	22
23	·0871	·0880	·0608	·0608	23
24	·0879	·0885	·0610	·0611	24
25	·0880	·0002	5·50	·0890	·0006	1·50	·0611	·0002	5·00	·0615	·0004	1·50	25
26	·0882	·0896	·0613	·0619	26
27	·0888	·0903	·0618	·0624	27
28	·0894	·0910	·0624	·0628	28
29	·0905	·0918	·0630	·0634	29
30	·0910	·0011	1·82	·0926	·0009	1·56	·0630	·0010	...	·0640	·0006	1·50	30
31	·0921	·0935	·0640	·0646	31
32	·0936	·0945	·0644	·0653	32
33	·0939	·0956	·0649	·0660	33
34	·0953	·0968	·0659	·0669	34
35	·0968	·0020	1·30	·0981	·0014	1·50	·0670	·0009	2·00	·0678	·0009	1·67	35
36	·0988	·0995	·0679	·0687	36
37	·1003	·1010	·0694	·0698	37
38	·1022	·1027	·0711	·0709	38
39	·1042	·1045	·0720	·0722	39
40	·1062	·0026	1·23	·1065	·0021	1·48	·0735	·0018	1·17	·0735	·0015	1·47	40
41	·1088	·1086	·0753	·0750	41
42	·1114	·1109	·0767	·0765	42
43	·1141	·1134	·0784	·0782	43
44	·1167	·1160	·0807	·0801	44
45	·1201	·0032	1·38	·1189	·0031	1·42	·0823	·0021	1·19	·0820	·0022	1·32	45
46	·1233	·1220	·0844	·0842	46
47	·1267	·1254	·0870	·0864	47
48	·1309	·1289	·0901	·0889	48
49	·1351	·1328	·0925	·0915	49
50	·1385	·0044	1·30	·1368	·0044	1·25	·0957	·0025	1·20	·0943	·0029	1·28	50
51	·1429	·1412	·0982	·0972	51
52	·1473	·1458	·1008	·1004	52
53	·1518	·1506	·1042	·1037	53
54	·1571	·1557	·1079	·1071	54
55	·1619	·0057	...	·1610	·0055	...	·1115	·0030	...	·1107	·0037	...	55
56	·1676	·1665	·1145	·1144	56
57	·1728	·1722	·1186	·1182	57
58	·1784	·1779	·1211	·1220	58
59	·1832	·1836	·1256	·1258	59
60	·1893	·1893	·1295	·1295	60
Σ	4·8323	·1026	...	4·8323	·1026	...	3·3300	·0696	...	3·3303	·0696	...	Σ
Σ^2	...	2·9290	2·9290	1·9795	1·9792	...	Σ^2
A_t	·082,562,2	·057,047,2	A_t
B_t	·000,692,327,4	·000,477,476,8	B_t
C_t	—·000,000,954,47	—·000,000,668,73	C_t

TABLE I (continued).

Showing the true, and approximate, values of $\bar{\phi}_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0[M]

 $2\frac{1}{2}$ per-cent.

Age at Entry [x]	t=6						t=7						Age at Entry [x']
	True Values			Approximate Values			True Values			Approximate Values			
	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	0385	0385	0002	1.00	0219	0004	1.25	0219	0001	1.00	20
21	0383	0387	0223	0220	21
22	0392	0388	0223	0221	22
23	0390	0390	0217	0222	23
24	0387	0392	0221	0223	24
25	0394	0395	0002	2.00	0218	0005	...	0224	0001	2.00	25
26	0390	0397	0223	0225	26
27	0398	0400	0221	0227	27
28	0398	0403	0220	0228	28
29	0395	0406	0224	0230	29
30	0408	0410	0004	1.50	0222	0003	3.33	0232	0002	1.50	30
31	0403	0414	0225	0234	31
32	0409	0418	0229	0236	32
33	0417	0422	0233	0238	33
34	0422	0428	0237	0241	34
35	0423	0011	...	0433	0006	1.50	0239	0010	...	0244	0003	1.67	35
36	0434	0439	0249	0247	36
37	0447	0445	0247	0250	37
38	0447	0452	0249	0254	38
39	0459	0460	0260	0258	39
40	0472	0006	2.17	0468	0009	1.44	0264	0003	3.67	0262	0005	1.40	40
41	0478	0477	0267	0267	41
42	0490	0487	0278	0272	42
43	0505	0497	0274	0278	43
44	0510	0508	0284	0284	44
45	0523	0013	1.08	0521	0013	1.38	0295	0011	1.8	0290	0007	1.43	45
46	0536	0534	0306	0297	46
47	0557	0548	0310	0304	47
48	0571	0563	0322	0312	48
49	0586	0579	0327	0321	49
50	0608	0014	1.86	0596	0018	1.36	0339	0002	7.00	0330	0010	1.30	50
51	0622	0614	0341	0340	51
52	0637	0634	0355	0351	52
53	0662	0654	0368	0362	53
54	0688	0676	0374	0373	54
55	0696	0026	...	0698	0023	...	0388	0014	...	0386	0013	...	55
56	0722	0721	0402	0399	56
57	0742	0745	0419	0412	57
58	0771	0770	0428	0426	58
59	0793	0794	0441	0440	59
60	0818	0818	0454	0454	60
Σ	2.1168	0433	...	2.1166	0433	...	1.1835	0235	...	1.1833	0235	...	Σ
Σ^2	...	1.2370	1.2372	6779	6781	...	Σ^2
A'	036,759.2	020,995.7	A'
B'	000,291,251.0	000,151,164.1	B'
C'	-000,000,390.24	-000,000,181.07	C'

TABLE I (continued).

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0^[M]2 $\frac{1}{2}$ per-cent.

Age at Entry [x]	t = 8						t = 9						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·0099	·0002	...	·0099	·0000	...	·0025	·0001	...	·0025	·0000	...	20
21	·0101	·0099	·0026	·0025	21
22	·0097	·0100	·0027	·0025	22
23	·0099	·0100	·0021	·0025	23
24	·0094	·0100	·0027	·0025	24
25	·0097	·0001	2·00	·0101	·0000	...	·0027	·0025	·0001	...	25
26	·0098	·0101	·0023	·0026	26
27	·0095	·0102	·0028	·0026	27
28	·0103	·0103	·0023	·0026	28
29	·0097	·0103	·0024	·0026	29
30	·0099	·0002	...	·0104	·0001	1·00	·0025	·0000	...	·0026	·0000	...	30
31	·0101	·0105	·0025	·0026	31
32	·0103	·0106	·0025	·0026	32
33	·0099	·0106	·0025	·0027	33
34	·0106	·0108	·0026	·0027	34
35	·0108	·0001	2·00	·0109	·0001	2·00	·0026	·0000	...	·0027	·0000	...	35
36	·0109	·0110	·0026	·0027	36
37	·0110	·0111	·0027	·0028	37
38	·0113	·0112	·0027	·0028	38
39	·0114	·0114	·0027	·0028	39
40	·0116	·0002	...	·0116	·0002	1·00	·0029	·0000	...	·0028	·0001	1·00	40
41	·0118	·0118	·0029	·0029	41
42	·0119	·0120	·0028	·0029	42
43	·0120	·0122	·0030	·0030	43
44	·0123	·0124	·0030	·0030	44
45	·0131	·0001	2·00	·0127	·0002	2·00	·0030	·0009	...	·0031	·0001	1·00	45
46	·0132	·0129	·0039	·0032	46
47	·0135	·0132	·0032	·0032	47
48	·0137	·0136	·0033	·0033	48
49	·0139	·0139	·0031	·0034	49
50	·0148	·0002	6·00	·0143	·0004	1·25	·0033	·0008	...	·0035	·0001	2·00	50
51	·0150	·0147	·0041	·0036	51
52	·0161	·0151	·0034	·0037	52
53	·0155	·0156	·0044	·0038	53
54	·0167	·0161	·0035	·0039	54
55	·0169	·0012	...	·0167	·0005	...	·0044	·0000	...	·0040	·0002	...	55
56	·0181	·0172	·0044	·0042	56
57	·0182	·0178	·0046	·0044	57
58	·0189	·0185	·0046	·0046	58
59	·0194	·0192	·0049	·0048	59
60	·0199	·0199	·0050	·0050	60
Σ	·5207	·0100	...	·5207	·0100	...	·1287	·0025	...	·1287	·0025	...	Σ
Σ ²	...	·2952	·2952	·0763	·0763	...	Σ ²
A _t	·009,556,8	·002,429,7	A _t
B _t	·000,057,227,1	·000,011,670,8	B _t
C _t	·000,000,045,48	·000,000,000,36	C _t

TABLE II.

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0^[M]

3 per-cent.

Age at Entry [x]	t=0						t=1						Age at Entry [x']
	True Values			Approximate Values			True Values			Approximate Values			
	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·2729	·0003	3·67	·2729	·0010	1·60	·1886	·0011	1·55	·1886	·0009	1·67	20
21	·2732	·2739	·1897	·1895	21
22	·2736	·2750	·1902	·1905	22
23	·2742	·2762	·1910	·1916	23
24	·2750	·2776	·1915	·1928	24
25	·2760	·0011	1·91	·2790	·0016	1·56	·1924	·0017	1·24	·1941	·0015	1·47	25
26	·2771	·2806	·1941	·1956	26
27	·2786	·2823	·1951	·1971	27
28	·2798	·2842	·1965	·1988	28
29	·2817	·2863	·1989	·2007	29
30	·2841	·0021	2·00	·2885	·0025	1·48	·2006	·0021	1·67	·2027	·0022	1·55	30
31	·2862	·2910	·2027	·2049	31
32	·2893	·2936	·2053	·2073	32
33	·2921	·2966	·2077	·2099	33
34	·2948	·2997	·2107	·2128	34
35	·2992	·0042	1·50	·3032	·0037	1·54	·2142	·0035	1·49	·2159	·0034	1·47	35
36	·3034	·3069	·2177	·2193	36
37	·3082	·3110	·2218	·2229	37
38	·3127	·3155	·2256	·2269	38
39	·3178	·3203	·2310	·2313	39
40	·3242	·0063	1·44	·3256	·0057	1·49	·2362	·0052	1·60	·2360	·0050	1·50	40
41	·3305	·3313	·2414	·2410	41
42	·3374	·3375	·2473	·2465	42
43	·3452	·3442	·2533	·2525	43
44	·3527	·3515	·2608	·2589	44
45	·3618	·0091	1·42	·3594	·0085	1·45	·2675	·0083	1·17	·2658	·0075	1·39	45
46	·3709	·3679	·2758	·2733	46
47	·3809	·3771	·2834	·2813	47
48	·3917	·3870	·2927	·2899	48
49	·4026	·3976	·3022	·2991	49
50	·4144	·0129	1·25	·4090	·0123	1·34	·3117	·0097	1·43	·3090	·0104	1·28	50
51	·4273	·4213	·3214	·3194	51
52	·4403	·4344	·3320	·3305	52
53	·4545	·4483	·3441	·3422	53
54	·4689	·4631	·3564	·3544	54
55	·4836	·0161	...	·4788	·0165	...	·3678	·0139	...	·3673	·0133	...	55
56	·4997	·4953	·3817	·3806	56
57	·5165	·5126	·3941	·3943	57
58	·5327	·5306	·4080	·4083	58
59	·5504	·5492	·4219	·4223	59
60	·5683	·5683	·4362	·4362	60
Σ	14·7044	·2954	...	14·7043	·2954	...	10·8022	·2476	...	10·8020	·2476	...	Σ
Σ²	...	8·5959	8·5960	7·0820	7·0822	...	Σ²
A _t	·262,005,5	·178,661,9	A _t
B _t	·001,821,295,6	·001,657,567,5	B _t
C _t	— 000,001,925,72	— 000,002,196,42	C _t

TABLE II (continued).

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0^[M]

3 per-cent.

Age at Entry [x]	t=2						t=3						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·1426	·0005	1·60	·1426	·0006	1·83	·1080	·1080	·0005	1·80	20
21	·1431	·1432	·1078	·1085	21
22	·1434	·1441	·1083	·1091	22
23	·1436	·1450	·1089	·1097	23
24	·1446	·1459	·1093	·1104	24
25	·1456	·0008	2·50	·1470	·0011	1·61	·1096	·0008	1·12	·1112	·0009	1·33	25
26	·1464	·1481	·1104	·1121	26
27	·1475	·1493	·1116	·1130	27
28	·1492	·1507	·1122	·1140	28
29	·1502	·1521	·1139	·1151	29
30	·1516	·0020	1·10	·1537	·0018	1·67	·1150	·0009	2·11	·1163	·0012	1·67	30
31	·1536	·1555	·1159	·1175	31
32	·1555	·1574	·1174	·1190	32
33	·1579	·1594	·1195	·1206	33
34	·1603	·1617	·1210	·1223	34
35	·1632	·0022	1·86	·1641	·0027	1·44	·1230	·0019	1·74	·1241	·0020	1·50	35
36	·1654	·1668	·1249	·1261	36
37	·1683	·1696	·1276	·1282	37
38	·1724	·1728	·1302	·1306	38
39	·1759	·1762	·1336	·1332	39
40	·1801	·0041	1·61	·1799	·0039	1·51	·1361	·0033	1·27	·1359	·0030	1·43	40
41	·1842	·1838	·1394	·1389	41
42	·1891	·1882	·1435	·1422	42
43	·1940	·1928	·1461	·1457	43
44	·1989	·1979	·1502	·1494	44
45	·2046	·0066	1·35	·2033	·0059	1·39	·1552	·0042	1·31	·1535	·0043	1·40	45
46	·2112	·2092	·1594	·1578	46
47	·2170	·2154	·1645	·1626	47
48	·2246	·2222	·1697	·1676	48
49	·2315	·2292	·1741	·1730	49
50	·2394	·0089	1·03	·2370	·0082	1·24	·1814	·0055	1·36	·1788	·0060	1·27	50
51	·2483	·2452	·1869	·1848	51
52	·2564	·2538	·1924	·1913	52
53	·2648	·2629	·1992	·1981	53
54	·2732	·2724	·2072	·2051	54
55	·2840	·0092	...	·2824	·0102	...	·2133	·0075	...	·2126	·0076	...	55
56	·2932	·2926	·2208	·2202	56
57	·3027	·3032	·2281	·2281	57
58	·3136	·3139	·2357	·2360	58
59	·3242	·3246	·2436	·2439	59
60	·3351	·3351	·2516	·2516	60
Σ	8·2504	·1925	...	8·2502	·1925	...	6·2265	·1436	...	6·2261	·1436	...	Σ
Σ²	...	5·4887	5·4889	4·0891	4·0895	...	Σ²
A _t	134,807,0	102,083,1	A
B _t	·001,305,731,9	·000,981,866,9	B
C _t	—·000,001,784,03	—·000,001,363,51	C

TABLE II (continued.)

Showing the true, and approximate, values of $\bar{\Phi}_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0^[M]

3 per-cent.

Age at Entry [x]	t=4						t=5						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·0788	·0003	3·33	·0788	·0004	1·50	·0550	·0000	...	·0550	·0002	2·00	20
21	·0791	·0792	·0550	·0552	21
22	·0792	·0796	·0555	·0555	22
23	·0799	·0801	·0554	·0559	23
24	·0799	·0806	·0558	·0562	24
25	·0805	·0010	1·40	·0812	·0006	1·67	·0561	·0003	2·33	·0566	·0004	1·50	25
26	·0815	·0818	·0564	·0570	26
27	·0819	·0825	·0567	·0574	27
28	·0828	·0832	·0576	·0579	28
29	·0837	·0840	·0578	·0585	29
30	·0839	·0014	1·36	·0849	·0010	1·50	·0585	·0007	...	·0591	·0006	1·67	30
31	·0853	·0859	·0592	·0597	31
32	·0866	·0869	·0600	·0604	32
33	·0873	·0880	·0607	·0612	33
34	·0886	·0893	·0613	·0620	34
35	·0899	·0019	1·42	·0906	·0015	1·47	·0627	·0006	...	·0629	·0010	1·40	35
36	·0918	·0921	·0633	·0639	36
37	·0930	·0937	·0646	·0649	37
38	·0956	·0954	·0659	·0661	38
39	·0968	·0972	·0672	·0673	39
40	·0994	·0027	1·33	·0992	·0022	1·45	·0694	·0004	8·00	·0387	·0014	1·57	40
41	·1021	·1014	·0698	·0701	41
42	·1040	·1038	·0720	·0717	42
43	·1066	·1064	·0741	·0734	43
44	·1101	·1091	·0754	·0753	44
45	·1128	·0036	1·08	·1121	·0032	1·34	·0775	·0032	...	·0772	·0022	1·36	45
46	·1164	·1153	·0807	·0794	46
47	·1201	·1187	·0820	·0817	47
48	·1229	·1223	·0852	·0841	48
49	·1276	·1262	·0876	·0867	49
50	·1317	·0039	1·28	·1304	·0043	1·26	·0901	·0025	1·40	·0895	·0030	1·23	50
51	·1356	·1347	·0926	·0925	51
52	·1395	·1394	·0970	·0956	52
53	·1448	·1443	·0988	·0989	53
54	·1492	·1494	·1026	·1023	54
55	·1548	·0050	...	·1546	·0054	...	·1062	·0035	...	·1059	·0037	...	55
56	·1598	·1600	·1097	·1096	56
57	·1652	·1656	·1134	·1133	57
58	·1707	·1711	·1170	·1171	58
59	·1762	·1766	·1208	·1208	59
60	·1818	·1818	·1245	·1245	60
Σ	4·5374	·1030	...	4·5374	·1030	...	3·1311	·0695	...	3·1310	·0695	...	Σ
Σ²	...	2·9164	2·9164	1·9734	1·9735	...	Σ²
A _t	·074,485,8	·052,107,7	A _t
B _t	·000,720,655,1	·000,480,813,6	B _t
C _t	-·000,001,051,07	-·000,000,685,46	C _t

TABLE II (continued.)

Showing the true, and approximate, values of $\bar{\phi}_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

0·M]

3 per cent.

Age at Entry [x]	t=6						t=7						Age at Entry [x]
	True Values			Approximate Values			True Values			Approximate Values			
	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	$\bar{\phi}_{[x]+t}$	Δ	$\frac{\Delta_{+5}}{\Delta}$	
20	·0353	·0005	...	·0353	·0002	1·50	·0199	·0000	...	·0199	·0001	2·00	20
21	·0358	·0355	·0199	·0200	21
22	·0356	·0356	·0198	·0201	22
23	·0355	·0358	·0204	·0202	23
24	·0359	·0361	·0203	·0203	24
25	·0363	·0363	·0003	1·33	·0203	·0005	...	·0204	·0002	1·00	25
26	·0360	·0366	·0208	·0206	26
27	·0369	·0368	·0200	·0207	27
28	·0366	·0371	·0205	·0209	28
29	·0368	·0375	·0210	·0211	29
30	·0377	·0003	3·00	·0378	·0004	1·50	·0215	·0213	·0002	1·50	30
31	·0380	·0382	·0214	·0215	31
32	·0382	·0387	·0212	·0217	32
33	·0388	·0392	·0216	·0220	33
34	·0392	·0397	·0221	·0222	34
35	·0400	·0009	1·78	·0402	·0006	1·67	·0226	·0226	·0003	1·67	35
36	·0409	·0408	·0223	·0229	36
37	·0410	·0415	·0227	·0232	37
38	·0418	·0422	·0240	·0236	38
39	·0434	·0430	·0237	·0241	39
40	·0435	·0016	...	·0438	·0010	1·30	·0242	·0013	1·03	·0245	·0005	1·40	40
41	·0451	·0448	·0255	·0250	41
42	·0460	·0458	·0251	·0256	42
43	·0468	·0468	·0264	·0262	43
44	·0485	·0480	·0269	·0268	44
45	·0493	·0010	3·00	·0492	·0013	1·38	·0274	·0014	...	·0275	·0007	1·43	45
46	·0503	·0505	·0288	·0282	46
47	·0530	·0520	·0294	·0290	47
48	·0539	·0535	·0299	·0298	48
49	·0558	·0551	·0305	·0307	49
50	·0569	·0030	...	·0569	·0018	1·28	·0321	·0007	1·71	·0317	·0010	1·30	50
51	·0599	·0587	·0328	·0327	51
52	·0609	·0607	·0344	·0338	52
53	·0632	·0627	·0352	·0349	53
54	·0652	·0649	·0364	·0361	54
55	·0673	·0022	...	·0671	·0023	...	·0376	·0012	...	·0373	·0013	...	55
56	·0695	·0694	·0388	·0386	56
57	·0717	·0718	·0400	·0399	57
58	·0741	·0741	·0412	·0412	58
59	·0763	·0765	·0426	·0426	59
60	·0787	·0787	·0439	·0439	60
Σ	1·9951	·0434	...	1·9949	·0434	...	1·1151	·0240	...	1·1153	·0240	...	Σ
Σ²	...	1·2316	1·2318	·6848	·6846	...	Σ²
A _t	·033,494,2	·018,893,6	A _t
B _t	·000,300,616,3	·000,163,398,5	B _t
C _t	— ·000,000,428,76	— ·000,000,224,83	C _t

TABLE II (continued.)

Showing the true, and approximate, values of $\Phi_{[x]+t} = (\bar{a}_{[x]+t} - \bar{a}_{x+t})$; also, at every fifth age, the values of the first differences of these quantities, and of the ratio of increase of such differences.

$Q^{[M]}$ 3 per-cent.

Age at Entry [<i>x</i>]	<i>t</i> = 8						<i>t</i> = 9						Age at Entry [<i>x</i>]
	True Values			Approximate Values			True Values			Approximate Values			
	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	$\Phi_{[x]+t}$	Δ	$\frac{\Delta+5}{\Delta}$	
20	·0088	·0088	·0000	...	·0022	·0000	...	·0022	·0000	...	20
21	·0085	·0088	·0025	·0022	21
22	·0092	·0089	·0023	·0022	22
23	·0089	·0089	·0023	·0022	23
24	·0092	·0090	·0024	·0022	24
25	·0094	·0090	·0001	1·00	·0018	·0007	...	·0022	·0000	...	25
26	·0090	·0091	·0025	·0022	26
27	·0092	·0092	·0026	·0022	27
28	·0094	·0092	·0026	·0023	28
29	·0097	·0093	·0020	·0023	29
30	·0092	·0002	1·00	·0094	·0001	2·00	·0021	·0000	...	·0023	·0000	...	30
31	·0094	·0095	·0021	·0023	31
32	·0096	·0096	·0022	·0024	32
33	·0098	·0098	·0029	·0024	33
34	·0101	·0099	·0022	·0024	34
35	·0095	·0002	1·00	·0100	·0002	1·00	·0023	·0008	...	·0025	·0000	...	35
36	·0097	·0102	·0031	·0025	36
37	·0107	·0103	·0024	·0026	37
38	·0102	·0105	·0024	·0026	38
39	·0104	·0107	·0025	·0026	39
40	·0106	·0002	1·50	·0109	·0002	1·50	·0025	·0001	...	·0027	·0001	1·00	40
41	·0108	·0111	·0026	·0028	41
42	·0118	·0114	·0026	·0028	42
43	·0121	·0116	·0027	·0029	43
44	·0114	·0119	·0037	·0030	44
45	·0125	·0003	1·00	·0122	·0003	1·33	·0037	·0030	·0001	1·00	45
46	·0128	·0125	·0029	·0031	46
47	·0130	·0129	·0029	·0032	47
48	·0132	·0132	·0040	·0033	48
49	·0146	·0137	·0031	·0034	49
50	·0139	·0003	2·00	·0141	·0004	1·50	·0031	·0005	...	·0035	·0001	1·00	50
51	·0142	·0145	·0036	·0036	51
52	·0151	·0150	·0038	·0038	52
53	·0155	·0155	·0039	·0039	53
54	·0161	·0160	·0040	·0040	54
55	·0165	·0006	...	·0166	·0006	...	·0041	·0001	...	·0042	·0001	...	55
56	·0171	·0172	·0042	·0043	56
57	·0176	·0177	·0044	·0045	57
58	·0182	·0183	·0045	·0046	58
59	·0188	·0189	·0047	·0047	59
60	·0194	·0194	·0049	·0049	60
Σ	·4951	·0106	...	·4947	·0106	...	·1230	·0027	...	·1230	·0027	...	Σ
Σ²	...	·3003	·3007	·0779	·0779	...	Σ²
<i>A</i> _{<i>t</i>}	...			·008,360,1			...			·002,033,3			<i>A</i> _{<i>t</i>}
<i>B</i> _{<i>t</i>}	...			·000,073,641,3			...			·000,019,545,6			<i>B</i> _{<i>t</i>}
<i>C</i> _{<i>t</i>}	..			-·000,000,105,53			...			-·000,000,030,07			<i>C</i> _{<i>t</i>}

TABLE III.

Showing the approximate method of Valuation, with allowance for selection; also a comparison of the results with those deduced by a detailed Valuation.

OMI (MODEL OFFICE No. 2).

2½ PER-CENT.

Duration t	Sums Assured S_t	Average Age $[w]$	$\phi_{(w)+t}$	$S_t \cdot \phi_{(w)+t}$ $(2) \times (4)$	Δ VALUE OF SUMS ASSURED		$\pi_{[w]}$	Δ VALUE OF NET PREMIUMS		Δ NET LIABILITY	Error in Net Liability $(11)-(12)$	Duration t
(1)	(2)	(3)	(4)	(5)	(5) \times δ	True Value	(8)	(5) \times (8)	True Value	(6) + (9)	(12)	(1)
0	1,962,500	38.4	.3400	667,250	16,476	£ 16,431	.02464	£ 16,441	£ 16,215	£ 32,917	£ 32,646	0
1	1,903,402	38.4	.2460	468,237	11,562	£ 11,449	.02464	£ 11,537	£ 11,407	£ 23,099	£ 22,856	1
2	1,759,092	38.5	.1873	329,478	8,136	£ 8,026	.02472	£ 8,145	£ 8,046	£ 16,281	£ 16,072	2
3	1,663,137	38.5	.1414	235,168	5,807	£ 5,743	.02472	£ 5,813	£ 5,755	£ 11,620	£ 11,498	3
4	1,590,049	38.6	.1034	164,411	4,060	£ 3,995	.02480	£ 4,077	£ 3,995	£ 8,137	£ 7,990	4
5	1,531,766	38.5	.0715	109,521	2,704	£ 2,662	.02472	£ 2,707	£ 2,664	£ 5,411	£ 5,326	5
6	1,479,717	38.5	.0453	67,031	1,655	£ 1,639	.02472	£ 1,657	£ 1,633	£ 3,312	£ 3,272	6
7	1,433,223	38.5	.0255	36,547	902	£ 889	.02472	£ 903	£ 883	£ 1,805	£ 1,772	7
8	1,390,511	38.4	.0113	15,713	388	£ 385	.02464	£ 287	£ 383	£ 775	£ 768	8
9	1,351,152	38.3	.0027	3,648	90	£ 75	.02456	£ 90	£ 79	£ 180	£ 154	9
Totals Error	16,064,549 ...	— —	— —	2,097,004 ...	51,780 +486	51,204 ...	— —	51,757 +697	51,060 ...	103,537 +1,183	102,354 ...	Totals Error

TABLE IV.

Showing the approximate method of Valuation, with allowance for selection; also a comparison of the results with those deduced by a detailed Valuation.

0^m (MODEL OFFICE No. 2).

3 PER-CENT.

Duration t	Sums Assured S_t	Average Age [w]	\bar{q}_{w+t}	$S_t \cdot \bar{q}_{w+t}$ (2) \times (4)	Δ VALUE OF SUMS ASSURED		$\pi_{[w]}$	Δ VALUE OF NET PREMIUMS		Δ NET LIABILITY		Error in Net Liability (11) - (12)	Duration t
					(5) \times (6)	True Value		(5) \times (8)	True Value	(6) + (9)	True Value (7) + (10)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(1)
0	1,962,500	38.4	.3147	617,599	18,255	18,147	.02327	14,372	14,206	32,627	32,353	+274	0
1	1,903,402	38.4	.2278	433,595	12,817	12,656	.02327	10,090	10,018	22,907	22,674	+233	1
2	1,759,092	38.5	.1742	306,433	9,058	8,924	.02335	7,155	7,097	16,021	16,021	+192	2
3	1,663,137	38.5	.1319	219,368	6,484	6,368	.02335	5,122	5,072	11,606	11,440	+166	3
4	1,590,049	38.6	.0963	153,122	4,526	4,441	.02342	3,587	3,535	8,112	7,976	+136	4
5	1,531,766	38.5	.0666	102,016	3,015	2,958	.02335	2,382	2,348	5,397	5,306	+91	5
6	1,479,717	38.5	.0426	63,036	1,863	1,841	.02335	1,472	1,451	3,335	3,292	+43	6
7	1,433,223	38.5	.0239	34,254	1,013	1,017	.02335	800	798	1,813	1,815	-2	7
8	1,390,511	38.4	.0102	14,183	419	410	.02327	330	330	749	740	+9	8
9	1,351,152	38.3	.0025	3,378	100	94	.02319	78	79	178	173	+5	9
Totals Error	16,064,549 ...	- -	- -	1,946,984 ...	57,550 + 694	56,856 ...	- -	45,387 + 453	44,934 ...	102,937 + 1,447	101,790 ...	+ 1,147 ...	Totals Error

TABLE V.

Tabular Values of $S \times c^{-x-40}$, where $\log_{10} c = .039$, applicable to valuations at all rates of interest, and in respect of Assurances of all durations.

Entry age x	100	200	300	400	500	600	700	800	900	1000
15	11	21	32	42	53	64	74	85	95	106
16	12	23	35	46	58	70	81	93	104	116
17	13	25	38	51	63	76	89	101	114	127
18	14	28	42	55	69	83	97	111	125	139
19	15	30	46	61	76	91	106	121	137	152
20	17	33	50	66	83	100	116	133	149	166
21	18	36	54	73	91	109	127	145	163	182
22	20	40	60	79	99	119	139	159	179	199
23	22	43	65	87	109	130	152	174	196	217
24	24	48	71	95	119	143	166	190	214	238
25	26	52	78	104	130	156	182	208	234	260
26	28	57	85	114	142	171	199	228	256	284
27	31	62	93	124	156	187	218	249	280	311
28	34	68	102	136	170	204	238	272	306	340
29	37	74	112	149	186	223	261	298	335	372
30	41	81	122	163	204	244	285	326	367	407
31	45	89	134	178	223	267	312	357	401	446
32	49	98	146	195	244	293	341	390	439	488
33	53	107	160	213	267	320	373	427	480	533
34	58	117	175	233	292	350	408	467	525	583
35	64	128	191	255	319	383	447	511	574	638
36	70	140	209	279	349	419	489	559	628	698
37	76	153	229	306	382	458	535	611	687	764
38	84	167	251	334	418	501	585	668	752	836
39	91	183	274	366	457	548	640	731	823	914
40	100	200	300	400	500	600	700	800	900	1,000
41	109	219	328	438	547	656	766	875	985	1,094
42	120	239	359	479	598	718	838	957	1,077	1,197
43	131	262	393	524	655	786	916	1,047	1,178	1,309
44	143	286	430	573	716	859	1,003	1,146	1,289	1,432
45	157	313	470	627	783	940	1,097	1,253	1,410	1,567
46	171	343	514	686	857	1,028	1,200	1,371	1,543	1,714
47	188	375	562	750	937	1,125	1,312	1,500	1,687	1,875
48	205	410	615	820	1,026	1,231	1,436	1,641	1,846	2,051
49	224	449	673	898	1,122	1,346	1,571	1,795	2,020	2,244
50	245	491	736	982	1,227	1,473	1,718	1,964	2,209	2,455
51	269	537	806	1,074	1,343	1,611	1,880	2,148	2,417	2,685
52	294	588	881	1,175	1,469	1,763	2,056	2,350	2,644	2,938
53	321	643	964	1,285	1,607	1,928	2,250	2,571	2,892	3,214
54	352	703	1,055	1,406	1,758	2,109	2,461	2,812	3,164	3,516
55	385	769	1,154	1,538	1,923	2,308	2,692	3,077	3,461	3,846
56	421	841	1,262	1,683	2,104	2,524	2,945	3,366	3,787	4,207
57	460	921	1,381	1,841	2,301	2,762	3,222	3,682	4,142	4,603
58	504	1,007	1,510	2,014	2,518	3,021	3,524	4,028	4,532	5,035
59	551	1,102	1,652	2,203	2,754	3,305	3,856	4,406	4,957	5,508
60	603	1,205	1,808	2,410	3,013	3,615	4,218	4,820	5,423	6,026
61	659	1,318	1,978	2,637	3,296	3,955	4,614	5,273	5,933	6,592
62	721	1,442	2,163	2,884	3,606	4,327	5,048	5,769	6,490	7,211
63	789	1,578	2,367	3,155	3,944	4,733	5,522	6,311	7,100	7,889
64	863	1,726	2,589	3,452	4,315	5,178	6,041	6,904	7,767	8,630
65	944	1,888	2,832	3,776	4,720	5,664	6,608	7,552	8,497	9,441
66	1,033	2,066	3,098	4,131	5,164	6,197	7,229	8,262	9,295	10,328
67	1,130	2,260	3,389	4,519	5,649	6,779	7,909	9,038	10,168	11,298
68	1,236	2,472	3,708	4,944	6,180	7,416	8,652	9,881	11,124	12,359
69	1,352	2,704	4,056	5,408	6,760	8,112	9,464	10,817	12,169	13,521
70	1,479	2,958	4,437	5,916	7,396	8,875	10,354	11,833	13,312	14,791

TABLE VI.

Showing the average values of the valuation functions specified, when "weighted" by the Sums Assured, in respect of each of the first ten years of duration; also the average entry ages corresponding to such values: as deduced from the $O^{(M)}$ Model Office No. 2, with interest (where applicable) at $2\frac{1}{2}$, and 3, per-cent.

AVERAGE VALUES OF FUNCTIONS				CORRESPONDING AVERAGE AGES													
Duration <i>t</i>	<i>e^w</i>	<i>e^{2w₁}</i>	Interest at 2½ per-cent			Interest at 3 per-cent			<i>w₀</i>	<i>w₁</i>	Interest 2½ per-cent			Interest 3 per-cent			Duration <i>t</i>
			$\pi[w_2]$	$\pi[w_3]e^{w_3}$	$\pi[w_4]e^{2w_4}$	$\pi[w_2]$	$\pi[w_3]e^{w_3}$	$\pi[w_4]e^{2w_4}$			<i>w₂</i>	<i>w₃</i>	<i>w₄</i>	<i>w₂</i>	<i>w₃</i>	<i>w₄</i>	
			(4)	(5)	(6)	(7)	(8)	(9)			(10)	(11)	(12)	(13)	(14)	(15)	
(1)		(3)							(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(1)
0	31.591	2,428.4	.02305	1.0207	110.02	.02174	.9759	106.32	38.4	43.4	36.3	40.6	45.4	36.3	40.7	45.5	0
1	31.598	2,423.1	.02306	1.0200	109.60	.02174	.9754	105.95	38.4	43.4	36.3	40.6	45.4	36.3	40.7	45.5	1
2	31.825	2,445.5	.02312	1.0287	110.53	.02181	.9837	106.87	38.5	43.4	36.4	40.7	45.4	36.5	40.8	45.5	2
3	31.904	2,445.7	.02315	1.0305	110.35	.02183	.9857	106.73	38.5	43.4	36.4	40.7	45.4	36.5	40.8	45.5	3
4	31.913	2,437.6	.02316	1.0296	109.76	.02184	.9845	106.10	38.6	43.4	36.5	40.7	45.4	36.5	40.8	45.5	4
5	31.857	2,416.5	.02316	1.0253	108.42	.02184	.9803	104.74	38.5	43.4	36.5	40.7	45.4	36.5	40.7	45.4	5
6	31.739	2,384.5	.02314	1.0180	106.44	.02182	.9732	102.86	38.5	43.3	36.4	40.6	45.3	36.5	40.7	45.3	6
7	31.650	2,360.5	.02312	1.0123	105.03	.02181	.9677	101.45	38.5	43.3	36.4	40.6	45.2	36.5	40.6	45.3	7
8	31.485	2,321.5	.02309	1.0026	102.59	.02177	.9581	99.10	38.4	43.2	36.4	40.5	45.1	36.4	40.6	45.2	8
9	31.323	2,285.6	.02306	.9936	100.55	.02174	.9492	97.06	38.3	43.1	36.3	40.4	45.0	36.3	40.5	45.1	9
Average Values	31.694	2,399.8	.02311	1.0190	107.67	.02179	.9742	104.65	38.5	43.3	36.4	40.6	45.3	36.4	40.7	45.4	Average Ages.

TABLE VII.

Showing the Average Values of the valuation factors specified, when "weighted" by the Net Premiums, in respect of each of the first ten years of duration; also the Average Entry Ages corresponding to such values; as deduced from the $O^{[M]}$ Model Office No. 2, with interest at $2\frac{1}{2}$ and at 3 per-cent.

Duration	AVERAGE VALUES OF FUNCTIONS				AVERAGE ENTRY AGES				Duration
	2½ per-cent		3 per-cent		2½ per-cent		3 per-cent		
	c^{w_5}	c^{2w_6}	c^{w_5}	c^{2w_6}	w_5	w_6	w_5	w_6	
t									t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)
0	44.282	4,773.1	44.898	4,891.2	42.2	47.2	42.4	47.3	0
1	44.240	4,753.2	44.860	4,873.0	42.2	47.1	42.4	47.3	1
2	44.491	4,780.2	45.112	4,900.9	42.3	47.2	42.4	47.3	2
3	44.514	4,766.5	45.142	4,888.1	42.3	47.2	42.4	47.3	3
4	44.457	4,739.1	45.071	4,857.1	42.3	47.1	42.4	47.3	4
5	44.279	4,682.5	44.887	4,796.3	42.2	47.1	42.4	47.2	5
6	44.001	4,600.6	44.602	4,714.0	42.1	47.0	42.3	47.1	6
7	43.782	4,542.5	44.379	4,652.7	42.1	46.9	42.2	47.0	7
8	43.420	4,442.9	44.004	4,551.3	42.0	46.8	42.1	46.9	8
9	43.089	4,360.3	43.655	4,463.8	41.9	46.7	42.1	46.8	9
Average Values	44.096	4,659.1	44.703	4,774.5	42.2	47.0	42.3	47.2	Average Ages

TABLE VIII.

Showing the Average Values, and corresponding Average Entry Ages, as deduced by dividing the actual values of the net premiums, $\Sigma(S_{[x]+t}\pi_{[x]}\Phi_{[x]+t})$, (1) by the total sums assured; (2) by the total net premiums; according to the $O^{[M]}$ Model Office No. 2, in respect of each of the first ten years of duration; with interest at $2\frac{1}{2}$ and 3 per-cent.

Duration	AVERAGE VALUES OF FUNCTIONS				AVERAGE ENTRY AGES				Duration
	2½ per-cent		3 per-cent		2½ per-cent		3 per-cent		
	$\pi_{[w_7]}\bar{\Phi}_{[w_7]+t}$	$\bar{\Phi}_{[w_8]+t}$	$\pi_{[w_7]}\bar{\Phi}_{[w_7]+t}$	$\bar{\Phi}_{[w_8]+t}$	w_7	w_8	w_7	w_8	
t									t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)
0	·00826	·3585	·00724	·3330	38·1	41·4	38·2	41·4	0
1	·00599	·2599	·00526	·2421	38·2	41·0	38·3	41·1	1
2	·00457	·1978	·00403	·1850	38·2	40·9	38·3	41·2	2
3	·00346	·1495	·00305	·1397	38·3	41·0	38·3	41·1	3
4	·00251	·1085	·00222	·1018	38·1	40·9	38·3	40·9	4
5	·00174	·0751	·00153	·0702	38·1	40·9	38·2	41·2	5
6	·00110	·0477	·00098	·0449	38·2	40·8	38·3	40·9	6
7	·00062	·0267	·00056	·0255	38·2	40·8	38·8	42·3	7
8	·00028	·0119	·00024	·0109	38·5	42·3	38·5	41·1	8
9	·00006	·0025	·00006	·0027	(38·2)	(41·2)	38·8	(41·2)	9
Average Values	—	—	—	—	38·2	41·2	38·4	41·2	Average Ages

TABLE IX.

Summary of Average Functions deduced in the course of the Paper, arranged according to value of Average Ratios, and Entry Ages. (The values involving $\pi_{[x]}$ are taken at 3 per-cent interest.)

DATA SUMMED FOR ALL VALUES OF x FROM 20 TO 65 INCLUSIVE, IN RESPECT OF A GIVEN VALUE OF t		Resulting Average Function	Average Ratio of Increase	Average Entry Age	Reference		
"Weighted" Function.	Divisor				Formula	Table	Column
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$S_{[x]+t}\pi_{[x]}$	$S_{[x]+t}$	$\pi_{[w_2]}$	1.0515	36.4	(18)	VI	(15)
$S_{[x]+t}\phi_{[x]+t}$		$\phi_{[w_2]+t}$	1.0776	38.0	(15)	Table in	§(7)
$S_{[x]+t}\pi_{[x]}\phi_{[x]-t}$		$\pi_{[w_2]}\phi_{[w_2]}$	1.0904	38.2	(27)	VIII	(8)
$S_{[x]+t}c^x$		c^{w_2}	1.0940	38.5	(16)	VI	(10)
$S_{[x]+t}\pi_{[x]}c^x$		$\pi_{[w_2]}c^{w_2}$	1.1379	40.7	(18)	VI	(16)
$S_{[x]+t}c^{2x}$		c^{2w_2}	1.1967	43.3	(16a)	VI	(11)
$S_{[x]+t}\pi_{[x]}c^{2x}$		$\pi_{[w_2]}c^{2w_2}$	1.2455	45.4	(18)	VI	(17)
$S_{[x]+t}\pi_{[x]}\phi_{[x]+t}$	$S_{[x]+t}\pi_{[x]}$	$\phi_{[w_3]+t}$	1.0776	41.2	(28)	VIII	(9)
$S_{[x]+t}\pi_{[x]}c^x$		c^{w_3}	1.0940	42.3	(23)	VII	(8)
$S_{[x]+t}\pi_{[x]}c^{2x}$		c^{2w_3}	1.1967	47.2	(23)	VII	(9)

APPENDIX.

The function to be investigated is equal to

$$\phi_{[x]+t} = \langle \bar{a}_{[x]+t} - \bar{a}_{x+t} \rangle = \int_{r=0}^{r=\omega-x-t} \left[v^r ({}_np_{[x]+t} - {}_np_{x+t}) \right] dr.$$

Now, from the formulas employed by Mr. G. F. Hardy in the graduation of the O^[M] Table (see Account of Principles and Methods, etc., p. 158, formulas 21 to 23), we have

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t. \quad (1)$$

where $f_t = m(10-t)^2 + m'(c')^t$, and $\psi_t = n(10-t)^2$,

also $m = .000040955$, $m' = .00112$, $n = .02386$, $c' = .24$

$\beta = .000046635$, and $c = 1.0939564$.

It follows from formula (1) that—

$$\log_{10} l_{[x]+t+r} = \log_{10} l_{x+t+r} - f_{t+r} - \beta c^x \psi_{t+r} \quad . \quad . \quad . \quad (2)$$

where the two last terms disappear for values of $(t+r) = \text{or} > 10$.

Deducting (1) from (2), we have—

$$\log_{10} ({}_r p_{[x]+t}) = \log_{10} ({}_r p_{x+t}) + (f_t - f_{t+r}) + \beta c^x (\psi_t - \psi_{t+r}) \quad . \quad . \quad (3)$$

$$\text{But } \log_{10} ({}_r p_{x+t}) = \log_{10} p_{x+t} + \log_{10} p_{x+t+1} + \quad . \quad . \quad + \log_{10} p_{x+t+r-1}$$

which (since $\log_{10} p_{x+t} = -a - \beta c^{x+t}$, where $a = .0026111$),

$$= -ra - \beta c^{x+t} \frac{c^r - 1}{c - 1},$$

and formula (3) becomes

$$\begin{aligned} \log_{10} ({}_r p_{[x]+t}) = & -ra + (f_t - f_{t+r}) \\ & - \beta c^x \left[\frac{c^t (c^r - 1)}{c - 1} - \psi_t + \psi_{t+r} \right] \quad . \quad . \quad . \quad (4) \end{aligned}$$

Similarly, for the ultimate function, we have

$$\log_{10} ({}_r p_{x+t}) = -ra - \beta c^{x+t} \frac{c^r - 1}{c - 1} \quad . \quad . \quad . \quad (5)$$

Representing now, for conciseness

$-ra$	as	a
$(f_t - f_{t+r})$	as	f
$-\beta c^x$	as	C
$\frac{c^t (c^r - 1)}{c - 1}$	as	κ
$-(\psi_t - \psi_{t+r})$	as	ψ
$\log_e 10$	as	M^{-1} ,

and

formulas (4) and (5) become, respectively—

$$\log_{10} ({}_r p_{[x]+t}) = a + f + C(\kappa + \psi) \quad . \quad . \quad . \quad (6)$$

and

$$\log_{10} ({}_r p_{x+t}) = a + C\kappa \quad . \quad . \quad . \quad (7)$$

Expressing these formulas in exponential form, we have—

$$\begin{aligned} {}_r p_{[x]+t} &= 10^{a+f+C(\kappa+\psi)} = e^{[a+f+C(\kappa+\psi)]M^{-1}} \\ &= 1 + [a+f+C(\kappa+\psi)]M^{-1} \\ &\quad + [a+f+C(\kappa+\psi)]^2 \frac{M^{-2}}{2} + \quad . \quad . \quad . \quad (8) \end{aligned}$$

and

$$\begin{aligned} {}_r p_{x+t} &= 10^{a+C\kappa} = e^{[a+C\kappa]M^{-1}} \\ &= 1 + [a+C\kappa]M^{-1} + [a+C\kappa]^2 \frac{M^{-2}}{2} + \quad . \quad . \quad . \quad (9) \end{aligned}$$

Deducting (9) from (8), we have—

$$({}_rp_{[x]+t} - {}_rp_{x+t}) = (f + C\psi)M^{-1} + [2af + f^2 + C(2a\psi + 2f\psi + 2f\kappa) + C^2(2\kappa\psi + \psi^2)] \frac{M^{-2}}{2} \dots \quad (10)$$

+ the difference of terms in (8) and (9), involving the third and higher powers. Arranging formula (10) according to ascending powers of C , we have—

$$\begin{aligned} ({}_rp_{[x]+t} - {}_rp_{x+t}) = & f \cdot M^{-1} + (2af + f^2) \frac{M^{-2}}{2} + \dots \\ & + C \left[\psi \cdot M^{-1} + 2(a\psi + f\psi + f\kappa) \frac{M^{-2}}{2} + \dots \right] \\ & + C^2 \left\{ [2\kappa\psi + \psi^2] \frac{M^{-2}}{2} + \dots \right\} \\ & + \dots \dots \dots \quad (11) \end{aligned}$$

As $C = -\beta c^x$, this expression is in the form

$$F_0 c^0 + F_1 c^x + F_2 c^{2x} + \dots$$

where F_0, F_1, F_2, \dots are each functions of both t and r . (See Mr. Lidstone's demonstration of a general formula for F_n , on p. 83).

If now the values of the quantities a, f, C, κ , and ψ be introduced in formula (11), and the whole expression be multiplied by v^r , and the several terms be then summed for successive values of r , from 0 to $\omega - x - t$, we shall obtain an approximate value of $\bar{\phi}_{[x]+t}$, assuming that the series in formula (11) are convergent within practical limits, in respect of the terms involving ascending powers both of C and of $\frac{1}{M}$. I find by calculation that in practice a correct numerical value of $\bar{\phi}_{[x]+t}$ cannot be obtained from the terms to the extent given above in formula (11), where the probabilities are taken throughout the whole of life; and it would appear that an extension, and probably a considerable extension, of the terms as stated in the formula, would be required, to bring out the desired value with even approximate accuracy.

If, however, the period of selection be alone dealt with, that is, by evaluating the functions of r in formula (11), for values of r from 0 to $(10 - t)$ inclusive, I find that an approximate value can be deduced for $|_{10-t} \bar{\phi}_{[x]+t} = |_{10-t} \bar{u}_{[x]+t} - |_{10-t} \bar{u}_{x+t}$, taking the terms specified above in formula (11), up to those involving the second power only. The method adopted was to deal separately

with each term specified in formula (11), as a function of r , say u_r , and then ascertain the value of $\int_0^{10-t} v^r u_r dr$, in each case. This is equivalent to the present value of a continuous variable term annuity-certain; and the general formula adopted was:

$$\int_0^n v^r u_r dr = (v\bar{a})_n = \frac{u_0}{\delta} + \frac{1}{\delta^2} \cdot \frac{d}{dr} u_0 + \frac{1}{\delta^3} \frac{d^2}{dr^2} u_0 + \dots$$

$$- v^n \left\{ \frac{u_n}{\delta} + \frac{1}{\delta^2} \frac{d}{dr} u_n + \frac{1}{\delta^3} \frac{d^2}{dr^2} u_n + \dots \right\} \dots \dots \dots (12)$$

(This useful and powerful formula does not appear to be given in the standard Text-Books on Interest and Annuities-Certain, though readily deduced from first principles; but the analogous formula for the present value of a variable term annuity payable yearly, expressed in terms of the initial payment and its successive finite differences

$$(va)_n = \frac{u_1}{i} + \frac{\Delta u_1}{i^2} + \frac{\Delta^2 u_1}{i^3} + \dots - v^n \left\{ \frac{u_{n+1}}{i} + \frac{\Delta u_{n+1}}{i^2} + \frac{\Delta^2 u_{n+1}}{i^3} + \dots \right\}$$

is, of course, well known).

The several terms of formula (11) having each been dealt with as above, the value of ${}_{10-t}\bar{\Phi}_{[x]+t}$ was approximately deduced as the sum of a series of continuous variable term annuities-certain, calculated at various special rates of interest (arising from combinations of v with c , c' , and c'^2), and with variations in the amounts and incidence of the successive payments, expressed in the form of functions of r of the first four orders (since the term f^2 includes r^4). Taking the particular case of $x=40$, $t=0$, with interest at 3 per-cent, an approximate value of ${}_{10-t}\bar{\Phi}_{[x]+t}$ was thus deduced of .1156, as compared with the value of .1166, calculated from the O^[M] Tables.

The above process is laborious, and not very satisfactory. The formulas can, however, be much simplified, and the work reduced, by eliminating the terms involving a in formulas (8) to (11). Since $a = -ra = r \log_{10} s$ (where s is the Makeham constant) we have, in formulas (8) and (9),

$${}_r p_{[x]+t} = 10^{a+f+C(\kappa+\psi)} = s^r 10^{f+C(\kappa+\psi)},$$

and

$${}_r p_{x+t} = 10^{a+C\kappa} = s^r 10^{C\kappa};$$

so that, in both cases (as is well known), the annuity-value can be computed at the special rate i' , where $(1+i')^{-1} = sv$; and, similarly, in applying the method illustrated by formula (12),

$\delta = \log_e(1+i)$ will be replaced by $\delta' = \log_e(1+i') = aM^{-1} + \delta$. By thus eliminating the terms involving a , the series given in formulas (10) and (11) will be much simplified, and, when arranged in ascending powers of C , that is of $-\beta e^x$, will run according to the following scheme :

	$C^0 \times$	$+ C^1 \times$	$+ C^2 \times$	$+ C^3 \times$	$+ C^4 \times$	\dots
$\frac{M^{-1}}{1} \left[\begin{array}{c} 1 \\ 1 \end{array} \right]$	f	$(\kappa + \psi) - \kappa$				\dots
$+$						
$\frac{M^{-2}}{2} \left[\begin{array}{c} 2 \\ 2 \end{array} \right]$	f^2	$2f(\kappa + \psi)$	$(\kappa + \psi)^2 - \kappa^2$			
$+$						
$\frac{M^{-3}}{3} \left[\begin{array}{c} 3 \\ 3 \end{array} \right]$	f^3	$3f^2(\kappa + \psi)$	$3f(\kappa + \psi)^2$	$(\kappa + \psi)^3 - \kappa^3$		
$+$						
$\frac{M^{-4}}{4} \left[\begin{array}{c} 4 \\ 4 \end{array} \right]$	f^4	$4f^3(\kappa + \psi)$	$6f^2(\kappa + \psi)^2$	$4f(\kappa + \psi)^3$	$(\kappa + \psi)^4 - \kappa^4$	
$+$						
\dots	\dots	\dots	\dots	\dots	\dots	\dots
$+$						
$\frac{M^{-m}}{m} \left[\begin{array}{c} m \\ m \end{array} \right]$	f^m	$\frac{m}{m-1} f^{m-1}(\kappa + \psi)$	$\frac{m}{m-2} f^{m-2}(\kappa + \psi)^2$	$\frac{m}{m-3} f^{m-3}(\kappa + \psi)^3$	$\frac{m}{m-4} f^{m-4}(\kappa + \psi)^4$	\dots
$+$						

This shows clearly the law followed by the successive terms; and the general term, say the coefficient of C^n involving the m th power of M^{-1} , will be equal, where $m > n$, to

$$\frac{M^{-m}}{[m]} \cdot \frac{[m]}{[m-n][n]} f^{m-n} (\kappa + \psi)^n = \frac{M^{-m}}{[m-n][n]} f^{m-n} (\kappa + \psi)^n,$$

and, where $m = n$, to $\frac{M^{-n}}{[n]} [(\kappa + \psi)^n - \kappa^n]$.

Summing these expressions for all values of m from n to ∞ , we have

$$\sum_{m=n}^{\infty} \left\{ \frac{M^{-m}}{[m-n][n]} f^{m-n} \right\} \frac{(\kappa + \psi)^n}{[n]} - \frac{M^{-n}}{[n]} \kappa^n,$$

which can be shown to be equal to

$$\begin{aligned} \frac{M^{-n}}{[n]} \left[\left\{ \frac{f^0 M^0}{[0]} + \frac{f M^{-1}}{[1]} + \frac{f^2 M^{-2}}{[2]} + \dots \right\} (\kappa + \psi)^n - \kappa^n \right] \\ = \frac{M^{-n}}{[n]} [e^{f M^{-1}} (\kappa + \psi)^n - \kappa^n]. \end{aligned}$$

This is the coefficient of C^n , and, multiplying this coefficient by $(-\beta)^n$, we have, as the coefficient of $c^{n,x}$,

$$F'_n = \frac{(-\beta)^n M^{-n}}{[n]} [e^{f M^{-1}} (\kappa + \psi)^n - \kappa^n],$$

this expression agreeing with that deduced by Mr. Lidstone, after eliminating from the latter the term involving a .*

It may be added that the quantity $C\kappa$, which enters, as shown in formulas (6) and (7) above, into the values of both select and ultimate probabilities, will be gathered up and commuted, as it were, if the select probability is expressed in terms of the ultimate probability. Thus, we have—

$${}_r p_{[x]+t} = 10^{a+f+C(\kappa+\psi)}; \quad {}_r p_{x+t} = 10^{a+C\kappa};$$

whence

$${}_r p_{[x]+t} = {}_r p_{x+t} (10^{f+C\psi})$$

$$\text{and} \quad ({}_r p_{[x]+t} - {}_r p_{x+t}) = {}_r p_{x+t} (10^{f+C\psi} - 1) \quad . \quad . \quad . \quad (13)$$

Here the quantity by which ${}_r p_{x+t}$ is multiplied can be expanded, by the exponential theorem, in a series of ascending powers of C , that is of c^x , which series must be, ultimately, convergent; but,

* The general formula, deduced as above, was arrived at after I had seen Mr. Lidstone's more elegant demonstration of the same result.

as c^x enters also in ascending powers into the ultimate probability ${}_r p_{x+t}$, this would not satisfactorily express the function under investigation in the form of a series in ascending powers of c^x .

I have calculated the numerical values of the quantity by which ${}_r p_{x+t}$ is multiplied in formula (13), for different values of x , t , and r . Inserting the full values of f , c , and ψ , the exponent becomes

$$f + C\psi = (m + n\beta c^x)[2r(10-t) - r^2] - m'(c'^t)(c'^r - 1) \quad (14)$$

for values of r from 0 to $(10-t)$; whilst for higher values of r , the exponent becomes

$$f + C\psi = (m + n\beta c^x)(10-t)^2 + m'(c'^t) \quad (15)$$

which is constant, and independent of r . Introducing interest, and summing for all integral values of r to the end of life, we then have

$$\begin{aligned} \phi_{[x]+t} = & \sum_{r=1}^{r=10-t} [v^r {}_r p_{x+t} (10^{(m+n\beta c^x)[2r(10-t)-r^2]-m'(c'^t)(c'^r-1)} - 1)] \\ & + {}_{10-t}|a_{x+t} (10^{(m+n\beta c^x)(10-t)^2+m'(c'^t)} - 1) \quad (16) \end{aligned}$$

I have computed the value of the exponent of 10, in formula (16), over the first $(10-t)$ years, and taken out its antilogarithm, for values of $x=20, 40$ and 60 , and for values of $t=0, 1, 2 \dots 9$, in respect of each value of r from 1 to $(10-t)$; and also the value of the constant exponent and its antilogarithm, applicable to all higher values of r , for the same values of x and t .

The results are given in Table X., and represent, during the period of selection, the quantities by which the values of $\frac{D_{x+t+r}}{D_{x+t}}$, deduced, at any rate of interest, from the ultimate table, may be multiplied, to give, as it were, the varying life annuity payments, the value of which would be equal to that of a uniform life annuity, according to the select table;—also, after the expiration of the period of selection, the constant quantity by which the value of a deferred annuity (at any rate of interest) according to the ultimate table, may be multiplied, to give the value of the deferred annuity by the select table.

It will be seen from formula (16) that, if we deduct unity from the tabular results in Table X, we shall obtain the quantities by which $\frac{D_{x+t+r}}{D_{x+t}}$, during the period of selection, and ${}_{10-t}|a_{x+t}$,

TABLE X.

Showing the Values, according to the O^[M] Table, of

$$10^{(m+n\beta c^x)(2r(10-t)-r^2)-m'(c^t)\chi(c^t-1)} = \frac{D_{[x]+t+r}}{D_{[x]+t}} \cdot \frac{D_{x+t}}{D_{x+t+r}},$$

for all Values of r, from 1 to (10-t) inclusive; applicable to all rates of interest. $x=20.$

r	$t=0$	1	2	3	4	5	6	7	8	9
1	1.004053	1.002339	1.001761	1.001455	1.001214	1.000990	1.000769	1.000549	1.000329	1.00011
2	1.006402	1.004103	1.003218	1.002671	1.002205	1.001760	1.001319	1.000878	1.000439	...
3	1.008173	1.005564	1.004436	1.003666	1.002976	1.002309	1.001648	1.000988
4	1.009640	1.006785	1.005430	1.004435	1.003526	1.002629	1.001758
5	1.010866	1.007782	1.006203	1.004986	1.003857	1.002749
6	1.011866	1.008557	1.006756	1.005317	1.003967
7	1.012645	1.009110	1.007087	1.005428
8	1.013200	1.009443	1.007198
9	1.013534	1.009553
10	1.013645

 $x=40.$

r	$t=0$	1	2	3	4	5	6	7	8	9
1	1.005534	1.003662	1.002927	1.002465	1.002069	1.001689	1.001313	1.000937	1.000562	1.00018
2	1.009216	1.006600	1.005400	1.004540	1.003762	1.003004	1.002251	1.001500	1.000750	...
3	1.012170	1.009081	1.007480	1.006236	1.005079	1.003944	1.002814	1.001688
4	1.014666	1.011169	1.009182	1.007557	1.006021	1.004508	1.003002
5	1.016765	1.012877	1.010506	1.008501	1.006587	1.004696
6	1.018483	1.014207	1.011453	1.009068	1.006775
7	1.019819	1.015156	1.012022	1.009257
8	1.020775	1.015728	1.012212
9	1.021349	1.015913
10	1.021540

 $x=60.$

r	$t=0$	1	2	3	4	5	6	7	8	9
1	1.014506	1.011670	1.009985	1.008577	1.007236	1.005913	1.004595	1.003280	1.001966	1.0006
2	1.026346	1.021772	1.018647	1.015884	1.013191	1.010535	1.007890	1.005252	1.002623	...
3	1.036593	1.030535	1.026018	1.021881	1.017846	1.013849	1.009857	1.005911
4	1.045484	1.037991	1.032084	1.026756	1.021184	1.015842	1.010533
5	1.053048	1.044129	1.036826	1.029943	1.023192	1.016508
6	1.059274	1.048926	1.040226	1.031968	1.023863
7	1.064142	1.052366	1.042272	1.032644
8	1.067632	1.054435	1.042955
9	1.069731	1.055126
10	1.070432

Showing the Values, according to the O^[M] Table, of

$$10^{(m+n\beta c^x)(10-t)^2+m'(c^t)} = \frac{10-t}{10-t} \frac{a_{[x]+t}}{a_{x+t}},$$

applicable to the period after the expiration of selection, at all rates of interest. $x=20.$

r	$t=0$	1	2	3	4	5	6	7	8	9
(11-t) & upwards	1.013645	1.009553	1.007198	1.005428	1.003967	1.002749	1.001758	1.000988	1.000439	1.00011

 $x=40.$

(11-t) & upwards	1.021540	1.015913	1.012212	1.009257	1.006775	1.004696	1.003002	1.001688	1.000750	1.00018
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 $x=60.$

(11-t) & upwards	1.070432	1.055126	1.042955	1.032644	1.023863	1.016508	1.010533	1.005911	1.002623	1.0006
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after that period, may be multiplied, to arrive at the terms, which, when summed for values of r , will give the value of

$$\phi_{[x]+t} = a_{[x]+t} - a_{x+t}.$$

This represents, by the method of finite integration followed above, the difference between the select and ultimate *yearly* annuities; but the value of $\phi_{[x]+t}$, for continuous payments, can readily be obtained by the formula

$$\phi_{[x]+t} = \phi_{[x]+t} + \frac{\mu_x - \mu_{[x]}}{12} \quad . \quad . \quad . \quad (17)$$

Mr. Lidstone has kindly supplied the following demonstration, showing how the values of F_0 , F_1 , F_2 , . . . can be expressed in compact form by means of Taylor's Theorem. From formulas (6) and (7) in the above appendix, we have—

$${}_r p_{[x]+t} - {}_r p_{x+t} = e^{(a+f+C(\kappa+\psi))M^{-1}} - e^{(a+C\kappa)M^{-1}}.$$

Regarding this as a function of C , say u_C , and differentiating we shall have—

$$u'_C = (\kappa + \psi) M^{-1} e^{(a+f+C(\kappa+\psi))M^{-1}} - \kappa M^{-1} e^{(a+C\kappa)M^{-1}}$$

$$u''_C = (\kappa + \psi)^2 M^{-2} e^{(a+f+C(\kappa+\psi))M^{-1}} - \kappa^2 M^{-2} e^{(a+C\kappa)M^{-1}}$$

$$u'''_C = (\kappa + \psi)^3 M^{-3} e^{(a+f+C(\kappa+\psi))M^{-1}} - \kappa^3 M^{-3} e^{(a+C\kappa)M^{-1}},$$

and so on.

Putting $C=0$, we have—

$$u_0 = e^{(a+f)M^{-1}} - e^{aM^{-1}}$$

$$u'_0 = (\kappa + \psi) M^{-1} e^{(a+f)M^{-1}} - \kappa M^{-1} e^{aM^{-1}},$$

and so on.

Hence, by Taylor's Theorem, in the particular case known as Maclaurin's Theorem,

$$u_C = u_0 + C u'_0 + \frac{C^2}{2} u''_0 + \dots$$

$$= [e^{(a+f)M^{-1}} - e^{aM^{-1}}] + C[(\kappa + \psi) M^{-1} e^{(a+f)M^{-1}} - \kappa M^{-1} e^{aM^{-1}}] + \dots$$

$$= e^{aM^{-1}} [(e^{fM^{-1}} - 1) + C\{(\kappa + \psi) M^{-1} e^{fM^{-1}} - \kappa M^{-1}\}]$$

$$+ \frac{C^2}{2} \{(\kappa + \psi)^2 M^{-2} e^{fM^{-1}} - \kappa^2 M^{-2}\}$$

$$+ \dots \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad]$$

Thus, since $C = \beta e^x$, and F_n is the coefficient of e^{nx} ,

$$F_n = \frac{(-\beta)^n e^{aM^{-1}} M^{-n}}{[n]} [(\kappa + \psi)^n e^{fM^{-1}} - \kappa^n],$$

where it is to be remembered that a is a function of r , and κ , ψ , f , are functions of both t and r .

ABSTRACT OF THE DISCUSSION.

MR. G. J. LIDSTONE, in opening the discussion, said he thought that if the progress of actuarial science during the last fifty years were considered, nothing was more striking than the development in the theory and the practical application of Select Tables. The effects of selection, of course, were observed at a very early period in the history of the science. In 1850, Mr. J. A. Higham read his classic paper on the subject (*J.I.A.*, i, p. 179), which was followed in 1867 by Dr. Sprague's paper on the rate of mortality as influenced by the duration of assurance (*J.I.A.*, xv, p. 328). In 1876, the first analysed annuity-values were published by Mr. Berridge and Mr. King independently (*J.I.A.*, xix, p. 351 and p. 381), and in 1881 Dr. Sprague presented his well-known H^M Select Tables (*J.I.A.*, xxi, p. 229), the form of which was so excellent that up to the present it had not been improved upon, but formed the basis of the recently published British Offices Experience Tables. But all those tables up to Dr. Sprague's were formed on imperfect materials, and for a long time they were regarded rather as the latest ultra-scientific toy, than as having any particular relation to practical work. It had needed a long fight to get actuarial opinion to recognize, as he ventured to think it did recognize to-day, that the real value of a mortality experience was to be found in the Select mortality tables based upon it, and not in an aggregate table such as the OM . Up to the present there had been no really convenient way of applying the Select Tables to the practical work of valuation, but that evening there were no less than three methods presented, all of them good, and the date was one to be marked in the calendar of the Institute with a red letter.

Mr. King, who was one of the first to introduce analysed tables, was also one of the first to bring out a practical method of applying them. His method was so simple that in all essentials it could be described on the proverbial "half-sheet of note-paper." Perhaps the easiest way to regard it was to express the reserve value as the sum assured, less an annuity-due of the premium and the discount [$S.V = S - S(P+d)a$]. From that it followed that the difference between the sums assured and the reserve ($S - S.V$) was equal to the present value of the premium plus the discount [$S(P+d)a$], and if the classification were so arranged that the premium was a constant quantity, then by simple proportion that difference could be divided into the value of the premiums, $S.P.a$, and the value of the discount, $S.d.a$, and deducting the value of the discount from the sums assured the value of the sums assured ($S - S.d.a$) was obtained. That, he thought, was a striking example of the power of a really well-devised classification, based upon a common factor. The valuation ages, and the adjustment for the incidence of the premium income, were found by the well-known nearest anniversary method, which gave very accurate results. The adjustment for half-yearly and quarterly premiums, which, of course, was a comparatively minor matter, was accurate if the premiums were instalments of the annual premiums, the outstanding instalments being deducted from the sum assured when a claim arose. If, however, that were not

the case—*i.e.*, if they were true quarterly and half-yearly premiums,—Mr King's method slightly over-stated the values of the net premiums. In dealing with the office premiums, Mr. King suggested that an adjustment which he (Mr. Lidstone) introduced in dealing with the same matter in his paper on Endowment Assurances was unnecessary. The explanation, however, was very simple. Mr. King was dealing with the case in which the office premiums were valued as the annual premiums receivable, namely, twice the half-yearly, and four times the quarterly; whereas he himself was dealing with the case in which the office premiums, like the net, were valued on the basis of the equivalent annual office premiums, and in that case his adjustment was required. The advantages of Mr. King's method appeared to be, first, that it was accurate, and independent of any particular graduation formula; secondly, that it would be applicable to endowment assurances, if each maturity age were dealt with separately; and thirdly, that it obviated the calculation of the net premiums for each individual case. But there were also disadvantages. In the first place, there were those who thought—and he must confess he was amongst the number—that a certain amount of power and insight into the results, and the power of detecting gross errors, was lost if the reserves were dealt with, as distinct from finding the values of the sums assured and the values of the net premiums separately by direct calculation. However that might be, it was certain that the large number of sub-groups which were involved in Mr. King's method, and of the separate multiplications and adjustments, must involve a very great increase in the work, and particularly in the anxiety, of a valuation. With great deference to Mr. King, he was unable to see that the work of the valuation on his method would be no more than by an O^M and $O^{M(5)}$ valuation. There were of course a very large number of actuarial functions to be inserted in the sheets and carefully checked, and a very large number of individual multiplications. Furthermore, the number of sub-groups practically prevented a continuous classification and an annual valuation being made, with any degree of convenience; and, again, the method gave no convenient way of calculating expected deaths according to the Select Tables.

Mr. Diver's method and Mr. Ackland's method had very much in common. Both proceeded upon the same principle, which he believed to be the most advantageous for practical work. They first made an ordinary valuation, on the assumption that all the assurances had been more than ten years in force, and then applied a correction to those that had been less than ten years, and that correction being small in amount could be safely found by an approximate process. In each case, the approximation was effected by means of an auxiliary function, which was calculated once for all at the outset; and all that was required was the total of the sums assured, the total of that function (and in Mr. Diver's case the total of the net premiums also) for each duration of assurance under 10, but *without any sub-grouping at all as to age*. Those totals could be very easily obtained from year to year by a continuous classification, so that the method lent itself very admirably to an annual valuation. Mr. Ackland's method had the further advantage that the mean ages

brought out enabled the expected claims to be accurately calculated for each year of assurance. For practical purposes he thought both Mr. Diver's and Mr. Ackland's methods might be taken as exact, for, although Mr. Ackland brought out an error of about £1,000, that was an absolutely negligible quantity, compared with the total reserves of £14,000,000. Mr. Diver's very complicated mathematical analysis showed that his method depended upon the presence of a certain relation between the graduation constants, and it was not to be expected that the method would necessarily hold good with any table based on a Makeham graduation. Moreover, the mathematics did not apparently help to determine the auxiliary functions, which were of the nature of empirical constants, to be determined practically by trial. It, therefore, became of some importance to consider the way in which those constants were to be determined, and in that connection he should like to refer to p. 18 of Mr. Diver's paper, where the author said that the greatest weight was evidently given to the youngest ages, the argument apparently being that that followed because the largest coefficients occurred at the youngest ages. He ventured to think that a misconception was involved in that statement. In the first place, it could be shown that, if the constants were determined by a double summation, the same result was obtained, whether the series was summed from the top or from the bottom, but in the latter case the smallest coefficients would be obtained at the youngest ages; and if the line of argument were correct, it would follow that the series was most heavily weighted at the youngest and oldest ages at the same time, which was impossible. Then he thought the facts could hardly be said to be weighted at all when the primary equation (the first summation) was left unweighted, as it was in Mr. Diver's case; and it was a little difficult to say exactly what one obtained when one equation was weighted and the other was not. Theory required that all the equations should be weighted simultaneously. He also desired to ask Mr. Diver whether, now he had developed his original method by introducing a second constant, Y_t , as well as his constant multiplier of X_t , anything was gained by using $\phi_{[x]+1}$, and not $\phi_{[x]}$, as the standard of reference. Of course, if the empirical equation held strictly, it would be immaterial which was done. To use $\phi_{[x]}$ would be more symmetrical, and would have the advantage that the error for the year 0, which was the most important, would be absolutely zero; but there might be some other reason, which Mr. Diver had gone into, which made him still prefer $\phi_{[x]+1}$.

Mr. Ackland's method seemed to have the great advantage that it could be applied to any table graduated by Makeham's law, as he showed in the appendix. Mr. Ackland went into the theory of the method with very great fulness, and his various values of w (mean ages) were somewhat confusing owing to their number, until one saw that they followed one simple principle. Taking a series of which the first differences were in geometrical progression (that is to say, such that the series itself consisted of a constant plus a geometrical progression), if they found the mean age or centre of gravity of the series, then, as the common ratio of the geometrical progression increased the mean age also increased. That governed the whole of Mr. Ackland's mean ages, and if they were set out, with the common

ratios, it would be seen that the feature was so marked that it was almost possible to interpolate as though the mean ages were a simple function of the common ratios, even if those ratios were only approximately constant. That suggested that, if the error of Mr. Ackland's method were considered of any importance—though he himself considered it was not—it could be reduced practically to almost any point, by adopting a smaller ratio in getting out the auxiliary function. The tables of the values of r showed that a common ratio of about 1.08, instead of c , would probably produce very accurate results. He thought, as a matter of fact, it would be possible to proceed without any special auxiliary function at all. The underlying principle of the method was that for every value of t the first differences of $\phi_{[x]+t}$ progressed in the same ratio. If, therefore, $\phi_{[x]}$ were recorded on the cards, it would produce no error at all for duration 0, as regarded the sum assured, and would therefore at once introduce a substantial correction. From those values of $\phi_{[x]}$, they could obtain the mean ages for other durations, in precisely the same way as Mr. Ackland obtained them from his auxiliary function, and as the common ratios were seen from the table not to vary very much with t , and on the whole to be rather less than the ratio for $\phi_{[x]}$, he thought that that method, while being very simple, would be as accurate as, and probably rather more accurate than, Mr. Ackland's. It had also the advantage of forcing the table to give up its own law, without their having to investigate what the common ratio was.

He thought the best thanks of the Institute were due to the contributors of three such excellent papers, which would certainly pass into the literature of the subject. Their congratulations were especially due to Mr. Diver on his most successful first appearance, which he hoped would be only the first of many. In considering Mr. Diver's and Mr. Ackland's methods, they must not omit a note of gratitude to their absent friend, Mr. G. F. Hardy, to whose brilliant graduation of the tables they owed the possibility of those methods being developed.

MR. R. P. HARDY said the Institute that evening had spread before it so abundant and so variegated a banquet that it required an actuarial epicure to select from so generous a menu the particular dish which showed the highest triumph of art. The several writers had, with considerable diplomacy, carefully avoided raising the highly debateable question whether the particular adjustment was imperatively necessary, to exhibit the true financial position of an assurance company, or not, and they had wisely put their labours in the shape of samples of methods for conveniently obtaining some particular end, whether that end was of paramount importance or not. Anyone who had read the papers with the same care and attention he himself had given to them would admit that it was scarcely possible, at so short a notice, to speak positively upon their particular merits; but, to his mind, they all presented certain fundamental objections—objections not due to the writers, but to the subject itself, and which were, he believed, insuperable. He would not dwell too much on the large amount of additional work the methods entailed, for he thought all actuaries ought to be prepared to meet that; but it was an objection to have so much of

the necessary work postponed until the valuation schedules assumed their final shape. There was also to his mind—he spoke for nobody else—a great objection to having the values of the sums assured brought out in lump, and not attributable to any particular group of assurances, according to the scheduled age. One lost, in his experience, a number of excellent checks, and a number of most instructive views, but of course to have the particular adjustment made it necessary to abandon all such luxuries. But, dismissing these points as necessarily accidental to the matter, if he were called upon to make a preference of the particular methods, he inclined to that of his friend Mr. King. He thought that his method of scheduling the risks followed a recognized plan, and one that had been abundantly tested by Mr. Fraser (*J.I.A.*, xxxviii, p. 385), and consequently agreed with a standard with which everyone was familiar. Consequently, a great deal of the final work, such as the function ${}_nV_x$, could be inserted beforehand, and therefore the method seemed to present the minimum of difficulty, with a distinct achievement of accuracy in the final results. On the actual merits of the methods submitted he had nothing more to add, but he desired to say that had he not been familiar, from those studies that had occupied his mind for many years, with the many revealing sides a great central truth presented to an observant eye, by even the faint light of which the expert can be safely guided to the goal, he should be specially prepared to point to those three papers as bright illustrations of what a professional training could do, not only in imparting a manifest and admirable dexterity in operation, but at the same time expanding, although still controlling, the mental grasp, so that it soberly distinguished and maintained the limited essentials amidst other elements. All the authors, to his mind, had really succeeded, and the agreement of their practical results with the theoretical standard were ridiculously close. Hence the present and all succeeding generations would be the richer for the labours of the several authors, and the common stock of recorded and freely available knowledge had become sensibly increased. For that he was sure the members would agree with him that the thanks of the Institute were eminently due to Mr. King and Mr. Ackland, both of whom had already done so much imperishable work for the profession, and to Mr. Diver, a new contributor, but one who, it was hoped, would continue on that journey of which he had made so honourable a commencement.

MR. H. H. AUSTIN said that if there was any value in the methods submitted, one of the most important points was their applicability to what were ordinarily known as valuation or classification books. The point evidently had not been overlooked by the authors, since Mr. Diver, for instance, remarked in his paper, “We must have a separate list of policies arranged in durations,” while Mr. Ackland said “Most valuation class-books are so arranged as to enable the sums assured existing in respect of each year of duration to be readily extracted.” That would probably be so in the case of industrial assurance business, and an admirable example of that description had been given by Mr. Schooling in the *Journal of the Institute* (vol. xxxii, p. 302), but for the whole-life business of any ordinary office to be tabulated both in respect to ages and

durations seemed to him to involve a rather prohibitive amount of sub-division and number of headings. That, however, was not the point he wished to emphasise, which was that, by any of the correction methods, as those of Mr. Diver or Mr. Ackland might be called, there was no necessity for any change in the ordinary class-books, whatever system might be in use. They would be entered just as usual, with the substitution of select premiums for non-select, and the particulars extracted and valued in the ordinary way. The only difference that seemed to be necessary was that a separate book should be kept, a correction book with a separate opening for each of the ten durations, in which to enter the necessary correction factors, irrespective of age at entry. It would be a matter of individual choice, then, as to whether the entries should be subdivided into claims, surrenders and lapses. The total in force of correction factors for each duration would be ascertained at the end of each year, and carried forward for the succeeding year at the next higher duration, until the ultimate was attained. Taking Mr. Ackland's method, for example, in its simplest form as explained in paragraph 11, the only particulars to schedule in durations would be the policy number, the sum assured, and $S_{[x]}c^x$ or $S_{[x]}c^{x-40}$. The amount of extra work involved in that was so small as to be of practically no importance, and the only objection he could see was that the correction entries were absolutely divorced from the ordinary class-book entries. If, however, proper precautions were taken, he did not think any error need arise from that source. If Mr. Diver's method was in question, an extra factor would appear necessary, since both the functions $SG_{[x]}$ and $SP_{[x]}$ were required as shown in the fourth formula under Method 1. $SG_{[x]}$ could of course be taken direct from a table of prepared values, in the same way as is done in endowment assurances, when Mr. Lidstone's Z function is used.

There was one other matter in connection with the correction methods that appeared worth attention. It would seem desirable, if the valuation was based on select tables, that the expected claims, and the death strain at risk, should be also obtained on the same basis, and for that purpose he roughly investigated the difference between the select and ultimate values of q , and as might be anticipated a somewhat similar result was obtained to that shown by the differences in the annuity-values, and a series for each value of t could be obtained similar to Mr. Diver's Table I. In dividing those series by an appropriate set of values a table of fairly constant values on the lines of Table II was obtained. The chief difficulty was when $t=0$, where apparently the series did not run quite in conformity with those for the higher values of t . That peculiarity was shown to an extent, of course, in Mr. Diver's Table II. From that it would appear that if desired the expected deaths and the death strain at risk according to the select table could be easily obtained, if another constant were included in the correction book. He had not investigated the point, but it would seem probable that $(q_{w+t} - q_{[w]+t})$, where w was the average age as deduced by Mr. Ackland, would give a very close approximation to the correction, in which case no further constant need be included in the suggested

correction book, and the result would be obtained with a minimum of work.

Turning to Tables III and IV in Mr. Ackland's paper, he noticed that in column (4) when the average age w had been determined the true value of $\phi_{[w]+t}$ was used for the correction, as explained in paragraph 11. If those values in Tables I and II, however, were studied, it would be seen that, although fairly regular, the first differences did not form a good progression, and for that reason it appeared to him that some slight graduation should be made before a particular value was used, otherwise a certain amount of error might be the result, through an irregularity of the series at the point where the average age happened to fall. That was brought out by Mr. Diver, when he remarked that "Unless the ratio $\frac{\phi_{[x]+t}}{\phi_{[x]+1}}$ is varying very irregularly at this point it will generally give good results." For that reason, then, it appeared that when a particular value was required to give effect to an average, such particular value should be taken from a graduated and not an irregular series.

Mr. King's method was bound to command admiration; it was simple, clearly expressed, and beautifully worked out in detail. It was of course based on an old familiar formula in the *Text-Book*. Provided the particulars for the valuation had to be written up completely from the start, and given the select policy-values, it was simplicity itself and apparently not difficult in application, but for a valuation based on classification registers, where the facts are tabulated according to dates of birth, or what was the same in effect, valuation ages, it appeared to be quite unsuitable.

MR. E. C. THOMAS could not help thinking that Mr. Lidstone had done less than justice to the method of valuation put forward by Mr. King. From the fact that in America it was the usual custom to make a valuation by individual policies, it would seem that not only was it a convenient course to take, but that it would not involve a vast amount of unnecessary work, compared with the method of valuing in groups. Mr. King's method involved only a very slight addition to the work of an ordinary valuation made for individual policies, the extra work being represented for each group by the few calculations shown by Mr. King in his specimen figures in paragraph 26. That being so, some considerable value would seem to attach to the method. Mr. Lidstone had made a remark with regard to the instalment premiums included by Mr. King in his valuation. The fractional premiums included by Mr. King were the net premiums, and they were for the whole of the current outstanding instalments; the difference between him and Mr. Lidstone was in respect to the office premiums, Mr. Lidstone having taken into account a similar complete value of the unpaid instalments of office premium. Mr. Lidstone remarked that he took in those office premiums simply because the premium that he had valued was the yearly premium corresponding to the fractional premium of the prospectus, and was not the fractional loaded premium itself. He (the speaker) considered that a certain portion of those outstanding instalments might have been included, even if the original fractional premium had been valued, that portion, namely, that would be due at

the end of the term. That would be represented, if complementary premiums were charged, by the unpaid instalments multiplied by $A_{x:\overline{n}|}$, or, if the premium were not a complementary one, by the same figures multiplied by $A_{x:\overline{n}|}^1$. He concluded by conveying to the three authors his high appreciation of their extremely interesting and valuable papers.

MR. S. G. WARNER thought that the complexity of the subject made it very difficult to add anything material or useful in the way of technical criticism, but it was easy to see that the papers fell practically into two classes, Mr. King's on the one hand, and Mr. Ackland's and Mr. Diver's on the other. With regard to Mr. King's paper, difference of opinion seemed to be chiefly centred on that feature of the method which involved dealing with the reserve as a single function, without separating the values of the sums assured and the premiums. Of course, in that connection, as was pointed out in the paper, there was the formidable barrier, first of all, of the statutory requirements, which necessitated showing the values separately, and involved the process of bringing the results into harmony with the form required by the Board of Trade returns, which would add to the labour and complexity of the valuation. But apart from the fact that the statutory form of returns required those particulars to be separately shown, there seemed to be a good deal to be said for the advisability of so showing them. Given the particulars as they appeared in the Board of Trade returns for any valuation, it was within the power of anyone having a little expert knowledge to make some kind of rough check on the results, and that was a point of some importance. With regard to the other two papers, which were built very much upon the same lines, Mr. Lidstone had pointed out the indebtedness of the authors to Mr. G. F. Hardy. Mr. Lidstone might also have pointed out, what naturally he would not call attention to, the great indebtedness to himself. Obviously the method adopted was substantially on the lines of that introduced by Mr. Lidstone with such great effect for the valuation of endowment policies. The method of ascertaining the ratios of the differences, and selecting an average value, was that upon which Mr. Ackland had proceeded in his paper. The difference between the two papers seemed to be that, while Mr. Ackland had succeeded in finding something like a general principle applicable to any table following Makeham's graduation, Mr. Diver had rather found his method in what might be described as more or less of an accident of the particular table in question, and had not altogether succeeded in his mathematical attempt to find some general reason for it. While they were all agreed as to the great value of the papers, they were irresistibly led to the further question of the advisability of making a Select Table valuation at all. He admitted that that was hardly before the meeting, but papers of the kind under discussion must make its consideration at some time or other very desirable. Personally, he should be somewhat disposed to doubt, with every respect to the great labour and ingenuity shown in the applications of the method, its general advisability. Mr. King said, in the opening of his paper, that he did not suggest that other bases of valuation resulting in a lower estimate of liability were

unsound, because the margins both of premium and interest were always so large that any practical differences in net premium valuation estimates were not of great consequence. "We naturally, however," continued Mr. King, "wish to know as nearly as may be what is the truth, and the opinion is general that the fundamental truth is best brought out by Select Tables." Obviously it was desirable to know the exact truth, but the truth all round, both as regarded the mortality, and the rate of interest employed; and the logical complement of a Select Table valuation, supposing it for the moment to be absolutely sound, would be the employment of a rate of interest representing to the nearest penny what had been actually realised during the quinquennium. So that an attempt to ascertain the exact truth as regarded the mortality, which admitted that more approximate methods were quite good enough, because the truth was departed from as regards the rate of interest, while probably practically satisfactory, was not strictly logical, in so far as it set itself different ideals in regard to the two vital factors in a valuation.

MR. JAMES BACON said that if, as was necessary under Mr. King's method, the facts for entry ages and for durations were to be scheduled, he could not see much practical gain from applying the method. The whole work of the valuation by Select Tables, of one class of whole-life policies, when the facts for the first ten durations were already scheduled in that way, would not occupy more than about thirty hours at the utmost, for the actual entries, multiplications, &c., and it seemed to him that the application of Mr. King's method would occupy nearly the same time. There were other difficulties also in the way. The bonuses and the sums assured had to be dwelt with separately under all three methods. Although perhaps they would not take a great deal of time in valuation, yet, when the difference in the time taken by a full and an approximate valuation was so small it seemed to him that this point should be considered. There was also the question of gross or office premium valuations, though, perhaps, in these days, it was of little more than academic interest. It seemed to him, however, that, where the premium valued was not a net premium, but a percentage of the office premium, Mr. Ackland's method was not applicable, although no doubt Mr. King's and Mr. Diver's methods could be used. There was one point in connection with Mr. Ackland's paper which he thought had not been fully brought out. For the valuation of the net premiums, Mr. Ackland took a net premium at an average age w , deduced from the sums assured by taking $\frac{\sum (Sc^x)}{\sum S} = c^w$, and it appeared that if the true net premiums were valued, a somewhat large error would result. The reason why the adoption of the net premium at the average age deduced as above gave such correct results was that that age was based on the assumption that $\Delta\phi_{[x]+t}$ increased in geometrical progression with the ratio c , as $[x]$ increased, the actual average ratios of increase for each value of t being shown in paragraph 7, and it would be found that if $\pi_{[x]}\phi_{[x]+t}$ were substituted for $\phi_{[x]+t}$, the resulting average ratio was not only more regular than that shown by Mr. Ackland, but was also nearer to the

assumed value c , and the same feature was shown in a more marked degree in an investigation into the rate of increase at individual ages, similar to that shown in Tables I and II, which had been made for values of $[x]$ from 20 to 60 and the value $t=0$.

MR. W. P. PHELPS said that they had before them three of the most important and most interesting papers, both from a purely academic and practical point of view, that had been read at the Institute for some time. Mr. King's method, he thought, might be called an exact method of valuation. His own impression was that the work involved in that method would be considerably greater than that involved in the methods of Mr. Ackland and Mr. Diver. The two latter methods were practically accurate; they were approximate methods, but the results brought out were so close to the true results that they could for all practical purposes be considered as accurate. He took it that Mr. King's paper had been written partly to bring out one fact which was not shown in the other papers, namely, the relative reserves by the various tables. In his tables at the end of the paper, Mr. King showed a fact which he did not think had been brought out before, that the $O^{(M)}$ Reserves of the Model Office were greater than the $H^{(M)}$ Reserves, and also than the O^M and $O^{M(5)}$ Reserves. There was no doubt that the O^M Tables were the truest measure of mortality at the present time, and a select valuation was the theoretically correct valuation to make. The objection in employing that method of valuation so far had been the labour involved, but he thought the members now had in their hands a method of making a select valuation which did not involve as much work as a valuation by the O^M and $O^{M(5)}$ Tables, and that was a very great gain. Mr. Ackland and Mr. Diver worked from the same principle, namely, that by the application of a correction the select valuation reserve could be obtained from the valuation made by the ultimate table. Mr. Diver's results, in the two examples he had worked out, did not vary from the true result to any appreciable extent. Mr. Ackland's did not differ from the true values to any extent of importance, but he hardly thought Mr. Ackland had quite done himself justice, for reasons he would explain later on.

Mr. Ackland's method had an advantage over Mr. Diver's method, in that it did not necessitate the scheduling of the net premiums; he had worked entirely from the sums assured. Mr. Diver had introduced two factors, X_t and Y_t , which, applied in the ingenious way explained in his paper, gave good results in the two examples he had worked out, but it did not appear that his method dealt automatically with the distribution of the business, as did the method employed by Mr. Ackland. The correction used by Mr. Ackland was really in two parts, a constant into ϕ in respect of the sum assured and a constant into $\phi\pi$ in respect of the premiums. Mr. Ackland, as he thought, wisely abandoned the attempt to get the mathematical law underlying the difference in the two annuity-values. He should not like to say that the problem could not be solved, but he was very glad he had not to solve it. At any rate, he thought that whatever law was brought out it would be somewhat complicated, and he very much doubted if any modification of that law, or any approximation from that law, would give better results than were

brought out by Mr. Ackland when he showed that ϕ might be approximately represented in the form $A + Bc^x$. He regretted that Mr. Ackland should have brought c into the expression. The law was equally represented by $A + Br^x$. He used r in contradistinction to c , which for the O^M Table has a value of 1.094. Mr. Lidstone had already pointed out that, by taking 1.094, Mr. Ackland had over-estimated the average age. The result was that the corrective quantity which was deducted from the liability on an ultimate basis, was overstated, with a consequent understatement of reserves. Mr. Ackland had given certain values of r in paragraph 7, which could be taken to represent the rate of increase in the function ϕ very closely. He had also worked entirely from the sums assured, which was a great advantage. But the premium portion consisted of $\pi\phi$. That could be called ϕ' and it could be shown that ϕ' followed approximately a similar law to that obtained by Mr. Ackland for ϕ . Mr. Macnaghten, who had had considerable experience in dealing with average ages in connection with Mr. Lidstone's method of valuing endowment assurances, had been good enough to make certain calculations, and he had also had the benefit of discussing the paper with him. Mr. Macnaghten had calculated the value of the increase in $\pi\phi$, and he showed a corresponding rate of increase, r' , which was on an average about .015 greater than the rate of increase r , in ϕ . But the point came out that both the rate of interest in ϕ and the rate of increase in ϕ' were less than c , and if Mr. Ackland had taken, instead of c , a value between those two he would have got very close to the figure of 1.08 mentioned by Mr. Lidstone. As a matter of fact, Mr. Macnaghten had worked out Mr. Ackland's example, taking r equal to 1.0875, and the results were exceptionally good. Mr. Lidstone had pointed out that by increasing r the average age was also increased. By taking an average age given by a rate of increase between the r derived from ϕ and the r' derived from ϕ' , the average age would be over-estimated in respect to the sum assured, but at the same time it would be under-estimated in respect of the premiums, and the net error would be the difference between a small positive error in the correction for the sums assured, and a small negative error in the correction for the premiums. Of course, more exact results could be obtained by dealing separately with the sums assured and the premiums; but there would be no necessity for this duplication of labour. Working with 1.0875 at 3 per-cent and the actual experience used by Mr. Ackland, the error in the correction for the sum assured was reduced from £694 to £329, but was still positive. The error in the correction for the premiums was reduced from +£453 to -£313. Adding those two errors together, there was a total error on the net liability of £16, which was even closer than that brought out by Mr. Diver.

MR. R. P. HARDY asked whether that was not upon the assumption of a uniform distribution of risks?

MR. PHELPS said it was from the material actually used by Mr. Ackland in his paper. He might mention that, taking the value 1.0875 instead of 1.094, the average age was reduced by about a third a year. With that modification it struck him that Mr. Ackland's method gave practically an exact result with a minimum of work.

THE PRESIDENT, in moving a hearty vote of thanks to the authors of the papers, was sure that the members would better appreciate the true value of the communications when they had had a little more time to consider them, particularly in the light of the discussion that had taken place.

The resolution was carried unanimously.

MR. GEORGE KING, in replying on the discussion, said that when Dr. Sprague's paper "On Select Tables" was read at the Institute almost exactly twenty-five years ago, he foreshadowed, in the discussion, the paper that he had submitted that evening, and a week or two later he sent a letter to the *Insurance Record*, which gave the method exactly as he had given it now, except that now he had gone into a little more detail, and had given an illustration. So far as his own paper was concerned, the criticisms had been confined almost entirely to the question of the amount of work involved, and he thought the different speakers had rather exaggerated what was required. As a matter of fact, the cards were not more complicated than the ordinary valuation cards. It was quite true that valuation registers in the old form could not be used, but, once they were in the new form, there would be no greater difficulty than at present. Then as to scheduling the cards, in the first preliminary detailed schedule there was not a single figure more required in the writing than by the other method of valuing by age attained. It was only that there was an intermediate scheduling according to duration for each age at entry. Having completed the schedules by age at entry and duration, it was a very short work to pass to the Board of Trade returns, showing the ages attained. It would be found there was scarcely any more work involved in the scheduling than under the ordinary plan. If he had placed in his hands cards for a large valuation of 15,000 or 20,000 policies, he would undertake without undue strain to send in his report and the Board of Trade returns complete in about a fortnight. Mr. Lidstone had spoken as to the history of the idea of select tables, and had mentioned, as an early contribution to the subject, the late Mr. J. A. Higham's paper. But so long ago as 1815 Joshua Milne in his book spoke strongly on the subject, and severely criticised De Parcieux's tables because they were not select. That showed how very long ago the idea was held that select tables were the best. Mr. Lidstone had spoken of the method of correcting for the fractional durations of assurance and outstanding premiums. That was referred to more particularly in paragraphs 20, 23 and 24 of his paper, and he would call attention to the fact that the correction for outstanding premiums was not because they were instalment premiums, but because it was fundamental to the formula. The valuation was by duration, and the ordinary policy-value assumed, when multiplying the premium by the annuity, that the full annual premium was paid at the beginning of the year, and it reserved the proportionate part of that full annual premium for the unexpired term of the year. Therefore, if the full annual premium had not been received, the difference had to be taken off, and those were the outstanding half-yearly and quarterly premiums he had ventured to call fractional premiums. One apparent complication in the method he had submitted was in the

columns α and β ; that was to give an automatic adjustment for the incidence of premium income. In all his recent valuations he had adopted it, because it was so very convenient and very accurate, but it was not necessary, and those who did not wish it need not adopt it, in connection with the method of valuation now suggested for select tables, but could proceed by an examination into the average incidence of the premium income. He would add a few words to paragraph 23 of his paper to make this point clear. As regards the reference in paragraph 25 to Mr. Lidstone's correction for fractional payments of premium, Mr. Lidstone had now explained that he referred to equivalent annual premiums, instead of multiples of the half-yearly and quarterly premiums.

MR. T. G. ACKLAND desired to express his appreciation of the successful efforts of Mr. King and Mr. Diver, the latter of whom was good enough to communicate to him, at a very early stage, some outline of his interesting method, before it was so fully developed as in the paper. Mr. Lidstone had gone deeply into the subject of his (Mr. Ackland's) paper, and he was sure Mr. Lidstone's remarks, when printed, would add much to their knowledge of the method and its applicability, both on the theoretical and practical side. He referred, as other speakers had done, to the possibility of getting out expected claims by the use of the approximate function employed. There would, he thought, be no difficulty, by following the method Mr. Lidstone had himself indicated in the case of endowment assurances, with necessary modifications, in getting out the expected claims, with due regard to selection, with very close accuracy. Attention had been also called to the fact, which indeed lay on the surface, that if a smaller ratio than c had been adopted, such a ratio, for example, as was indicated in paragraph 7 of the paper, or one derived in the interesting way referred to by Mr. Bacon and Mr. Phelps, the results would have been substantially closer. That was patent to his mind, but he had been much impressed by the fact that, when Mr. Lidstone investigated his approximate method of valuation of endowment assurances, he had the same indications before him that a lower value than c would have given a more accurate result, but nevertheless (and rightly as he thought) he adopted c as the ratio that could most conveniently be employed in practice. In that, Mr. Lidstone was supported by the high authority of Mr. G. F. Hardy. (See *J.I.A.*, vol. xxxiv., pp. 69, 70, 77, 78, 86.) It was quite evident that those who employed the method could use another ratio than c , but from the point of view of theory it was convenient to adhere to that well-known value, and it had also other practical advantages. Mr. Lidstone made the interesting suggestion that by putting the value of the correct function $\phi_{[x]}$ at the point of selection, on the cards, instead of the value of c^x , a very accurate result would be attained, and, he might add, with the addition of some little convenience, in respect of the fact that the number of figures entered on the card would be very much less than by the employment of c^x . The value of the function ϕ for duration 0, as given in his Tables I and II, varied from 0 to '60, and therefore for £100 assured one would never have a larger value, within practicable limits, than £60. That

suggestion was undoubtedly worth following up, and might be a better practical way of applying the method than that indicated in his paper. Mr. Austin had suggested that there was some error in using the true values of the corrective function given in Tables I and II, because of accidental irregularities in the tabular values of that function. He would, however, point out that the function ϕ was based on a mathematical law, and therefore it was to all intents and purposes a graduated function, and if there were any accidental irregularities in the tabular values, they arose, as pointed out in paragraph 6 of his paper, mainly from the fact that there were no means readily available of getting out the correct annuity-values, either on a select or an ultimate basis, even to three places of decimals, and that the fourth and fifth places could not be relied upon at all. If those functions were more accurately taken out by construction of more extended D and N Tables, it would be found that the results ran practically as graduated values, as indeed they must do, because they were based on a mathematical law. Mr. Warner had rightly said that a good deal in his paper, and perhaps in Mr. Diver's paper also, was due to Mr. Lidstone himself, and in this connection he might say that his own paper would never have been written, if it had not been for the interesting and valuable contributions of Mr. Lidstone on the approximate method of getting out endowment assurance reserves. Mr. Bacon had referred to the impossibility of getting out the values of the office premiums by his method. He had not gone into that point, but he thought there would be no difficulty, having ascertained the value of the office premiums and net premiums on an ultimate basis, in determining from the value of the net premiums on a select basis the proportionate value of the office premiums. Mr. Bacon and Mr. Phelps had both referred to the point, which was fundamental to his method, of the employment of the net premium at the average age, and the fact that it was not necessary to schedule the net premiums from the valuation class-books. When he at first employed the true net premiums, he found a substantial error, and the method was, he thought, not worth following up on those lines; but he stumbled as it were, in the course of his work on the fact that by taking the net premium at the average age a much closer result was arrived at. That arose from the fact that the average net premium multiplied into the function ϕ gave an average age which did not substantially differ from the age deduced from the sums assured weighted with c^x . He should like to add that the mathematics in his appendix did not conclusively or quite satisfactorily prove the law followed by the corrective function ϕ . He believed Mr. Lidstone had been more successful in that respect, and had succeeded in getting out, by Taylor's theorem, a form that was exceedingly neat and simple, and he hoped that these results deduced by Mr. Lidstone would be included in the report of the discussion later on.

MR. O. F. DIVER did not propose to do much more than express his deep gratitude for the kind reception given to his paper. He had no doubt he should find Mr. Lidstone's remarks exceedingly useful, and he was much obliged to him for pointing out what was certainly a fallacy in the paper, where he had used the word "weight" in a sense in which it should not be used. He had no great acquaintance with modern

statistical methods, and he had hoped that a better method of obtaining X_t and Y_t might be supplied in the course of the discussion. But as he gathered that the method was not considered likely to be practically useful, that was not now a matter of importance. He should have been glad to hear whether it was considered that the method he used for obtaining X_t and Y_t was fallacious in itself, apart from the theory associated with it. With regard to the use of the function which he had called $\phi_{[x]+1}$, instead of $\phi_{[x]}$, he had considered the point whether it would not be better to discard $\phi_{[x]+1}$ in the first method, but it seemed to him that it was the duration of one year which was the most important. In the example he had taken, and, he should think, in most other practical examples, in duration 0 were included only the entrants for about the last six months, and the total sums assured in duration 0 would on the average come out at about half those for succeeding durations. Therefore he thought it was better to retain $\phi_{[x]+1}$ in his first method.

At the close of the discussion, Mr. C. H. L. CATCHLOVE, of the Australian Mutual Provident Society of Sydney, conveyed to the Institute, on behalf of the profession in Australia, an expression of deep attachment and regard. It was not, he said, given to many Australian actuaries to have the privilege of personally attending these meetings, but great interest was taken in all the proceedings, and the various publications were carefully perused. There existed in Australia a very deeply-rooted admiration and respect for the Institute, and for all matters appertaining to it. The solid basis on which it rested, the high public spirit and lofty purpose of its members, were readily recognized, as well as the ability and the devotion manifested by those who contributed to its proceedings. It was regarded, indeed, as a high honour to be connected with the Institute, and its degrees were highly esteemed. He belonged at present to the class that was sometimes referred to as the "unfortunate students", and would take the opportunity of reminding the Institute of the difficulties under which they laboured in the Colonies. He had crossed the seas to make himself proficient in his profession, and was hoping to carry back considerable knowledge. He wanted to bear testimony to the kindly welcome he had received from the members of the Institute, and he recognized that it was largely due not to himself but to the Colonies, and he felt sure the profession in the Colonies would be duly appreciative. A great deal was heard in these days about drawing the Colonies in closer relation to the Motherland. The attitude of the Australian actuary towards the Institute was one of deep attachment and loyalty, and he sincerely trusted it would so continue. He therefore asked the President to accept his expression of deep regard and goodwill.

On the motion of Mr. R. P. HARDY, seconded by Mr. KING and supported by the PRESIDENT, it was unanimously resolved that Mr. Catchlove's remarks should be recorded on the minutes.

THE PRESIDENT said the members of the Institute had felt assured for a long time past of the loyalty of the members in Australia, and of the interest taken by them in the proceedings at Staple Inn Hall. He hoped Mr. Catchlove would convey to his colleagues in Australia the most fraternal greetings of the members.

Report of the Departmental Committee on Bond Investment Companies.

[The following is the full text of the Report (Cd. 2769) of the Departmental Committee appointed by the Board of Trade to enquire and report as to the operations of Bond Investment Companies (see *J.I.A.*, xxxix, 336). The marginal references are to the Appendices to the Report, published as a separate Blue-book (Cd. 2770) and including the full minutes of evidence, with exhibits and Tables.—
ED. *J.I.A.*]

To the Most Hon. the MARQUESS OF SALISBURY, C.B.,
President of the Board of Trade.

My Lord,

1. This Committee was appointed on May 31st, 1905, “to enquire as to the operation of Companies (not being Life Assurance Companies) which collect periodical payments from the industrial classes in return for benefits promised in the future, and whether it is desirable that there should be any restrictions imposed on such Companies or any Government supervision of their transactions, and to report to the Board of Trade.”

2. The Committee has sat eighteen days, and examined eighteen witnesses. They have also had before them the Memoranda and Articles, Prospectuses, and other papers relating to a considerable number of Companies of the class under consideration.

3. The Companies, to which this enquiry has been directed, may be described as Companies which issue bonds or certificates to members of the public, by which the Company, in return for fixed monthly subscriptions payable over a fixed period of years, contracts to pay the subscriber a lump sum at the end of the period, and to give him certain incidental advantages, including in many cases the right to an Advance on certain conditions.

4. Companies of this character appear to have existed for some time past in England, one of those brought to our notice having been formed in the year 1884, but during the last few years there has been a large increase in their number. The earlier Companies for the most part appear to have made a practice of issuing bonds or certificates payable by monthly instalments over a long period of years (usually 30 years). The oldest of these Companies, and certain others also, put forward as a prominent feature of their business the advancing to bondholders of moneys at interest for the purpose of house purchase, the moneys being advanced on the security of the house purchased, with the partly-paid bond or certificate as collateral security. The house purchase element is usually absent from the more modern

type of Bond Investment Company, which may be said to date from the year 1898 or 1899, and which for the most part issues bonds or certificates payable at the end of 10 or 12 years.

5. The more modern type of Bond Investment Company referred to in the preceding paragraph appears to be an importation from the United States, where, we are told, large numbers have been formed during the past 20 years. They appear to have been the subject of legislation in the States of Missouri, Tennessee and Texas in 1897, in the State of Ohio in 1900, and in the Commonwealth of Massachusetts in 1904, and copies of the Acts referred to will be found in an Appendix to this Report.

Appendix II.

Q. 857, 1076,
1807, 1811,
1863, 1948,
2118, 2209,
&c., 2590-2,
2872.

6. There is evidence to show that subscribers to Bond Investment Companies in the United Kingdom are drawn not only from the working or industrial classes, but from the lower middle and the professional classes.

7. We have endeavoured to hear evidence from all persons concerned in the direction or management of Bond Investment and House Purchase Companies who were willing to give us their assistance. Mr. W. W. Rae, one of the managing directors of the State Banking Corporation, Limited (registered in Guernsey), and Mr. J. H. Kilby, a director of the same company, after expressing a desire to give evidence, ultimately declined unless they were paid remuneration for so doing in addition to their travelling and out-of-pocket expenses. This, under the standing regulations of the Treasury, we were unable to give them, and there seemed to us to be no ground for making a special application to the Treasury in this respect. In like manner, Mr. Jeffery Power, who was the promoter and at one time the managing director of one Bond Investment Company, and was until recently the managing director of another such Company, expressed his desire to be heard, and an appointment was given to him for this purpose. On the day appointed he telegraphed from an address in London requesting that the appointment might be postponed to the following day. This was done, and the Committee sat specially for the purpose of hearing him, but he did not appear and has failed to communicate any further with the Committee.

Q. 315, 348.

TWO MAIN CLASSES OF BOND INVESTMENT COMPANIES.

8. These Companies naturally divide themselves into two main classes, viz.:

(A) Companies (hereinafter called Companies of Class A) which in return for the monthly subscriptions contract to pay the subscriber at the end of the period a lump sum equivalent to the amount

of the subscriptions together with compound interest at a low rate, varying from $1\frac{3}{4}$ per cent. to $2\frac{1}{2}$ per cent., together with, in some cases, bonuses periodically allotted out of the annual profits of the Company as ascertained by an actuarial valuation.

Q. 2441.
Q. 1003-1008.
Q. 2714-2720.

(B) Companies (hereinafter called Companies of Class B) which, in return for the monthly subscriptions, contract to pay the subscriber a lump sum, which is in excess, and sometimes considerably in excess, of the amount of the subscriptions together with compound interest calculated at 3, or even at 4 or 5 per cent.

TERMS AND CONDITIONS OF BONDS.

9. The terms and conditions on which the bonds are issued may be classified as follows :

(a) *Lapse*. If the monthly subscriptions are not paid within a period varying in different Companies from 24 to 60 days after the due date, the Company has the right to declare the bond to be lapsed or forfeited, in which case all the subscriptions previously paid are forfeited to the Company. Most Companies, however, reserve to themselves the power to re-instate the bond within a further period (varying in different companies from 3 months to a year) on payment of the instalments due and a fine. The bondholders have, as a rule, no legal right to receive any notice as to the payment of their instalments or the re-instatement of their bonds. But it appears that, as a matter of grace, many of the Companies give such notices to their bondholders and give them an opportunity of having their bonds re-instated, after the expiration of the period during which the bondholder has a legal right to keep the bond alive.

Q. 572-578,
633-637, 773-
793, 1055, 1510-
1511, 2192-
2201, 2530-
2535.

Cf. as to
notice before
forfeiture,
Section 3 of
the Collecting
Societies and
Industrial
Assurance
Companies
Act, 1896, 53
and 60 Vict.,
cap. 26.

(b) *Surrenders and Surrender-Values*. Until the bond has been maintained for a period of from 3 to 5 years no return at all is made to the subscriber who desires to discontinue his subscriptions. After the expiration of that period the bond acquires with most Companies a surrender-value; and if the bondholder in such a case desires to discontinue his subscriptions, he will receive from the Company an immediate cash payment equal to the surrender-value. The amount of such surrender-value is a percentage of the subscriptions actually paid without interest; in the earlier years of the bond it is 20 or 30 per cent. of the subscriptions paid, but in some cases it increases in the later years.

The effect of such surrender is that the Company gains, and the subscriber loses, a sum equal to the difference between the total amount of subscriptions actually paid with compound interest and the amount of the surrender-value.

(c) *Death of Bondholder.* It is usual on the death of a bondholder to permit his legal personal representative, at his option, either to step into the shoes of the deceased bondholder and to complete the bond, or to receive the amount of the subscriptions then actually paid, but without interest.

(d) *Transfer.* It is usual to allow the bondholder, at any rate after the bond has been in force for some time, to transfer his bond on payment of a small fee to the Company.

(e) *Advances.* Some of the Companies into which we have inquired give their bondholders a right, after the bond has been maintained for a certain minimum number of years (usually five years but in some cases less) to obtain from the Company, if the funds of the Company admit, and the security is approved, an Advance carrying interest at 5 per cent. per annum equal to the amount of the bond, secured on house or shop property and in some cases collaterally secured on the bond itself. In the case of the two oldest Companies which offer these Advances, the evidence shows that only one-fifth, or less than one-fifth, of the bondholders avail themselves of this offer.

Most of the other Companies offer to make the bondholders, whose bonds have been maintained for a certain minimum number of years (usually three years, but in some cases less) an Advance, provided the funds of the Company admit. The Advance so made has nothing to do with house or shop purchase, and is not secured on any house or shop property. The amount of this Advance varies according to the number of years for which the bond has been maintained, but it does not in any case exceed the surrender-value of the bond. The result of this transaction is that, if the bondholder should subsequently discontinue his subscriptions, the amount of the Advance is set off against the surrender-value of the bond, and the transaction is closed.

(f) *Redemption.* In most of the modern Companies, a system exists, by virtue of which the holders of certain bonds, which are from time to time selected by a method which will be more fully described below, receive payment in cash of the whole or a part of their bonds before the period of

Mutual
Property
Investment
and Accident
Co., Ltd.
Q. 1068-1072.
Anchor
Savings and
Investment
Corporation,
Ltd.
Q. 1972-1975.
Provident
Association of
London, Ltd.
Q. 2423-2424.
Provincial
Homes
Investment
Co., Ltd.
Q. 2730-2733.

Provident
Association of
London, Ltd.
Q. 2423-2424.
Provincial
Homes
Investment
Co., Q. 2714-
2716.

maturity of the bonds arrives. If the whole of the bond is thus paid off, there is obviously no reason for the bondholder to continue his subscriptions and he thus receives under the name of "Redemption" a payment far in excess of the amount which his subscriptions have earned. If only a portion of the bond is paid off, it is optional to him to continue his subscriptions or not. If he does continue them he will receive at the end of the period the amount of the bond less the amount already received under the name of "Redemption."

(g) *Advance without Interest.* In the case of one Company of which we have had evidence, the holders of certain bonds which are from time to time selected by ballot, receive an "Advance without interest"; in other words, a loan without interest for the remainder of the period during which the bond is current. These operations deserve separate and careful consideration, and are dealt with in a subsequent part of this Report.

Provident
Association of
London,
Q. 2460, &c.

SCOPE OF THE ENQUIRY.

10. We conceive it to be no part of our duty to enquire whether contracts of the kind we have described are in themselves advantageous or not. That is a matter for the intending subscriber to decide.

It is, however, obvious that in contracts of this character each of the parties to the contract, the Company and the bondholder, is in a position of a somewhat exceptional character. As regards the Company, it incurs, by the nature of the contract, heavy obligations, which do not come up for fulfilment till the end of ten, twenty, or thirty years, according to the duration of the bond. As regards the bondholder, he is not, *quâ* bondholder, a shareholder, and he is not, pending the maturity of the bond, a creditor of the Company, and has none of the usual rights of a shareholder or creditor. At the same time, he is by virtue of his position deeply concerned in the good management and financial soundness of the Company, and may fairly claim to be entitled to receive full information as to the Company's affairs.

11. Having regard to these considerations, we propose to direct our enquiry to the five following points, viz.:

(1) The provision made for paying the expenses of management.

(2) The provision made for meeting the obligations of the Companies to their bondholders.

(3) The accounts kept by the Companies and their procedure for ascertaining their financial position and informing their bondholders of the result.

(4) The system of "Redemption" or "Advances without Interest."

(5) General considerations as to the legal position of the bondholders.

THE PROVISION MADE FOR PAYING THE EXPENSES OF MANAGEMENT.

12. It is obvious that, before the funds of the Company can be made available for paying the bondholders, provision must be made for paying "Expenses of Management," under which term we include payments to agents and persons introducing business, payments to directors and servants of the Company, and, generally speaking, working expenses.

Q. 750-755,
1352-1364,
1620-1626,
1845, 2070-
2089, 2204.

13 Omitting the receipts from fines and transfer fees (which are relatively small and may be practically neglected) the main sources on which the Companies must rely for funds to meet the expenses of management and to pay off the bonds are as follows:

(a) The monthly or other subscriptions paid by the bondholders whose bonds are kept up.

(b) The interest earned by the Company on the investments of these subscriptions.

(c) Capital paid up and uncalled (if any).

(d) The gains arising from lapses and surrenders of bonds. In the case of lapses the Company is relieved from all liability on the lapsed bonds and gains the benefit of the whole of the subscriptions paid on these bonds. In the case of surrenders, the Company is relieved from all liability on the surrendered bonds, and gains the benefit of a percentage (usually a large one) of the amount of the subscriptions paid on such bonds, together with any interest that has been earned.

Expenses of Management,
Q. 1540-1541,
3061.

British Equitable Corporation,
Q. 1441,
Purtell, A.

Mutual Property Investment Co., Q. 1089-1096.

Provincial Homes Investment Co., Q. 2840-2851.

14. In all the Companies which we have examined, the expenses of management are large, especially in the first few years of a Company's existence, when the cost of procuring new business, and finding and extending a connection is necessarily heavy. In the case of one Company, the expenses of management for the first five years were (in round figures) respectively 94, 41, 42, 31 and 27 per-cent of the total amount of the subscriptions received from bondholders. In the case of another Company, the expenses of management for the first three years were respectively 67, 72 and 48 per-cent of the total amount of the subscriptions. In the case of a third Company the amounts estimated as sufficient to meet the expenses of management in each year were the first twelve months' subscriptions paid on the bonds issued in

that year, together with 20 per-cent of the subscriptions on the bonds previously issued ; but it turned out that in each of the first five years of that Company's existence these estimates were largely exceeded ; and it was only in the sixth year of the Company's existence that the amount of the subscriptions mentioned above commenced to be in excess of the amount actually expended on management.

15. In the older Companies, there is no special allocation of funds to meet the expenses of management. In most of the modern Companies, the entrance fee of 5s. and the first three monthly subscriptions of 5s. each, and 20 per cent. of the subsequent monthly subscriptions, are allocated to meet these expenses. We are satisfied on the evidence before us that, at any rate in the first few years of a Company's existence, the expenses of management will largely exceed any sum which can be safely allocated out of subscriptions, and that it would not be safe to rely only on lapses and surrenders to make up the deficiency. It would therefore be necessary, in order to meet these expenses, to draw on the paid-up capital of the Company. In most of the modern Companies, which we have considered, the paid-up capital has been small, and in some cases it was all exhausted, and more than exhausted, in payment of management expenses during the first few years of the Company's existence.

16. One of the witnesses called before us, Mr. Harmood Banner, M.P., who not only is an ex-President of the Institute of Chartered Accountants, but who also acted for a time as auditor of one of the Companies under consideration, gave it as his opinion, that in all these Companies there ought to be ample capital subscribed to meet any possible deficiency in management expenses. We agree in principle, and we think that, in order to secure the existence of a fund to meet these early expenses, and to prevent the formation of unsubstantial Companies, every Company of this class should, in future, before commencing business, be compelled to deposit a substantial sum in Court, which should not be paid out until a fund is set apart and secured for bondholders amounting to (say) double the sum originally deposited. A provision of this sort was made in the case of Life Assurance Companies by the Act of 1870, and it is worthy of note that in the debate in the House of Commons in Committee of that Bill, it was expressly stated that the object of the deposit was to prevent the formation of bubble companies. (See *Hansard*, 3rd series, vol. 202, p. 1171.)

Q. 3062.

Recommendation.

THE PROVISION MADE FOR MEETING THE OBLIGATIONS OF THE COMPANIES TO THEIR BONDHOLDERS.

17. In considering whether the funds of the Company, after payment of the expenses of management, will be

sufficient to pay off the bondholders, it is necessary to consider separately the case of the Companies of Class A and Class B, referred to in paragraph 8 of this Report.

Companies of Class A.

Provident
Association of
London,
Q. 2554-2574.
Provincial
Homes
Investment
Co.
Reports and
Balance
Sheets.

18. In the case of the two oldest Companies of Class A an actuarial valuation of the liability of the Company to its bondholders is made annually, and care is taken to keep funds at least equal to the amount of such liability invested in the names of trustees ready to meet such liability. We consider that this is a sound system. We have not had any evidence given before us as to other Companies of this class, but we think that in all such Companies a similar system should be adopted.

Companies of Class B.

19. As regards Companies of Class B, different considerations arise. In these Companies the most usual form of bond is one by which, in consideration of an entrance fee of 5s. and 120, 144, 147, or 150 monthly subscriptions of 5s. each, as the case may be, the Company at the end of the period contracts to pay the bondholder £50. The entrance fee and subscriptions are dealt with as follows:—

The entrance fee and the first three subscriptions and 20 per-cent of the remaining subscriptions are placed to a fund for expenses of management.

40 per-cent of the subscriptions (after the first three) are placed to a Reserve Fund, and 40 per-cent are placed to a Redemption Fund.

This system of book-keeping, however plausible and well intentioned, cannot in itself ensure, or be relied on for solvency.

20. A simple actuarial calculation makes it clear that, in the case of Companies which promise £50 in return for 120, 144, 147, or even 150, monthly subscriptions of 5s. each, 80 per-cent of the subscriptions after the first three, together with compound interest at 3, 4, or even 5 per-cent, is insufficient to provide for payment of the bonds. The Tables furnished by Mr. Leveaux, of the Actuarial Department of the Friendly Societies' Central Office, which appear in the evidence, establish this. The basis on which these Tables are founded is as follows:

Mr. Leveaux's
Table, p. 111.

21. Mr. Leveaux takes the amount of the annual contribution of the subscriber at £3, and deducts 20 per-cent, which is allocated to expenses of management. This leaves £2. 8s., payable in monthly instalments. Assuming (in favour of the Company) that £2. 8s. is payable in the first year as well as in subsequent years, and that this £2. 8s. is payable in one sum at the

beginning of each year instead of by monthly instalments throughout the year, the calculations work out as follows :

(1) *Ten-year Bonds.* The amount of an annual payment of £2. 8s., payable for 10 years, with compound interest at 3 per-cent, is £28. 8s. The amount payable on the bond at maturity is £50. There is, therefore, in this case, a deficiency of £21. 12s.

If interest be allowed at 4 per-cent, the amount of the annual payments, with compound interest, is £30. 1s. 5d., leaving a deficiency of £19. 18s. 7d.

If interest be allowed at 5 per cent, the deficiency is £18. 1s. 7d.

(2) *Twelve-year Bonds.* Making the calculations in the same way, the deficiency is £14. 16s. 6d., allowing interest at 3 per-cent, £12. 6s. 11d. allowing interest at 4 per-cent, and £9. 11s. 1d. allowing interest at 5 per-cent.

(3) *Twelve-and-a-half-year Bonds.* The deficiency is £13. 1s. 1d., allowing interest at 3 per-cent, £10. 16s. 7d. allowing interest at 4 per-cent, and £7. 4s. 1d. allowing interest at 5 per-cent.

(4) *Fourteen-year Bonds.* The deficiency is £7. 12s. 8d., allowing interest at 3 per-cent, £4. 2s. 6d. allowing interest at 4 per-cent., and 1s. 2d. allowing interest at 5 per-cent.

22. Having regard to these figures, and to the difficulty of earning interest with safety at 5 or even 4 per-cent per annum, it is manifest that, in order to make up the deficiency and to provide for payment of the bonds in full, these Companies must rely on some source or sources of income other than the subscriptions and the interest earned on them. The only available sources are uncalled capital, if any, and lapses and surrenders (see par. 13).

23. In the case of one Company which came under our notice, provision is made for making good this deficiency by means of uncalled capital. The practice in this Company is not to issue more bonds than can be paid in full by means of the subscriptions on the issued bonds with the interest thereon, and the uncalled capital. It works out thus. The £50 bond issued by this Company is practically a twelve-year bond. The subscriptions, with compound interest at 4 per-cent, amount to £37. 13s. 1d., leaving a deficiency of £12. 6s. 11d. The practice is to issue two £10 shares, with £1 called for each bond subscribed, so that there is for each bond subscribed £18 uncalled capital, which is more than enough to meet the deficiency above mentioned.

We consider that this system, provided the shares are held by solvent persons, is a proper and sound one.

Q. 553, 928,
1365, 1724,
1853, 1861-
1867, 2089,
2305, 3266.

24. In all the other Companies of this class which we examined, there was no uncalled capital, at any rate of any substantial amount. These Companies must therefore for solvency rely to a material extent on lapses and surrenders, and the evidence shows beyond doubt that this is so.

Q. 903-909,
1611-1614.

25. A curious suggestion was made to us by more than one witness to the effect that the funds necessary to meet the admitted deficiency on any one bond or series of bonds would be supplied by the subscriptions paid in respect of subsequent bonds; or, in other words, that increase of business in itself supplied the means of making good deficiencies which could not otherwise be made good. This was elaborated in considerable detail by one witness.

Q. 3157-3218.

A fallacy, of course, lies in the assumption that the business will go on indefinitely, and will go on continually increasing. It is evident, too, that if one contract involves a loss of £20, a thousand similar contracts involve a loss of £20,000, and that increase of business of this character merely means the increase of liabilities which the Company is unable to meet, and that the final collapse will be all the greater for being postponed.

26. Having arrived at the conclusion that Companies of Class B must, and do, to a greater or less extent, according to the shorter or longer duration of their bonds, rely for solvency on lapses and surrenders, it becomes material to consider how far this is a sound or desirable basis of finance.

LAPSES AND SURRENDERS.

27. We have had a considerable amount of evidence as to the proportion of lapses which takes place in the earlier years of the currency of a bond. The result of such evidence is as follows:—

See Statements
of Mr. Bewley,
Mr. Partell,
Mr. Haynes,
Mr. Benham,
and Q. 2091,
2190, 2600,
2852-2858,
3396-3399.

The proportion of lapses of bonds during the first year of their currency varies from about 12 per-cent to 54 per-cent. of the total number of bonds issued in that year. The proportion of lapses during the second year varies from about 9 per-cent to 25 per-cent of the total number of bonds in existence at the beginning of the second year. During the third and subsequent years the proportion tends to diminish considerably. One reason for this diminution is that the bond has by that time acquired a value which the holder is unwilling to forfeit; but another and more important reason is that after a bond has acquired a surrender value (which it does usually at the end of three years or thereabouts) surrenders take the place of lapses. The proportion of surrenders is difficult to ascertain, but possibly is not very large.

Q. 2586-2587.

28. In our judgment, and as the result of the evidence, the number of lapses and surrenders which will occur is necessarily an uncertain and fluctuating quantity. It will vary with the character of the subscribers, the locality, the nature of the Company, and, in the case of lapses, with the efforts which the Company may make to prevent or to condone lapses.

Q. 706-710,
1882, 2344-
2345, 3147-
3151.
Q. 2191, 2343

29. An interesting calculation was submitted to us by Mr. Finney, Chartered Accountant, of Liverpool, for the purpose of showing the *lowest* number of lapses and surrenders which would suffice to make it safe to issue bonds for £50 in return for monthly subscriptions of 5s. continued for 10 years and 12 years respectively. In view, however, of the uncertainty as to the number of lapses and surrenders which will actually occur, such calculations do not appear to us of much practical utility. The general conclusions to which we have arrived are as follows:—

Mr. Finney's
Evidence,
2306, *et seq.*,
and papers
handed in.

30. Any system under which, as is admitted by the Companies concerned, a fraction only of the subscribers can hope to obtain payment in full at maturity, and that only through the remainder allowing their contracts to lapse, and thereby suffering loss, must of necessity be a precarious and speculative system of finance, ill adapted to further the cause of true thrift—the advantages of which, nevertheless, are largely advertised in the literature of most of the Companies referred to.

General
conclusions as
to lapses and
surrenders.

31. It is also possible, that as time goes on, and the nature of the business is better understood by the public, the persons subscribing for bonds may tend to belong to a different class, exhibiting a much lower rate of lapse; and in that case the basis of the transaction would be seriously affected.

32. In our opinion, lapses and surrenders should not be relied upon as an element of, or as necessary to, solvency, nor should profit therefrom be in any case depended on, or treated in advance as realized profit. Such profit should be dealt with only after being actually realized; that is to say, the sum to be regarded as profit should be such sum as may remain over after setting aside a proper reserve for all liabilities, estimated on a sound actuarial basis.

Q. 1970, 2346-
2364, 3149.

THE ACCOUNTS KEPT BY THE COMPANIES, AND THEIR PROCEDURE FOR ASCERTAINING THEIR FINANCIAL POSITION AND INFORMING THEIR BONDHOLDERS.

33. The Companies into which we have enquired are almost invariably (though not always) required by their Articles of Association to lay a balance-sheet and profit and loss account before their shareholders at their annual meeting, accompanied by a report of the directors, and a certificate and report from the auditors, as required by Statute (Companies Act, 1900, s. 23).

Q. 427-429.

Q. 427-429. These accounts are in most cases open to the inspection of shareholders, and copies of the accounts are sent to them, though in one case that has come before us, the accounts are not open to the inspection of the shareholders unless a general meeting of the shareholders should determine that they shall be so open. In no case that has come before us are the accounts sent to, or open to the inspection of, the bondholders, who, it must be remembered, are most deeply and materially concerned with the financial soundness of the Company.

Q. 3124. 34. In no case, with one single exception, that has come to our knowledge is any proper revenue account prepared such as is required in the case of Life Assurance Companies under the Life Assurance Companies Act, 1870. The (so-called) Profit and Loss Accounts submitted by certain Companies only deal with management expenses, and in no way indicate the profit or loss made upon the business.

Q. 1013, &c., 2720. 35. Two or three of the Companies which we have considered prepare annual actuarial valuations of assets and liabilities (including liabilities on current bonds). But in most cases no such actuarial valuations are ever prepared, and no case has come before us in which the Memorandum or Articles of the Company make specific provision for having such valuations prepared.

36. The evils which may result from a small capital, from the absence of proper valuations and accounts, and the consequent ignorance of the shareholders and bondholders as to the financial position of the Company, are strongly illustrated by the case of the Perseverance Home Assurance Company, Ltd.

That Company was formed in May 1898, with a nominal capital of £105,000, of which only about £6,000 was issued and paid for in cash. Prospectuses were issued, inviting the attention of "all who desire to invest their savings to the very best advantages with a safe and certain security." Advances on the security of house property were offered, and it was stated that "the terms and conditions of purchasing property under the certificates of this Company are believed to be unequalled by any existing institution in the Kingdom." Bonus investment bonds or certificates were issued, and it was stated that "they offered a very desirable and safe investment for various purposes," such as paying off existing mortgages on property, and providing a fund for enabling parents to article or apprentice their children.

Q. 169. The Company carried on business for about five years. Nearly 5,000 bonds or certificates were taken up by the public in the first four and a half years, of which 2,225 lapsed. Four Reports were issued by the directors to the shareholders, the last being dated in December 1903, and containing the accounts for the year ending

31 December 1902. In each one of these Reports it was stated that the condition of the Company's business was such as to "entitle the shareholders to look forward with confidence to the future," and dividends of 6 and 5 per cent per annum were regularly paid to the preference and ordinary shareholders.

In May 1904, the Company was wound up. It appeared from the evidence of the liquidator, who gave evidence before us, that practically the whole of the £54,000 which had been received by the Company during its existence (being £48,000 from subscribers for bonds and £6,000 from shareholders) had (with the exception of £3,000) gone in payment of management and other expenses, and that the assets consisted of this £3,000, while the admitted debts amounted to between £20,000 and £30,000. It appeared, further, that the house-purchase part of the scheme had been a failure, inasmuch as the Company throughout its whole existence made only ten advances on security of house property.

37. We consider that it is essential in the interests of bondholders that a proper actuarial valuation of assets and liabilities and an abstract thereof should be prepared at intervals of not more than five years, that a proper revenue account and balance sheet should be prepared annually; and that such abstract, revenue account and balance-sheet should be deposited with the Board of Trade, and that printed copies thereof should be forwarded by the Company on application to every shareholder and bondholder. In making this recommendation we act on the precedent made by the Life Assurance Companies Act, 1870; and it is satisfactory to note that the officials of the Companies who gave evidence before us have expressed their readiness to submit to regulations of the character suggested.

REDEMPTION AND ADVANCES WITHOUT INTEREST.

38. We have already stated (par. 9 (*f*)) that, in most of the Companies under consideration, a system exists under which the whole or a part of certain selected bonds is paid to the bondholders in advance of the period of maturity of the bonds.

This system is somewhat inaccurately described as "Redemption." Its effect is to give a considerable, sometimes a very large, gain to the bondholder, and to cause a corresponding loss to the Company.

The bondholder, it should be noted, is under no obligation to continue his subscriptions or to repay to the Company the amount he has received by way of "Redemption." If the bondholder is paid the whole of the amount of his bond, he will of course immediately discontinue his subscriptions, and the Company will lose

Q. 280-289, 290.

Q. 233.

Q. 226, 227.

Recommendation.

Q. 1405, 1790,
2019, 2050,
2090, 2948,
3413.

the benefit of all subscriptions remaining unpaid. If he is only paid a part, he may, if he chooses, discontinue, and if he does, the Company in that case also loses the benefit of all subscriptions remaining unpaid. On the other hand, if the bondholder, after being paid a part of the amount of his bond, continue his subscriptions, the Company loses the interest on the sum so paid.

39. The actual sum applied to Redemptions, and the times at which these take place, appear to be entirely at the will and discretion of the directors.

40. We do not propose to deal with the loss to the Company beyond pointing out that, if the amount of "Redemptions" is large, it must necessarily weaken the financial position of the Company, and may have a serious effect on its solvency. But, apart from the question of loss to the Company, it may fairly be considered whether the system is in itself a desirable one.

41. The selection of bonds for Redemption is usually determined by the application of what is called by the Companies a "mathematical rule," and for simplicity of explanation we will suppose there are only 1,000 bonds of £50 each.

Each bond has 10 coupons of £5 each. The bonds are numbered consecutively from 1 to 1,000. The coupons of Bond No. 1 are numbered from 1 to 10, the coupons of Bond No. 2 from 11 to 20, and so on, so that if there are 1,000 bonds, the highest numbered coupon is 10,000.

"Redemptions" take place periodically, in many cases at intervals of a month, and the holder of a coupon that is "called for redemption" receives £5 in cash. The coupons are "called for redemption" in the following order, viz. :—The first coupons to be called are Coupons 1, 3, 9, 27, 81, 243, 729, 2,187, 6,561, the successive numbers being multiples of three. If the funds of the Company admit of further "Redemptions," the next coupons to be called are determined by reverting back, and are Coupons 2, 6, 18, 54, 162, 486, 1,458, 4,374. The next coupons to be called are determined by reverting back again, and are Coupons 4, 12, 36, &c.

But it is provided that at any one Redemption no bond receives payment of more than one coupon, and that at each successive Redemption a start is made from the lowest numbered coupon in the lowest numbered bond for the time being unredeemed.

It is obvious that, by this system, the bonds bearing the lowest numbers have a far better chance of Redemption than bonds bearing higher numbers, and bonds which have coupons containing high multiples of three have a better chance than those containing lower multiples of three.

The actual "Redemptions" of coupons of £5 each in one of the Companies under review were as follows :

Bonds Numbered	Total Number of Coupons Redeemed
1 — 100	96
101 — 200	12
201 — 300	9
301 — 400	3
401 — 500	3
501 — 600	4
601 — 700	4
701 — 800	2
801 — 900	2
901 — 1,000	1
1,001 — 2,000	10
2,001 — 2,300	4
...	Total ... 150

It will be seen from the above table that while 108 coupons were "Redeemed" on the first 200 bonds, only 42 were "Redeemed" in all on the remaining bonds, which were more than ten times as numerous.

In the case of another Company, it is sought to diminish the advantage enjoyed by bonds bearing low numbers, by starting on the occasion of each successive periodical Redemption with the coupon at which the previous Redemption left off, instead of with the lowest numbered coupon in the lowest numbered unredeemed bond. Q. 1920, &c.

42. The allotment of bonds is by ballot, the numbers being drawn out of a hat, and the selection of coupons for Redemption by the method just described is obviously purely speculative and dependent on chance. The question who are to be the fortunate recipients of the sums paid at the successive Redemptions is, in fact, determined not by any skill or merit on the part of the recipients but by pure chance. Q. 1930, 3371, et seq.

43. The gains accruing to the bondholders under this system are at once speculative and attractive. We have evidence to show that the prospect of being fortunate in the Redemptions operates as a strong inducement to make persons subscribe, and the failure to obtain an early or lucky bond leads persons at once to abandon their subscriptions. Q. 2353, 3080-3083.

44. In the case of one Company, Advances free of interest are allotted from time to time to bondholders, to an extent decided by the Directors, and the selection of the bondholders to receive this benefit is determined by ballot. In this case also it is chance which decides who shall be the recipient of this benefit. Q. 2460. Q. 2479-2498.

45. We have grave doubts whether this system of Redemptions and Advances free of interest does not fall within the prohibition of the Lottery Acts, and is not illegal. The opinions of Sir George Jessel in *Sykes v. Beadon*, 11 Ch. D. 185 and 190, and of Lord Justice Stirling in *Barclay v. Pearson*, 1892, 2 Ch. 152, point strongly in that direction. But whether the system be illegal or not, we think that it is within the mischief of the Lottery Acts, and tends to discourage real saving or thrift. It introduces a purely gambling or speculative element into the transaction, and encourages persons to take up bonds and to subscribe for a limited time, not from any desire to save, but with the hope of making a speculative profit.

Recommendation.

46. The inexpediency of permitting these speculative methods of gain has recently been recognized by the legislature, and Sec. 12 of the Building Societies Act, 1894, prohibits Building Societies established after the passing of the Act from making the granting of an Advance depend on any chance or lot. Following this precedent, we think that it would be desirable to prohibit by legislation the system of Redemptions, or Advances without interest, as described above, at any rate in the case of future Companies.

GENERAL CONSIDERATIONS AS TO THE LEGAL POSITION OF THE BONDHOLDERS.

Recommendation.

47. It has been already pointed out that the bondholders are not in the position either of shareholders or (pending the maturity of the bonds) of creditors of the Companies. One result of this is that, no matter how insolvent the Company may be, they cannot petition for winding up the Company. We think that it is not reasonable that the bondholders should be under this disability, and that it would be proper to give to bondholders the right to petition for winding up the Company on the ground of insolvency. This would be in accordance with the views of the legislature as expressed in the Life Assurance Companies Act, 1870, which by Sec. 21 gave policyholders a similar right to that which we propose should be given to bondholders. It would also, we think, be desirable to lay down by legislation the principles on which bondholders would be entitled to prove in a winding up on lines similar to those on which policyholders are entitled to prove under the Life Assurance Companies Act, 1872, Sec. 5.

RECOMMENDATIONS.

48. (a) We recommend that every Company of the class under consideration which may hereafter commence to carry on business in the United

Kingdom shall be required to deposit in Court a substantial sum, of not less than £10,000, which shall not be paid out until a fund is set apart and secured for bondholders amounting to double the sum deposited. (*See* par. 16.)

(b) We recommend that every such Company shall be required annually to prepare a statement of its revenue account, and of its balance-sheet in a form to be prescribed, and similar, with the necessary modifications, to that prescribed by the Life Assurance Companies Act, 1870, and once at least in every five years to cause an investigation to be made into its financial position by an actuary, and an abstract of such actuary's report to be made in a form to be prescribed, and that every such statement and abstract shall be deposited with the Board of Trade, and printed copies thereof shall be forwarded by the Company, on application, to every shareholder and bondholder. (*See* par. 37.)

(c) We recommend that every such Company which may be formed in the future shall be prohibited from adopting any system of Redemption or Advances without interest such as that which we have described above, or any other system, in which bonds are wholly or partially paid off before maturity or Advances are made without interest according to a scheme founded on chance or lot. (*See* par. 46.)

(d) We recommend that power should be given to the Court to order the winding up of any such Company on the application of one or more bondholders, upon its being proved that the Company is insolvent, and that the claims of bondholders in a winding up should be valued on the same lines as are laid down for valuing the claims of policyholders under the Life Assurance Companies Act, 1872, Sec. 5. (*See* par. 47.)

(e) We think that it would be undesirable for the Government, or any Government department, to undertake the responsibility of any special or direct supervision of Companies of this class, whether by means of Government auditors or otherwise.

49. We desire to place on record the obligations we are under to our Secretary, Mr. H. A. Payne, of the Companies Department of the Board of Trade, for the unremitting industry and ability which he has displayed in the discharge of his duties, and which have materially assisted our labours. Our thanks are also due to Mr. Leveaux for the tables furnished by him, which are referred to in the body of our Report, and to Mr. Coombs, of the Companies Department of the Board of Trade, for

the valuable index to the evidence which he compiled, and the other work which he has done during the temporary absence of Mr. Payne.

We have the honour to be,

Your Lordship's obedient, humble servants,

J. G. BUTCHER (*Chairman*).

HENRY COCKBURN.

J. D. STUART SIM.

G. S. BARNES.

H. A. PAYNE,

Secretary,

30 October 1905.

ACTUARIAL NOTE.

On an alternative type of formula for Approximate Summation.

By W. PALIN ELDBERTON, F.I.A.

I.

IT is sometimes useful, especially when dealing with short periods, to express an area in terms of ordinates both within and without the base on which the area stands.

Let $u_x = a + bx + cx^2 + dx^3$

then $\int_0^1 u_x dx = a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4}.$

But $u_0 = a$

$$u_1 = a + b + c + d$$

$$u_2 = a + 2b + 4c + 8d$$

$$u_3 = a + 3b + 9c + 27d$$

Let $\int_0^1 u_x dx = hu_0 + ku_1 + lu_2 + mu_3.$

Then, equating coefficients of a , b , c and d , after substituting the values of u_0 , u_1 , u_2 and u_3 , in the last expression, the following equations are formed :

$$h + k + l + m = 1$$

$$k + 2l + 3m = \frac{1}{2}$$

$$k + 4l + 9m = \frac{1}{3}$$

$$k + 8l + 27m = \frac{1}{4}.$$

The solution of these equations gives

$$m = \frac{1}{24}, \quad l = -\frac{5}{24}, \quad k = \frac{19}{24} \quad \text{and} \quad h = \frac{9}{24};$$

and

$$\therefore \int_0^1 u_x dx = \frac{1}{24} \{9u_0 + 19u_1 - 5u_2 + u_3\} \quad . \quad . \quad . \quad . \quad . \quad (a)$$

If we had used the same method, but had only taken

$$u_x = a + bx + cx^2$$

the formula
$$\int_0^1 u_x dx = \frac{1}{12} \{5u_0 + 8u_1 - u_2\} \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

would have been obtained.*

The application of these formulæ to the ordinary cases in which approximate summation is required by actuaries would be by taking intervals of say n years and writing

$$\int_0^\infty u_x dx = \int_0^n u_x dx + \int_n^{2n} u_x dx + \int_{2n}^{3n} u_x dx + \dots$$

Then, using (a) this can be written

$$\int_0^\infty u_x dx = \frac{1}{24} \{9u_0 + 28u_1 + 23u_2 + 24u_3 + 24u_4 + 24u_5 + \dots\}$$

or, by using (β)

$$\int_0^\infty u_x dx = \frac{1}{12} \{5u_0 + 13u_1 + 12u_2 + 12u_3 + 12u_4 + \dots\}$$

*Formula (β) has already been given by me, together with some other expressions of a like nature in a recent contribution to *Biometrika*, Vol. iv, Part III, p. 379.

II.

The formulæ of approximate summation (α) and (β) can be applied for finding values for such functions as ${}_{n-1}q_{xy}^1$ in the following way :

$$\begin{aligned} {}_{n-1}q_{xy}^1 &= \int_{n-1}^n t p_{xy} \mu_{x+t} dt \\ &= \frac{5}{12} {}_{n-1}p_{xy} \mu_{x+n-1} + \frac{8}{12} {}_n p_{xy} \mu_{x+n} - \frac{1}{12} {}_{n+1}p_{xy} \mu_{x+n+1} \end{aligned}$$

by the direct application of (β).

As μ is now a regularly tabulated function it seems unnecessary to express it in terms of l .

There would be no difficulty in using the expansion with select functions

$$\begin{aligned} {}_{n-1}q_{[x]:[y]}^1 &= \frac{5}{12} {}_{n-1}p_{[x]:[y]} \mu_{[x]+n-1} \\ &\quad + \frac{8}{12} {}_n p_{[xy]} \mu_{[x]+n} - \frac{1}{12} {}_{n+1}p_{[xy]} \mu_{[x]+n+1}. \end{aligned}$$

III.

The single premium for a short term assurance payable at the moment of death would be

$$\begin{aligned} \int_0^1 v^t {}_t p_{[x]} \mu_{[x]+t} dt &= \frac{5}{12} \mu_{[x]} + \frac{8}{12} {}_1 p_{[x]} \mu_{[x]+1} - \frac{1}{12} v^2 {}_2 p_{[x]} \mu_{[x]+2} \\ &= \frac{5}{12} \mu_{[x]} + \frac{8}{12} \frac{D_{[x]+1}}{D_{[x]}} \mu_{[x]+1} - \frac{1}{12} \frac{D_{[x]+2}}{D_{[x]}} \mu_{[x]+2} \end{aligned}$$

if the term be one year, while a two-year term can be approximated to by taking

$$\int_0^1 v^t {}_t p_{[x]} \mu_{[x]+t} dt + \int_1^2 v^t {}_t p_{[x]} \mu_{[x]+t} dt$$

and approximating to each integral by (α) or (β) ; thus with (β) we get

$$\begin{aligned} &\frac{5}{12} \mu_{[x]} + \frac{8}{12} \frac{D_{[x]+1}}{D_{[x]}} \mu_{[x]+1} - \frac{1}{12} \frac{D_{[x]+2}}{D_{[x]}} \mu_{[x]+2} \\ &\quad + \frac{5}{12} \frac{D_{[x]+1}}{D_{[x]}} \mu_{[x]+1} + \frac{8}{12} \frac{D_{[x]+2}}{D_{[x]}} \mu_{[x]+2} - \frac{D_{[x]+3}}{D_{[x]}} \mu_{[x]+3} \\ &= \frac{1}{12} \left\{ 5\mu_{[x]} + 13 \frac{D_{[x]+1}}{D_{[x]}} \mu_{[x]+1} + 7 \frac{D_{[x]+2}}{D_{[x]}} \mu_{[x]+2} - \frac{D_{[x]+3}}{D_{[x]}} \mu_{[x]+3} \right\}. \end{aligned}$$

Or we might have written the second integral as

$$-\frac{1}{12}\mu_{[x]} + \frac{8}{12}\frac{D_{[x]+1}}{D_{[x]}}\mu_{[x]+1} + \frac{5}{12}\frac{D_{[x]+2}}{D_{[x]}}\mu_{[x]+2},$$

and have obtained the same approximation as that resulting from the application of formula (35) of Chap. xxiv of *Text-Book*, Part II.

IV.

The formulæ (a) and (β) can be applied to find values for A_{xy}^1 for

$$\begin{aligned} A_{xy}^1 &= v \int_0^1 t p_{xy} \mu_{x+t} dt + v^2 \int_1^2 t p_{xy} \mu_{x+t} dt + \dots \\ &= \frac{1}{12} [v \{5u_0 + 8u_1 - u_2\} \\ &\quad + v^2 \{5u_1 + 8u_2 - u_3\} \\ &\quad + v^3 \{5u_2 + 8u_3 - u_4\}, \text{ etc.}] \end{aligned}$$

by (β) where $u_n = {}_n p_{xy} \mu_{x+n}$

$$\begin{aligned} &= \frac{5}{12} v u_0 + \frac{1}{12} u_1 + \frac{5v^2 + 8v - 1}{12v} \{v u_1 + v^2 u_2 + v^3 u_3 + \dots\} \\ &= \frac{5}{12} v \mu_x + \frac{1}{12} p_{xy} \mu_{x+1} + \frac{V}{12 D_y l_x} \{D_{y+1}(l\mu)_{x+1} \\ &\quad + D_{y+2}(l\mu)_{x+2} + \dots\} \\ &= \frac{1}{12 D_y l_x} [5v D_y (l\mu)_x + (1+i) D_{y+1}(l\mu)_{x+1} \\ &\quad + V \{D_{y+1}(l\mu)_{x+1} + \dots\}] \end{aligned}$$

$$\text{where } V = \frac{5v^2 + 8v - 1}{v}.$$

The value of V is independent of x and y ; the two terms $5v D_y (l\mu)_x$ and $(1+i) D_{y+1}(l\mu)_{x+1}$ could be easily calculated, and from the remainder of the expression it is clear that a commutation column $(l\mu)$ would enable us to make a table of A_{xy}^1 with little more trouble than the present method which assumes an even distribution of deaths.

It would be as satisfactory to form a commutation column $(D\mu)_x$ for the expression for u_n can be written $\frac{l_{y+n}(D\mu)_{x+n}}{l_y D_x}$.

V.

Another use to which formulæ (α) and (β) can be put is to approximate to continuous annuities, thus :

$$\begin{aligned}\bar{a}_x &= \int_0^\infty v^t p_x dt \\ &= \frac{1}{12} \{ 5 + 13vp_x + 12v^2_2 p_x + 12v^3_3 p_x + \dots \} \text{ by } (\beta) \\ &= a_x + \frac{1}{2} - \frac{1}{12} \{ 1 - vp_x \} \\ &= a_x + \frac{1}{2} + \frac{1}{12} \frac{\Delta D_x}{D_x} \\ \text{or } \bar{a}_x &= \frac{1}{24} \{ 9 + 28vp_x + 23v^2_2 p_x + 24v^3_3 p_x \\ &\quad + 24v^4_4 p_x + \dots \} \text{ by } (\alpha) \\ &= a_x + \frac{1}{2} - \frac{1}{24} \{ 3 - 4vp_x + v^2_2 p_x \} \\ &= a_x + \frac{1}{2} + \frac{2\Delta - \Delta^2}{24D_x}\end{aligned}$$

which is the same as formula (21) in § 25 of Chap. ix of *Text-Book*, Part II.

CORRESPONDENCE.

ON THE DETERMINATION OF AVERAGE AGES BY METHODS OF WEIGHTING.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the discussion which arose on Mr. Ackland's paper on an approximate method of Valuation of Whole-life Assurances, with allowance for selection, the fact was commented on by more than one speaker that, in determining the mean age or centre of gravity of a series, the terms of which can be represented by the expression $A + Br^x$, an increase in the value of r will cause an increase in the mean age. As this method of finding an average age appears likely to become of general use, it may be of interest to readers of the *Journal* to have a mathematical proof of the following general proposition :

$$\text{Given } \bar{x} = \frac{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}}{S_1 + S_2 + \dots + S_n},$$

where x_1, x_2, \dots, x_n and S_1, S_2, \dots, S_n are any positive quantities independent of r , to prove that for all positive values of r an increase in the value of r will cause an increase in the value of \bar{x} .

Taking logarithms of both sides of the above equation and dividing by $\log r$, we have—

$$\bar{x} = \frac{\log(S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}) - \log(S_1 + S_2 + \dots + S_n)}{\log r}.$$

An increase in r in the above expression will cause an increase in \bar{x} if

$$\frac{d}{dr} \left[\frac{\log(S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}) - \log(S_1 + S_2 + \dots + S_n)}{\log r} \right]$$

is positive; i.e. if

$$\frac{(S_1 x_1 r^{x_1-1} + S_2 x_2 r^{x_2-1} + \dots + S_n x_n r^{x_n-1})}{(S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n})} \frac{\log r}{(\log r)^2} - \frac{\bar{x} \log r}{r (\log r)^2}$$

is positive; i.e. if

$$\left[\frac{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \dots + S_n x_n r^{x_n}}{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}} \log r - \bar{x} \log r \right]$$

is positive; i.e. if

$$r \frac{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \dots + S_n x_n r^{x_n}}{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}} > r \bar{x},$$

if

$$r \frac{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \dots + S_n x_n r^{x_n}}{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}} > \left(\frac{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}}{S_1 + S_2 + \dots + S_n} \right) S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}$$

$$) S_1 r^{x_1} (r^{x_2}) S_2 r^{x_2} (r^{x_3}) S_3 r^{x_3} \dots (r^{x_n}) S_n r^{x_n} > \left(\frac{S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}}{S_1 + S_2 + \dots + S_n} \right) S_1 r^{x_1} + S_2 r^{x_2} + \dots + S_n r^{x_n}$$

$$A^{S_1 A} B^{S_2 B} C^{S_3 C} \dots N^{S_n N} > \left(\frac{S_1 A + S_2 B + \dots + S_n N}{S_1 + S_2 + \dots + S_n} \right)^{S_1 A + S_2 B + \dots + S_n N}$$

where $A = r^{x_1}$, $B = r^{x_2}$, . . . $N = r^{x_n}$.

Now, it is a well-known theorem in algebra that if we have any number, say n , of positive quantities, $A, B, C \dots N$, which are not all equal, then

$$A^A B^B C^C \dots N^N > \left(\frac{A + B + C + \dots + N}{n} \right)^{A+B+C+\dots+N}$$

This inequality is perfectly general and will hold for any number of factors.

Take S_1 quantities each = A

.. S_2 = B

.

.

.. S_n = N .

The result of substituting $O^{[M]}$ net premiums for net premiums by the O^M Table in the O^M 3 per-cent valuation can be very easily obtained by the aid of a formula given by Mr. King (*J.I.A.*, xxxvii, p. 465). The result by the combined basis, which may be described as $O^{[M]}$ and O^M , 3 per-cent, is as follows:

Basis of Valuation	Value of Sums Assured	Value of Net Premiums	Actual Reserves
$O^{[M]}$ and O^M , 3 %	1,449,006	750,324	698,682

Here the value of the net premiums is practically the same as in the Select valuation. The difference in the value of the sums assured remains, and on the whole the result is a fairly close approximation to the Select valuation.

$O^{[M]}$ net premiums can be employed in other combinations, and various results for Model Office, No. 1, at the end of 50 years, are given in the following table in comparison with valuations involving O^M net premiums. The results of using ultimate factors after five years, and throughout, have been obtained by the aid of Mr. Diver's table of the values of $\phi_{[x]+t}$ (Table I in his paper). The valuations are all on the basis of interest at 3 per-cent.

Basis for Net Premiums	Basis for Valuation Factors	Actual Reserve	Comparative Reserve O^M , 3 % = 10,000	Comparative Reserve $O^{[M]}$, 3 % = 10,000
O^M	O^M	688,999	10,000	9,837
O^M	O^M first five years $O^{M(5)}$ after five years	693,884	10,071	9,907
O^M	$O^{M(5)}$ throughout	698,214	10,134	9,969
$O^{[M]}$	O^M	698,682	10,141	9,975
$O^{[M]}$	$O^{[M]}$	700,420	10,166	10,000
$O^{[M]}$	$O^{[M]}$ first five years Ultimate factors after five years	700,815	10,171	10,006
$O^{[M]}$	O^M first five years $O^{M(5)}$ after five years	703,580	10,212	10,045
$O^{[M]}$	Ultimate factors throughout . .	705,147	10,234	10,067
$O^{[M]}$	$O^{M(5)}$ throughout	707,911	10,274	10,107

It may be remarked in passing that, if the valuation be made by Select tables, the difference made by employing ultimate factors instead of Select after five years is less than the error that may be introduced by using nearest ages at entry and at valuation in place of exact ages, and is in fact for practical purposes inappreciable.

The combination of $O^{[M]}$ net premiums with $O^{M(5)}$ valuation factors

throughout may appear to be excessively stringent; but it differs from the $O^{(M)}$ Select valuation less than the latter from the $H^{(M)}$; and in the case of an office whose mortality has the same relation to the $O^{(M)}$ Table that the latter has to the $H^{(M)}$ Table the combination $O^{(M)}$ and $O^{(G)}$ throughout would therefore not be so severe as a Select net premium valuation based upon its own experience.

The conclusion may be suggested that in the case of valuations which aim at the highest standard, net premiums based upon Aggregate tables should be abandoned in favour of Select net premiums, whether the valuations are based on Select tables in other respects or not.

I am, Sir,

Yours faithfully,

DUNCAN C. FRASER.

1, North John Street,

Liverpool.

5 December 1905.

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## CONTENTS OF THIS NUMBER.

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List of Members, as on 1 January 1906.

---

- I. On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables. By George King, F.I.A., F.F.A., one of the Vice-Presidents of the Institute of Actuaries.
  - II. On a Property of the  $O^{[M]}$  Select Tables, and its Application to the Valuation of Whole-Life Policies. By O. F. Diver, M.A., F.I.A., of the Clerical, Medical and General Life Assurance Society.
  - III. Notes on an approximate Method of Valuation of Whole-Life Assurances, with Allowance for Selection. By Thomas G. Ackland, one of the Vice-Presidents of the Institute of Actuaries; Honorary Fellow of the Faculty of Actuaries; with Appendix.  
Abstract of the Discussion on the above three Papers.
  - IV. Report of the Departmental Committee on Bond Investment Companies.
  - V. Actuarial Note.
  - VI. Correspondence.
  - VII. The Institute of Actuaries.
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VOL. XL.

PART II.

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES.

No. 224.—APRIL 1906.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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VOL. XL. — PART II.

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## CONTENTS OF NO. 224.

|                                                                                                                                                                                                                                                                                                                                                                                                      | PAGE |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| Canadian Vital Statistics; with particular reference to the Province of Ontario. By M. D. Grant, B.A., F.I.A., Assistant Actuary, Government Insurance Department, Ottawa, Canada .....                                                                                                                                                                                                              | 125  |
| Abstract of the Discussion on the preceding.....                                                                                                                                                                                                                                                                                                                                                     | 143  |
| The Variations in Masculinity under different conditions. By John Norman Lewis, F.I.A., F.F.A., and Charles James Lewis, D.Sc., M.D., F.R.C.P. (Edinburgh) .....                                                                                                                                                                                                                                     | 154  |
| Abstract of the Discussion on the preceding .....                                                                                                                                                                                                                                                                                                                                                    | 181  |
| On some Special Features of Widows' and Orphans' Funds. By Ernest Charles Thomas, F.I.A., of the Gresham Life Assurance Society ..                                                                                                                                                                                                                                                                   | 188  |
| On the Calculation of the Contributions to be made to an Annuity Fund for Widows and Children, by the members of a Society, all of whom, whether married or single, are obliged to contribute. By Oscar Schjoll, formerly Manager (Kontorchef) of the "Idun" Life Insurance Company, Christiania; Manager of the Royal Office for Workmen's Assurances (Rigsforsikringsanstalten), Christiania ..... | 200  |
| Historical Memorandum on Friendly Societies in connection with Actuarial Certificates. (Extracted from the Report of the Chief Registrar of Friendly Societies for the year 1904) .....                                                                                                                                                                                                              | 212  |

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## JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

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*Canadian Vital Statistics; with particular reference to the Province of Ontario. By M. D. GRANT, B.A., F.I.A., Assistant Actuary, Government Insurance Department, Ottawa, Canada.*

[Read before the Institute, 18 December 1905.]

INVESTIGATIONS of the mortality prevailing in Canada, whether among assured lives or the population at large, have not up to the present been frequent. Such enquiries demand, first of all, a considerable extent and precision of data, and these are seldom furnished by the records of such private corporations as do business in a new country, or by the periodical censuses undertaken by Government. As to assured lives, most of the companies now transacting the business of life assurance in Canada are recent entrants into the arena, and of such of them as are not to be included in this class, it is yet true that most of their assurances have been effected at comparatively recent dates; so that an investigation into the mortality of the lives, whether undertaken by a single company (supposing its business to be of sufficient magnitude) or by any combination of companies, would be subject to the same defects as are chargeable against the Thirty American Offices' Experience, and against many others, for all of which this example—Titanic in all respects—may stand. Nevertheless, one Canadian office—the

largest and oldest domestic company—some years ago undertook such an investigation, and published the results thereof, embracing the period of forty-six years from its foundation in 1847 to the year 1893.\* Of the character and extent of this experience it will here be sufficient to say that it was confined to male lives ; that the preponderating mass of the assurances were upon the whole-life plan ; that the total number of entrants was 35,287, of whom approximately 8 per-cent had died, 37 per-cent had passed out by withdrawal or maturity, and the remaining 55 per-cent had emerged alive at the date when observations were closed ; and that the average period of exposure in the total experience was 8.36 years. Respectably high as the average period of exposure seems, too much must not be inferred from this single fact, for that high average has been largely contributed to by the young lives, among whom the mortality is of necessity light. Of the total years of exposure, more than one-half belongs to ages below 40, and four-fifths to ages below 50 ; whence results the small percentage of deaths for so high an average exposure, relatively to such *ripe* experiences as the Equitable, the Amicable, the Institute of Actuaries' H<sup>M</sup>, the Gotha Life, and the British Offices' O<sup>M</sup>. Apart from the mere magnitude of the data comprised in an experience, these two features, indeed, namely, the percentage of deaths and the average period of exposure, considered together, would seem to constitute the best rough test of the general sufficiency and distribution of the materials upon which any investigation is based. It is to be said of this well-conducted and, on the whole, valuable experience which we are now rapidly reviewing, that the relative inequality of the exposures, considered with reference to the several age groups, and the element of recency of selection, which is to a considerable extent inherent in very important tracts of the main aggregate table, must invalidate its claims to be considered completely representative of the true quality of assured lives in Canada. The truncated mortality table, founded upon an exclusion of the first five years of assurance, or about 43 per-cent of the total exposures, is necessarily of an indeterminate character.

The total number of deaths embraced in the above-noted experience amounts to approximately 71 and 74 per-cent of the number expected according to the H<sup>M</sup> and American Experience

\* *Mortality Experience of the Canada Life Assurance Company*. Times Printing Co., Hamilton, Ont., 1895. The general mortality table deduced from the observations was graduated in accordance with the first modification of Makeham's law.



Tables, respectively, and to about 79 per-cent of the number given by the O<sup>M</sup> Table; wherefore, taking the Company's Table as the standard, the mortality among assured lives in Canada runs from 20 to 40 per-cent, according to age, below that exhibited by the working tables of the British Isles and the United States. Corroborative evidence is also to be found in the fact that after the materials used in the Thirty American Offices' Experience had been collected and reduced, it was shown that the actual loss of the contributing companies in respect of their business in British America was only about 73 per-cent of the expected loss.\* So that, making all allowances proper to the case, and looking somewhat also to general causes, there seems substantial reason for the prevailing belief that the true mortality of Canadian assured, and therefore with strong presumption the mortality of the people as a whole, will be found to fall well below that of most other English-speaking countries, like classes being compared with like.

But whether life offices operating in Canada are wise in assuming the difference between their actual and expected losses, as determined by the standard tables of other countries, to be wholly of the nature of true profit, whereof no part need be set aside for "suspended mortality," is a question that deserves consideration.

Such being the condition of affairs in the field of life assurance, commonly most productive of authentic and precise materials, it remains to review, however slightly, the situation in respect of the general population. To this end, it will be expedient to preface the subject by a brief, but needful, reference to general conditions.

Canada is a federation of British States,† which occupy the northern half of the North American Continent, and stretch, north to south, from the 49th degree of north latitude to beyond the Arctic circle, and, east to west, from the Atlantic to the Pacific. Of

\* *System and Tables of Life Insurance*, p. 167. The "probable loss" there referred to—the "expected loss" of the text—was calculated by what Mr. Meech calls the "Common American Table of 1858." This is the table which Mr. Sheppard Homans constructed from the fifteen years' experience of the Mutual Life of New York, and is not, as some appear to have thought, the American Experience Table, which was not produced until 1867. The former table conformed more nearly than the latter to the experience of British companies.

† At present the political texture consists of nine Provinces and six Territories, whereof the Provinces of Quebec and Ontario possess seven-tenths of the total population of the country. Alberta and Saskatchewan were created Provinces by Act of Parliament, 1905.

this vast area, free from deserts as it is, and possessed of a singularly noble system of waterways, two-thirds at least is capable of sustaining civilized man in ease and abundance, and only the extreme hyperborean tracts are irreclaimable from Nature. In the Maritime Provinces of the east, the climate corresponds fairly well with the latitude; but as to the immense west and north-west, it were useless for one to sit down before his globes and hope, solely by measurement of distances from the equator, to arrive at correct conclusions respecting the climate. Generalizations of this nature, which omitted to take into account the Gulf Stream, would catalogue the British Isles as ice-bound during the greater part of the year. And the great Japan Current, which laves the north-western shores of America, is, like the Gulf Stream in northern Europe, a wonder-worker on this Continent. It gives to Southern British Columbia an almost sub-tropical vegetation; and its servants, the winds, leaping the barriers of the Rockies—the more easily in proportion to the decreasing altitude northwards—produce the most astonishing modifications of climate throughout an area fully twelve hundred miles square, extending eastwards as far as the shores of Hudson's Bay.\* It is impossible for any but the dullest or most atrabilious of mankind to dwell in mind upon the resources of this remarkable region without feeling that the time must come when busy populations shall possess it and civilizations flourish there.

After this somewhat exuberant, but truthful, description, it will seem like bathos to state that the population of Canada at the present day is probably little more than six millions. Such, however, is the fact. In reality, population is confined to a belt, generally narrow, but of varying width, spanning the Continent just north of the United States boundary. Up to the end of the century just closed, the stream of immigration, though fairly constant, was small, and was much more than offset by the heavy tribute of young blood exacted from Canada by the unparalleled industrial prosperity of the neighbouring Republic. The Eastern Provinces still pay this constant tribute, but in the West the tide has at last set the other way, and its rich alluvial plains exercise

\* The isothermal lines for this region do not even approximately follow the parallels of latitude, but run sharply north-west as one advances from the longitude of Lake Superior to the Pacific. Thus at Dawson City in the Yukon (latitude  $64^{\circ}+$ ), oats, turnips, celery, cauliflowers, etc., are raised in abundance, and at Fort Macpherson (some distance within the Arctic Circle), the potato is successfully cultivated to maturity.

a magnetic influence on the men of the south. Since the opening of the new century, however, the stream of immigration has begun to widen and deepen, and, what is of greater account, its flow is tolerably pure, being composed of persons chiefly of the Teutonic and Celtic races, with only a sprinkling of the parasitic classes and of nondescripts.\* I take it to be a scientific truth, however much Darwinians theorizing on the plastic and cosmopolitan nature of physiological man may dissent, that no race has ever yet been able to maintain itself for long when removed from its ancient habitat to one which differs widely in climate. Physicians tell us that there is no such thing as the acclimatization of an inhabitant of northern Europe to the malarious climates of the south: the fifth generation is quite as liable to febrile attacks as the original immigrants; whereas they of the soil always suffer incomparably less.† Conversely, it is fairly certain that the negro, for example, unsupported by immigration, cannot last beyond a few generations north of 45° N. lat.: a fact for which Canada should be thankful, as often as she remembers the proportions to which the negro problem has grown in the southern States of America. It should be part of the established polity of States which desire accessions of population so to guide or check the immigration that these high treasons against Nature may not be committed.

In view, then, of the conditions, everyone must be impressed at the outset with the great difficulties to be met with in a country of such wide extent, thinly peopled and with a percentage of the population always on the wing, in following accurately the movements of the people, registering their number and the births and deaths among them, and otherwise endeavouring to attain precision in the various departments of vital statistics. Effectively to do this would require the unceasing attention and

\* During the year ended June 30, 1903, 128,364 declared settlers entered the country, as against an average of less than 40,000 per annum for the previous decade. Of this number, approximately 32 per-cent came from the British Isles, and 38 per-cent from the United States. For the corresponding year ended 1904, 130,331 settlers entered: 39 per-cent being from the British Isles, and 35 per-cent from the United States. The figures for the year ended June 1905, are 146,266, with percentages of 45 and 30 from the British Isles and United States respectively. See also an article in the *Nineteenth Century* for December 1902; vol. lli, p. 910, *et seq.*

† Witness the mortality of the British in India, of the French in Morocco, of Europeans on the West Coast of Africa, &c. Almost unlimited examples might be given of the heavy mortality experienced by European troops serving in the tropics, the native troops suffering little in comparison. The capacity of resisting the diseases peculiar to any climate lies deep in the fibres of a race, and in the long run will give that race supremacy over the soil.

training to be found only in a permanent bureau, operating under enlightened methods. But a permanent bureau does not exist in Canada (although the nucleus of one has been provided for quite recently by Parliament), and for the solution of the complex problem we have as available apparatus, first, registration systems of births, deaths, and marriages in most of the political divisions;\* and secondly, a decennial census, taken synchronously with those of Great Britain and her other Colonies. The registration system of Ontario has been in operation for many years, and may now be said to be an efficient one; those of the other divisions have been instituted more recently, and are admittedly more imperfect. In respect of censal statistics, Canada is perhaps neither much better nor much worse off than other British countries. The New World indeed reached its zenith in this department many years ago under the Inca Emperors of Peru, when, according to the historian, Prescott, "a register was kept of all the births and deaths throughout the country, and *exact returns of the actual population were made to the Government every year.*" In view of such an ideally wise practice, the agitation in Britain and the United States for a quinquennial census seems modest indeed.

The census returns of Canada, besides giving the results of actual enumerations of the people, furnish statistics illustrative of the usual variety of subjects; but these need not concern us here.† The population is given by Provinces for groups of ages, and it has been customary also to compile, likewise for groups of ages, the deaths occurring in the year ended on Census day. For purposes of determining rates of mortality, it is notorious, of course, that while the population figures may on the whole be accepted, the deaths so given are untrustworthy, being generally much understated. In the census of 1901, however, the authorities undertook to determine with the utmost possible accuracy the deaths of the census year in the Province of Ontario—which Province alone contains about two-fifths of the total population of the Dominion. With this end in view, the census authorities collaborated with the Registrar of Ontario; the two

\* Registration is a matter lying solely within the jurisdictions of the Provincial and Territorial legislatures, but has not hitherto received from those bodies the attention which it deserves. Nova Scotia has no registration.

† Those desirous of general information as to the state of Canada should consult, in addition to census reports, the *Statistical Year Book of Canada*, published by the Government. This book, now in its twentieth year of issue, gives in synoptical form information regarding the resources, development and general condition of the country from time to time.



independent sets of death schedules were compared seriatim, and, where differences existed, the individual cases were patiently investigated until the truth was arrived at. There is, therefore, every reason to believe that considerable reliance may be placed upon the Ontario deaths for the year in question.

From what has been said, it will be understood that no data is in existence for the calculation of rates of mortality applicable to all Canada. Nor would it be desirable, I believe, to proceed to the calculation of such rates; for the French Province of Quebec has always exhibited a considerably higher mortality than the rest of Canada, while Manitoba, British Columbia, and the Territories are constantly receiving fresh accessions of young persons of the strongest physique, and consequently show abnormally low death rates. To investigate the cases of Quebec and Ontario separately, however, would be roughly equivalent to investigating the mortality of French-speaking and English-speaking Canadians. Leaving Quebec out of account, as at present affording no data, it is to be said of Ontario that it is broadly typical in ethnological respects of the remainder of Canada; that it contains between fifty and sixty per-cent of the *English-speaking* population of the country, has been comparatively long established, and is not subject to sudden increases of population—immigration being balanced, if not over-balanced, by emigration. For these reasons, the case of Ontario may be examined more particularly.

But any present determination of the mortality of Ontario must hinge upon the deaths of a single year, which, however, have been arrived at by methods which make for accuracy. These are given by quinary age-periods as far as age 15; from 15 to 74, by decennial age-periods, but beyond the latter age are lumped under the provoking sign-post, "75 and over." The population figures, on the other hand, are tabulated by quinary age-groups to the limit of life. Although the defects arising from this inconsistent usage may to some extent be overcome, there is much to be said against deriving mortality rates from the deaths of a single year, however accurate.\* If the selected year be in any way an abnormal one, specially healthy or specially unhealthy, what value shall be set upon the resulting figures? This being admitted, it can only be said that in Ontario the year in question was to all appearances a quite normal one, and that to

\*As an example of the employment of such material, all students of the Institute will recollect a paper of the late A. F. BurrIDGE, "On the Rates of Mortality in Victoria, &c.," *J.I.A.*, xxiii, 309.



know approximately the mortality at a certain epoch may subserve useful purposes, present and future.

### INFANTILE MORTALITY.

For determination of the rates of mortality at infantile ages, we must discard the population figures of the census, and deal with the births. These, as obtained from the reports of the Registrar-General for Ontario, are given for the requisite number of years in Table I, and along with them the infantile deaths of the census year. No figures are available for examination of the mortality in the several sections into which the first year of life is commonly divided.

TABLE I.—ONTARIO.  
*Births and Deaths as specified.*

| Year ended<br>31st March | BIRTHS |         | Ages         | DEATHS, CENSUS YEAR 1900-1 |         |
|--------------------------|--------|---------|--------------|----------------------------|---------|
|                          | Males  | Females |              | Males                      | Females |
| 1896                     | 21,534 | 20,238  | Under 1 year | 3,740                      | 2,957   |
| 1897                     | 25,200 | 23,399  | 1 to 2 years | 730                        | 633     |
| 1898                     | 24,068 | 22,710  | 2 to 3 "     | 343                        | 293     |
| 1899                     | 23,929 | 22,424  | 3 to 4 "     | 217                        | 218     |
| 1900                     | 23,062 | 21,717  | 4 to 5 "     | 179                        | 158     |
| 1901                     | 23,774 | 22,257  | Total ...    | 5,209                      | 4,259   |

If it suited the present purpose to employ the births by calendar years, the usual assumption regarding their uniform distribution throughout any one such year would, I find on investigation, be admissible. But where the unit of time most convenient for the purpose runs, as in the present case, from 1st April to 31st March, the births of the first half-year will considerably outnumber those of the second half-year, and an appreciable error would be introduced by proceeding upon the usual assumption. Having tabulated by months the births for the six years given above, and having combined all those which occurred between 1st April and 30th September during the six years, and likewise those which occurred between 1st October and 31st March, I found the result to be:—

|                                       | Males  | Females |
|---------------------------------------|--------|---------|
| Aggregate births first half-years ... | 72,728 | 68,179  |
| „ „ second half-years...              | 68,839 | 64,566  |

The ratio of excess of first half-years over second is thus approximately the same for both sexes, and instead of assuming, for example, that the births for the two years ended 31st March

1901, take place as at 1st April 1900, it would be more correct to refer them to a point of time five days earlier. In order to give effect to this conclusion, the rate of mortality at age 0 was derived from the expression,

$$q_0 = \frac{\frac{365}{370} \cdot (\text{Deaths under one year, 1900-1})}{\frac{1}{2} \cdot (\text{Births 1899-1900} + \text{Births 1900-1})}$$

The same principle was followed in determining  $q_1 \dots q_4$ , and no further adjustment was made except in the case of  $q_4$ , where, for the reason that the births of the year ended March 1896 are quite obviously too small,\* a lower rate was substituted, which gave a smoother junction with the main curve. The rates thus obtained are set forth in Table II, where are also given for general comparison the infantile rates of the "New English Life Table", derived from data of the intercensal period, 1881-90, and likewise of a "Life Table for Scotland," recently published, based upon data of the decennial period terminated 1900.†

TABLE II.—INFANTILE RATES OF MORTALITY, ONTARIO, ENGLAND AND WALES, AND SCOTLAND.

| Age | ONTARIO, 1901 |         | ENGLAND AND WALES,<br>1881-90 |         | SCOTLAND, 1891-1900 |         |
|-----|---------------|---------|-------------------------------|---------|---------------------|---------|
|     | Males         | Females | Males                         | Females | Males               | Females |
| 0   | ·15755        | ·13267  | ·16104                        | ·13113  | ·14343              | ·11789  |
| 1   | ·03640        | ·03262  | ·05730                        | ·05271  | ·05287              | ·05027  |
| 2   | ·01738        | ·01526  | ·02383                        | ·02300  | ·02214              | ·02188  |
| 3   | ·01090        | ·01032  | ·01539                        | ·01513  | ·01324              | ·01344  |
| 4   | ·00777        | ·00764  | ·01141                        | ·01102  | ·00934              | ·00970  |

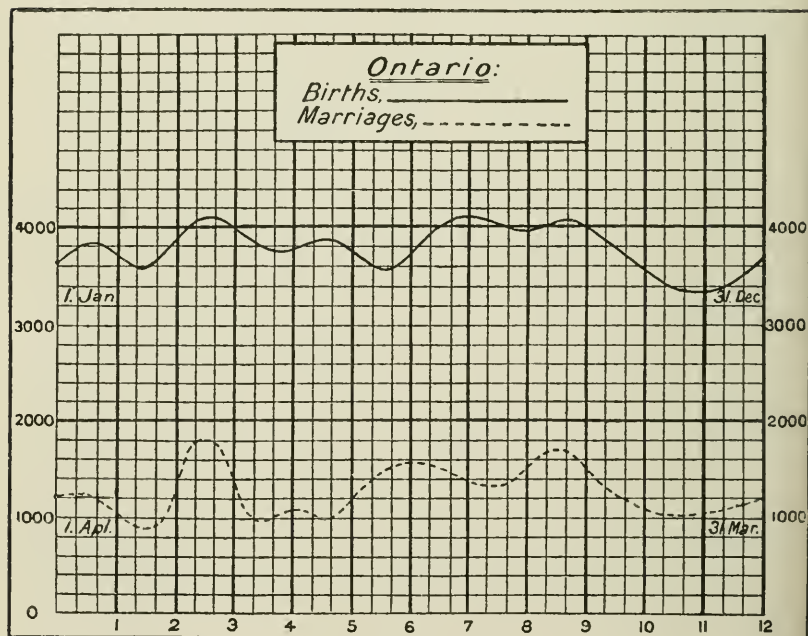
It is proper to point out that the rates for Ontario given above are in all probability overstated. The Registrar-General has frequently complained that, while the provisions of the Registration Act are in general well complied with in respect of marriages and deaths, there is more laxity in reporting births. Such a condition, if true, would operate to reduce the true denominator of the fraction, and thus give rise to rates greater than those actually prevailing. It will be observed, however, that, with certain exceptions in the first year of life, the rates as

\* In 1896 certain amendments were made in the Registration Act, and the direct result seems to have been, among other things, a better return of births for the subsequent years.

† "Life Table for Scotland, based on the Census Enumerations of 1891 and 1901, &c.," by Mr. T. Adam, M.A. *Journal of the Royal Statistical Society*, vol. lxvii, p. 448.

they stand run lower than those of the home countries, the differences being on the whole not inconsiderable. To the many who believe that the mortality among infants is to be regarded as a good criterion of the general health of a given population, this fact will seem to bear a broad application.

Reference has been made to the notably unequal distribution of births throughout the various months of the year, and before proceeding it may be well to add a further word upon this point. To re-state the fact,—on an average of the six yearly periods referred to above, out of every 1,000 births of both sexes, 513·7 took place during the first half, and 486·3 during the second half, of the year ending 31 March. The corresponding figures for England and Wales, for an average of eleven years ending with the same point of time, are 504·7 and 495·3 respectively. It is obvious, of course, that the cause of the disproportion in Ontario must be sought for in some accidental circumstance, quite remote from the influence of the seasons upon reproduction itself; and that it is in fact to be ascribed to the marriages being celebrated, for climatic and sentimental reasons, in much larger numbers at certain seasons than at others. The accompanying diagram exhibits graphically the yearly birth and marriage curves, being based on an average of the six years in question.



The origin of the marriage curve has been placed nine months earlier than that of the birth curve, in order to allow roughly for the period between time of marriage and birth of children. Comparison being thus facilitated, the similarity of type manifested by the two curves is seen to be as striking as could have been anticipated; showing clearly that that portion of the annual births which remains after elimination of the first-born must occur with approximately equal frequency in equal parts of any year.

The birth-rate of Ontario, measured as a percentage of the population, has been steadily declining for a number of years, a fact frequently commented upon by the Registrar-General and by many others who dislike the appearance of this sinister symptom. But whether the fact be attributable to a diminished number of births per marriage,—due weight being given to the relative numbers of wives in the various age-groups,—or to a gradual

TABLE III.—ONTARIO.  
*Population and Mean Population, 1901.*

| Ages         | POPULATION, 1901 CENSUS |           | MEAN POPULATION, CENSUS YEAR |           |
|--------------|-------------------------|-----------|------------------------------|-----------|
|              | Males                   | Females   | Males                        | Females   |
| 0            | 23,600                  | 22,813    | 23,635                       | 22,805    |
| 1            | 20,537                  | 19,671    | 20,567                       | 19,665    |
| 2            | 22,710                  | 22,334    | 22,743                       | 22,327    |
| 3            | 22,972                  | 22,710    | 23,005                       | 22,702    |
| 4            | 23,993                  | 23,242    | 24,027                       | 23,234    |
| 0-4          | 113,812                 | 110,770   | 113,977                      | 110,733   |
| 5-9          | 116,678                 | 113,651   | 116,846                      | 113,613   |
| 10-14        | 116,588                 | 113,024   | 116,768                      | 112,986   |
| 15-19        | 114,848                 | 114,281   | 115,014                      | 114,243   |
| 20-24        | 104,322                 | 111,146   | 104,473                      | 111,109   |
| 25-29        | 86,588                  | 91,540    | 86,713                       | 91,509    |
| 30-34        | 76,099                  | 78,365    | 76,209                       | 78,339    |
| 35-39        | 71,836                  | 71,761    | 71,939                       | 71,737    |
| 40-44        | 64,243                  | 62,470    | 64,336                       | 62,449    |
| 45-49        | 53,474                  | 50,681    | 53,551                       | 50,664    |
| 50-54        | 45,134                  | 43,757    | 45,199                       | 43,742    |
| 55-59        | 35,645                  | 34,780    | 35,696                       | 34,768    |
| 60-64        | 31,824                  | 30,674    | 31,870                       | 30,664    |
| 65-69        | 24,519                  | 22,616    | 24,555                       | 22,609    |
| 70-74        | 17,897                  | 16,685    | 17,922                       | 16,680    |
| 75-79        | 11,146                  | 10,109    | 11,162                       | 10,106    |
| 80-84        | 6,103                   | 5,461     | 6,112                        | 5,459     |
| 85-89        | 2,132                   | 1,986     | 2,135                        | 1,985     |
| 90-94        | 599                     | 612       | 600                          | 612       |
| 95 & over    | 186                     | 186       | 186                          | 186       |
| Unclassified | 2,965                   | 1,754     | ...                          | ...       |
| All Ages     | 1,096,638               | 1,086,309 | 1,095,263                    | 1,084,193 |

postponement of the average marriage-age, resulting directly in a shortening of the child-bearing period, has never been determined. Statistics as to the ages of the married,—a feature of all our censuses in the past,—were collected in 1901; but the census officials having concluded that they cannot spare the time necessary for their reduction, it is not possible for the present to institute a rigorous analysis such as would be likely to indicate the true solution.

#### MORTALITY AT AGES ABOVE FIVE YEARS.

Although the work of the Canadian census for 1901 is now completed, some of the figures proper to the present enquiry have not yet been made public. By the courtesy of the Census Commissioner, however, I was permitted to see the schedules. In Table III is given the population of Ontario according to the census. The "unclassified" having been distributed rateably over the various age-groups, the mean population of the census year was then computed,\* and is set down in the same Table.

Table IV exhibits the deaths of the year, and the central rates

TABLE IV.—ONTARIO.  
*Deaths and Central Death Rates, 1901.*

| Ages      | DEATHS, CENSUS YEAR, 1901 |         | CENTRAL DEATH RATE |         |
|-----------|---------------------------|---------|--------------------|---------|
|           | Males                     | Females | Males              | Females |
| 0         | 3,740                     | 2,957   | ...                | ...     |
| 1         | 730                       | 633     | ...                | ...     |
| 2         | 343                       | 293     | ...                | ...     |
| 3         | 217                       | 218     | ...                | ...     |
| 4         | 179                       | 158     | ...                | ...     |
| 0-4       | 5,209                     | 4,259   | ·04570             | ·03846  |
| 5-9       | 483                       | 478     | ·00413             | ·00421  |
| 10-14     | 339                       | 370     | ·00290             | ·00327  |
| 15-24     | 1,259                     | 1,343   | ·00574             | ·00596  |
| 25-34     | 1,123                     | 1,339   | ·00689             | ·00788  |
| 35-44     | 1,029                     | 1,184   | ·00755             | ·00882  |
| 45-54     | 1,220                     | 1,056   | ·01235             | ·01119  |
| 55-64     | 1,444                     | 1,312   | ·02137             | ·02005  |
| 65-74     | 2,132                     | 1,819   | ·05019             | ·04630  |
| 75 & over | 3,011                     | 2,863   | ·14910             | ·15604  |
| All Ages  | 17,249                    | 16,023  | ·01575             | ·01478  |

\* The factors employed were determined from the figures of the 1891 and 1901 censuses:

|                                                       |           |           |
|-------------------------------------------------------|-----------|-----------|
|                                                       | Males.    | Females.  |
| Ontario population as by census of 5 April 1891 ..... | 1,069,487 | 1,044,834 |
| " " " " 31 March 1901 .....                           | 1,096,638 | 1,086,309 |

Hence the multipliers necessary for bringing the population as of 31 March 1901, to a point of time six months earlier are ·9987456 for males and ·9980528 for females.



of mortality for the several age groups. There were 22 males and 19 females of doubtful age, and these I decided, after enquiry, to assign to the groups lying beyond age 34. The relative smallness of the numbers, however, makes this a matter of little moment.

A statement of the deaths relatively attributable to the various forms of disease is likely to be of general interest, apart from its obvious connection with the present subject. Accordingly, in presenting the figures for Ontario, it was thought advisable also to include those for Quebec and for Canada as a whole, which is done in Table V. The causes of death are registered in accordance with the Bertillon system of nomenclature, recommended for adoption by the International Commission which met at Paris in 1900.

TABLE V.—NOSOLOGICAL:

*Showing, out of 1,000 Deaths in Quebec, Ontario, and the whole of Canada, in 1901, the Numbers attributable to Various Causes.*

| Cause of Death                      | CANADA   |          | QUEBEC   |          | ONTARIO  |          |
|-------------------------------------|----------|----------|----------|----------|----------|----------|
|                                     | Males    | Females  | Males    | Females  | Males    | Females  |
| Typhoid . . . . .                   | 32·94    | 26·93    | 26·89    | 24·00    | 34·98    | 28·83    |
| Measles . . . . .                   | 16·44    | 15·94    | 29·34    | 24·43    | 6·55     | 9·29     |
| Scarlet Fever . . . . .             | 15·78    | 19·04    | 31·62    | 36·73    | 7·74     | 8·82     |
| Diphtheria and Croup . . . . .      | 49·50    | 51·55    | 73·75    | 73·98    | 27·68    | 30·03    |
| Influenza . . . . .                 | 32·19    | 35·05    | 35·37    | 36·39    | 21·95    | 28·12    |
| Tuberculosis . . . . .              | 131·89   | 175·72   | 113·35   | 171·71   | 118·55   | 154·96   |
| Cancer . . . . .                    | 30·98    | 46·91    | 18·17    | 26·68    | 39·07    | 51·23    |
| Apoplexy and Paralysis . . . . .    | 55·91    | 59·54    | 44·00    | 48·51    | 71·82    | 76·01    |
| Diseases of the Heart . . . . .     | 68·42    | 74·76    | 45·71    | 53·71    | 93·70    | 105·24   |
| Bronchitis and Pneumonia . . . . .  | 130·47   | 124·47   | 105·28   | 97·20    | 154·27   | 150·43   |
| Affections of the Stomach . . . . . | 13·15    | 11·93    | 10·92    | 10·14    | 12·88    | 11·86    |
| „ „ Intestines . . . . .            | 123·73   | 107·24   | 178·36   | 141·12   | 100·62   | 92·53    |
| „ „ Liver . . . . .                 | 10·28    | 9·30     | 8·07     | 8·23     | 13·77    | 11·04    |
| Diseases of the Kidneys . . . . .   | 31·61    | 18·28    | 16·95    | 9·36     | 39·67    | 25·97    |
| Puerperal Diseases . . . . .        | ...      | 22·98    | ...      | 21·14    | ...      | 20·57    |
| Congenital Debility . . . . .       | 97·86    | 80·74    | 155·31   | 126·05   | 70·11    | 53·85    |
| Senile Debility . . . . .           | 87·13    | 102·15   | 65·84    | 80·31    | 104·11   | 119·61   |
| Suicide . . . . .                   | 3·51     | 1·02     | 2·12     | ·35      | 4·24     | 1·27     |
| Accident . . . . .                  | 68·21    | 16·45    | 38·95    | 9·96     | 78·29    | 20·34    |
| All Causes . . . . .                | 1,000·00 | 1,000·00 | 1,000·00 | 1,000·00 | 1,000·00 | 1,000·00 |

It will not be amiss to note in passing that a heavier mortality, especially among infants, prevails in Quebec than in

any other part of Canada. The mortality per 1,000 of the general population has for this Province been set down at 18·54 for 1901, as against 15·27 for Ontario and 15·12 for all Canada. The high death-rate is, as usual, accompanied by a high birth-rate ; which, again, has had the looked-for effect of *flushing*, so to speak, the age-groups pertaining to infancy and childhood, as may be seen in Table VI.

TABLE VI.—QUEBEC AND ONTARIO.

*Showing the Population of each Province at early ages per-cent of its Total Population, 1901.*

| Ages  | Quebec<br>(Birth Rate 30·6 per 1,000) | Ontario<br>(Birth Rate 21·1 per 1,000) |
|-------|---------------------------------------|----------------------------------------|
| 0     | 3·05                                  | 2·13                                   |
| 1     | 2·85                                  | 1·85                                   |
| 2     | 2·91                                  | 2·07                                   |
| 3     | 2·85                                  | 2·10                                   |
| 4     | 2·75                                  | 2·16                                   |
| 0-4   | 14·41                                 | 10·31                                  |
| 5-9   | 12·74                                 | 10·57                                  |
| 10-14 | 11·55                                 | 10·54                                  |

Reverting to the subject of the Ontario mortality-rates, it was found that the only satisfactory method of dealing with the materials exhibited in Tables III and IV was unquestionably the original Graphic method of Milne. The well-known disadvantages of interpolation by a mathematical formula would become much accentuated in a case like the present ; nor, without preconceived notions as to the trend of the curves, could the most skilful draughtsman steer a good course with only the central decennial death-rates for stars of the firmament. But, by distributing the population and the deaths separately, one is enabled to make the most of the data. The central rates of mortality at consecutive ages were so obtained, and these having been plotted down on cross-section paper, any roughnesses were removed. From the  $m_x$  values of the final curve, the  $p_x$  values were derived by means of the well-known relationship subsisting between the two functions. The best practical test of the graduation seemed to consist in a comparison of the actual deaths of any age-group with the sum of the products of  $P_x.m_x$  for that group ; where  $P_x$  stands for the mean population at age  $x$ , as obtained from the Milne interpola-

tion curve. Table VII enables such a comparison to be made as far as age 75.

TABLE VII.—ONTARIO.

*"Expected" and Actual Deaths in 1901.*

| Ages   | MALES    |        | FEMALES  |        |
|--------|----------|--------|----------|--------|
|        | Expected | Actual | Expected | Actual |
| 5-9    | 486.8    | 483    | 480.3    | 478    |
| 10-14  | 340.1    | 339    | 367.9    | 370    |
| 15-24  | 1,264.1  | 1,259  | 1,343.7  | 1,343  |
| 25-34  | 1,112.5  | 1,123  | 1,340.8  | 1,339  |
| 35-44  | 1,033.8  | 1,029  | 1,188.4  | 1,184  |
| 45-54  | 1,214.4  | 1,220  | 1,056.6  | 1,056  |
| 55-64  | 1,450.3  | 1,444  | 1,309.6  | 1,312  |
| 65-74  | 2,130.8  | 2,132  | 1,816.3  | 1,819  |
| Totals | 9,032.8  | 9,029  | 8,903.6  | 8,901  |

The period from 75 upwards required somewhat arbitrary treatment. I terminated the male curve at age 102, and the female at 103. But the rates of mortality in the latter present the anomalous feature of being higher for this period than in the male curve; and since, apart from this, the figures would no doubt be looked upon as more or less fanciful, I refrain from giving the results. The mortality tables, therefore, are given only to age 75, and these will be found in Table VIII, where the  $l_x$ ,  $d_x$  and  $p_x$  columns are exhibited. For comparison, the probabilities of life of the "New Healthy Districts' Table" (England and Wales, 1881-90), and of the Scottish Table previously referred to, are added.

A brief mention of the more salient features of the mortality curves will be sufficient here. In the male curve the mortality attains a minimum at age 12, from which point it increases rapidly to age 20 or 21, and thence more slowly to age 27. After this age the rates begin to diminish, and continue so doing until age 31 or 32, and thereafter increase very gradually until the early forties. The feature of a higher rate of mortality being found to operate below age 25, or thereabouts, than for the subsequent few years of life, has recurred in so many experiences (based upon the returns both of special classes and of populations), that it no longer excites comment; and in the present

case the only surprising incident is the comparatively late ages at which the phenomenon appears. General reasoning leads to the conclusion that the healthier the body of lives under observation the more pronounced will this feature be—a view that receives some support from the observations on males in the Peerage, Gotha, Scottish Widows' Fund, and other experiences. It is conceivable also that certain climates, by retarding the period of pubescence and of functional maturity in general, may have the effect of prolonging to a later age the period of excess mortality which characterizes early manhood. At all events, the graduation could not in the present case have been successfully accomplished without introducing this characteristic.

The time of lightest mortality for females occurs at age 11. After this age the rates increase, gradually at first, then more sharply, to about age 19. For the next four or five years, corresponding to the short period between tolerably complete development and assumption of the duties of motherhood, there is a lull in the mortality. But in the vicinity of age 25 a sharp increase in the rates commences (the average age at marriage of Ontario females has been computed at 25·3 years), and continues for a few years, after which the increase becomes more gradual. The decennial period following age 35 is characterized, in the female as in the male curve, by a very slow rate of increase; and this feature must be allowed to remain, if we assume the data to be accurate.

It is proper to mention, before concluding, that the 1901 population of Ontario included nearly 20,000 Indians and about 5,000 Half-breeds—so called—among whom the death-rate is considerably heavier than among whites. This section of the population could not be eliminated from the data, as its age-distribution had not been investigated; but although it amounts to but little over one per-cent of the total population of the Province, the rates may be affected at certain points to an appreciable extent.

TABLE VIII.—MORTALITY TABLES, ONTARIO, 1901.

*Showing also the Probabilities of Life at recent epochs in the Healthy Districts of England, and in Scotland.*

## MALES

## FEMALES

| Age | ONTARIO, 1901 |        |        | HEALTHY DISTRICTS, ENGLAND AND WALES, 1881-90 | SCOTLAND, 1891-1900 | Age | ONTARIO, 1901 |        |        | HEALTHY DISTRICTS, ENGLAND AND WALES, 1881-90 | SCOTLAND, 1891-1900 |
|-----|---------------|--------|--------|-----------------------------------------------|---------------------|-----|---------------|--------|--------|-----------------------------------------------|---------------------|
|     | $l_x$         | $d_x$  | $p_x$  | $p_x$                                         | $p_x$               |     | $l_x$         | $d_x$  | $p_x$  | $p_x$                                         | $p_x$               |
| 0   | 100,000       | 15,755 | .84245 | .88085                                        | .85657              | 0   | 100,000       | 13,267 | .86733 | .90650                                        | .88211              |
| 1   | 84,245        | 3,067  | .96360 | .96679                                        | .94713              | 1   | 86,733        | 2,829  | .96738 | .97016                                        | .94973              |
| 2   | 81,178        | 1,410  | .98262 | .98684                                        | .97786              | 2   | 83,904        | 1,281  | .98474 | .98743                                        | .97812              |
| 3   | 79,768        | 870    | .98910 | .99096                                        | .98676              | 3   | 82,623        | 852    | .98968 | .99137                                        | .98656              |
| 4   | 78,898        | 613    | .99223 | .99288                                        | .99066              | 4   | 81,771        | 625    | .99236 | .99334                                        | .99030              |
| 5   | 78,285        | 469    | .99401 | .99444                                        | .99343              | 5   | 81,146        | 483    | .99405 | .99484                                        | .99322              |
| 6   | 77,816        | 351    | .99549 | .99567                                        | .99445              | 6   | 80,663        | 383    | .99525 | .99594                                        | .99413              |
| 7   | 77,465        | 300    | .99613 | .99660                                        | .99524              | 7   | 80,280        | 313    | .99610 | .99671                                        | .99493              |
| 8   | 77,165        | 257    | .99666 | .99727                                        | .99598              | 8   | 79,967        | 274    | .99657 | .99721                                        | .99554              |
| 9   | 76,908        | 239    | .99689 | .99770                                        | .99650              | 9   | 79,693        | 322    | .99696 | .99751                                        | .99614              |
| 10  | 76,669        | 215    | .99720 | .99793                                        | .99683              | 10  | 79,371        | 234    | .99705 | .99763                                        | .99638              |
| 11  | 76,454        | 200    | .99738 | .99800                                        | .99691              | 11  | 79,137        | 227    | .99714 | .99763                                        | .99647              |
| 12  | 76,254        | 198    | .99741 | .99793                                        | .99690              | 12  | 78,910        | 241    | .99694 | .99751                                        | .99646              |
| 13  | 76,056        | 226    | .99703 | .99777                                        | .99683              | 13  | 78,669        | 269    | .99658 | .99730                                        | .99641              |
| 14  | 75,830        | 271    | .99642 | .99753                                        | .99665              | 14  | 78,400        | 313    | .99601 | .99702                                        | .99625              |
| 15  | 75,559        | 311    | .99589 | .99724                                        | .99615              | 15  | 78,087        | 365    | .99533 | .99670                                        | .99535              |
| 16  | 75,248        | 354    | .99529 | .99692                                        | .99532              | 16  | 77,722        | 405    | .99478 | .99635                                        | .99496              |
| 17  | 74,894        | 389    | .99481 | .99660                                        | .99456              | 17  | 77,317        | 433    | .99441 | .99603                                        | .99476              |
| 18  | 74,505        | 414    | .99444 | .99631                                        | .99421              | 18  | 76,884        | 446    | .99420 | .99576                                        | .99460              |
| 19  | 74,091        | 437    | .99410 | .99605                                        | .99397              | 19  | 76,438        | 460    | .99398 | .99554                                        | .99446              |
| 20  | 73,654        | 453    | .99386 | .99582                                        | .99374              | 20  | 75,978        | 468    | .99384 | .99536                                        | .99434              |
| 21  | 73,201        | 465    | .99364 | .99560                                        | .99350              | 21  | 75,510        | 477    | .99368 | .99520                                        | .99420              |
| 22  | 72,736        | 474    | .99348 | .99537                                        | .99314              | 22  | 75,033        | 484    | .99355 | .99503                                        | .99404              |
| 23  | 72,262        | 480    | .99336 | .99513                                        | .99276              | 23  | 74,549        | 491    | .99342 | .99484                                        | .99382              |
| 24  | 71,782        | 483    | .99328 | .99486                                        | .99261              | 24  | 74,058        | 499    | .99325 | .99462                                        | .99360              |
| 25  | 71,299        | 484    | .99320 | .99459                                        | .99276              | 25  | 73,559        | 514    | .99302 | .99438                                        | .99345              |
| 26  | 70,815        | 489    | .99310 | .99433                                        | .99291              | 26  | 73,045        | 530    | .99274 | .99415                                        | .99331              |
| 27  | 70,326        | 490    | .99304 | .99410                                        | .99286              | 27  | 72,515        | 546    | .99247 | .99393                                        | .99309              |
| 28  | 69,836        | 481    | .99311 | .99389                                        | .99274              | 28  | 71,969        | 558    | .99225 | .99374                                        | .99269              |
| 29  | 69,355        | 473    | .99318 | .99370                                        | .99258              | 29  | 71,411        | 568    | .99205 | .99355                                        | .99222              |
| 30  | 68,882        | 465    | .99325 | .99353                                        | .99238              | 30  | 70,843        | 576    | .99186 | .99338                                        | .99185              |
| 31  | 68,417        | 457    | .99331 | .99336                                        | .99213              | 31  | 70,267        | 580    | .99175 | .99323                                        | .99154              |
| 32  | 67,960        | 455    | .99331 | .99320                                        | .99183              | 32  | 69,687        | 582    | .99165 | .99308                                        | .99126              |
| 33  | 67,505        | 454    | .99327 | .99302                                        | .99148              | 33  | 69,105        | 549    | .99157 | .99295                                        | .99101              |
| 34  | 67,051        | 455    | .99322 | .99282                                        | .99108              | 34  | 68,556        | 582    | .99151 | .99282                                        | .99079              |
| 35  | 66,596        | 457    | .99313 | .99262                                        | .99066              | 35  | 67,974        | 579    | .99148 | .99270                                        | .99059              |
| 36  | 66,139        | 462    | .99302 | .99240                                        | .99025              | 36  | 67,395        | 579    | .99142 | .99258                                        | .99039              |
| 37  | 65,677        | 466    | .99290 | .99216                                        | .98982              | 37  | 66,816        | 577    | .99136 | .99246                                        | .99019              |
| 38  | 65,211        | 473    | .99276 | .99190                                        | .98937              | 38  | 66,239        | 575    | .99132 | .99235                                        | .98999              |
| 39  | 64,738        | 479    | .99260 | .99162                                        | .98888              | 39  | 65,664        | 573    | .99128 | .99223                                        | .98977              |



TABLE VIII—continued.—MORTALITY TABLES, ONTARIO, 1901.

*Showing also the Probabilities of Life at recent epochs in the Healthy Districts of England, and in Scotland.*

## MALES

## FEMALES

| Age | ONTARIO, 1901 |       |        | HEALTHY DISTRICTS, ENGLAND AND WALES, 1881-90 | SCOTLAND, 1891-1900 | Age | ONTARIO, 1901 |       |        | HEALTHY DISTRICTS, ENGLAND AND WALES, 1881-90 | SCOTLAND, 1891-1900 |
|-----|---------------|-------|--------|-----------------------------------------------|---------------------|-----|---------------|-------|--------|-----------------------------------------------|---------------------|
|     | $l_x$         | $d_x$ | $p_x$  | $p_x$                                         | $p_x$               |     | $l_x$         | $d_x$ | $p_x$  | $p_x$                                         | $p_x$               |
| 40  | 64,259        | 486   | .99243 | .99132                                        | .98538              | 40  | 65,091        | 572   | .99120 | .99210                                        | .98954              |
| 41  | 63,773        | 496   | .99223 | .99100                                        | .98786              | 41  | 64,519        | 575   | .99110 | .99195                                        | .98929              |
| 42  | 63,277        | 506   | .99199 | .99066                                        | .98729              | 42  | 63,944        | 576   | .99098 | .99179                                        | .98903              |
| 43  | 62,771        | 523   | .99168 | .99031                                        | .98669              | 43  | 63,368        | 581   | .99084 | .99160                                        | .98875              |
| 44  | 62,248        | 542   | .99128 | .98993                                        | .98606              | 44  | 62,787        | 586   | .99066 | .99137                                        | .98846              |
| 45  | 61,706        | 571   | .99076 | .98953                                        | .98539              | 45  | 62,201        | 599   | .99038 | .99110                                        | .98814              |
| 46  | 61,135        | 598   | .99022 | .98909                                        | .98467              | 46  | 61,602        | 611   | .99007 | .99079                                        | .98779              |
| 47  | 60,537        | 629   | .98961 | .98861                                        | .98388              | 47  | 60,991        | 624   | .98978 | .99042                                        | .98707              |
| 48  | 59,908        | 662   | .98894 | .98806                                        | .98301              | 48  | 60,367        | 635   | .98948 | .98998                                        | .98636              |
| 49  | 59,246        | 699   | .98821 | .98745                                        | .98210              | 49  | 59,732        | 646   | .98919 | .98947                                        | .98518              |
| 50  | 58,547        | 736   | .98743 | .98677                                        | .98106              | 50  | 59,086        | 661   | .98882 | .98889                                        | .98445              |
| 51  | 57,811        | 775   | .98659 | .98603                                        | .97989              | 51  | 58,425        | 675   | .98844 | .98824                                        | .98350              |
| 52  | 57,036        | 813   | .98575 | .98523                                        | .97838              | 52  | 57,750        | 701   | .98787 | .98753                                        | .98211              |
| 53  | 56,223        | 849   | .98489 | .98436                                        | .97597              | 53  | 57,049        | 726   | .98727 | .98676                                        | .98135              |
| 54  | 55,374        | 883   | .98406 | .98342                                        | .97383              | 54  | 56,323        | 767   | .98638 | .98592                                        | .98017              |
| 55  | 54,491        | 897   | .98354 | .98239                                        | .97207              | 55  | 55,556        | 805   | .98551 | .98499                                        | .97880              |
| 56  | 53,594        | 908   | .98305 | .98124                                        | .97054              | 56  | 54,751        | 843   | .98460 | .98396                                        | .97768              |
| 57  | 52,686        | 927   | .98241 | .97994                                        | .96899              | 57  | 53,908        | 876   | .98376 | .98280                                        | .97635              |
| 58  | 51,759        | 944   | .98175 | .97846                                        | .96735              | 58  | 53,032        | 903   | .98297 | .98149                                        | .97480              |
| 59  | 50,815        | 980   | .98073 | .97680                                        | .96565              | 59  | 52,129        | 944   | .98188 | .98002                                        | .97311              |
| 60  | 49,835        | 1,035 | .97922 | .97494                                        | .96380              | 60  | 51,185        | 1,013 | .98022 | .97837                                        | .97009              |
| 61  | 48,800        | 1,129 | .97687 | .97291                                        | .96184              | 61  | 50,172        | 1,086 | .97835 | .97655                                        | .96804              |
| 62  | 47,671        | 1,224 | .97433 | .97071                                        | .95953              | 62  | 49,086        | 1,159 | .97639 | .97455                                        | .96590              |
| 63  | 46,447        | 1,310 | .97180 | .96833                                        | .95695              | 63  | 47,927        | 1,257 | .97376 | .97234                                        | .96230              |
| 64  | 45,137        | 1,395 | .96908 | .96575                                        | .95399              | 64  | 46,670        | 1,362 | .97083 | .96992                                        | .95944              |
| 65  | 43,742        | 1,471 | .96637 | .96293                                        | .95084              | 65  | 45,308        | 1,436 | .96831 | .96725                                        | .95679              |
| 66  | 42,271        | 1,544 | .96348 | .95981                                        | .94750              | 66  | 43,872        | 1,525 | .96522 | .96429                                        | .95407              |
| 67  | 40,727        | 1,597 | .96078 | .95636                                        | .94391              | 67  | 42,347        | 1,612 | .96194 | .96102                                        | .95133              |
| 68  | 39,130        | 1,677 | .95714 | .95252                                        | .94008              | 68  | 40,735        | 1,668 | .95906 | .95741                                        | .94830              |
| 69  | 37,453        | 1,756 | .95312 | .94827                                        | .93568              | 69  | 39,067        | 1,753 | .95513 | .95343                                        | .94446              |
| 70  | 35,697        | 1,830 | .94875 | .94354                                        | .93068              | 70  | 37,314        | 1,808 | .95154 | .94904                                        | .94007              |
| 71  | 33,867        | 1,915 | .94435 | .93830                                        | .92512              | 71  | 35,506        | 1,834 | .94835 | .94420                                        | .93604              |
| 72  | 31,952        | 1,981 | .93798 | .93250                                        | .91876              | 72  | 33,672        | 1,878 | .94422 | .93890                                        | .93107              |
| 73  | 29,971        | 2,027 | .93237 | .92609                                        | .91146              | 73  | 31,794        | 1,892 | .94049 | .93309                                        | .92660              |
| 74  | 27,944        | 2,072 | .92585 | .91901                                        | .90321              | 74  | 29,902        | 1,903 | .93638 | .92673                                        | .92107              |
| 75  | 25,872        | 2,114 | .91830 | .91122                                        | .89431              | 75  | 27,999        | 1,911 | .93175 | .91980                                        | .91407              |

## ABSTRACT OF THE DISCUSSION.

MR. A. G. HEMMING said that Mr. Grant's interesting paper would be welcomed by the Institute, both by reason of its having been contributed by one of their Colonial Fellows, and also because it threw some light on the subject of Canadian vital statistics, regarding which at present there was but little information. The paper divided itself conveniently into two sections, in the first of which Mr. Grant reviewed the only available local insurance experience, namely, that of the Canada Life, the second section being devoted to an endeavour to show the mortality prevailing amongst the general population. Looking at the first section, it was remarkable that, although the experience extended over fifty years, and the average period of exposure worked out at 8.36 years, which compared not unfavourably with the  $H^M$  figure of about nine and a quarter years, the resulting mortality table was not so reliable as one would expect at first sight, owing to the large proportion of young and recently-selected lives included in the observations; and when, therefore, Mr. Grant drew the conclusion that the true mortality prevailing amongst Canadian lives assured, and presumably, therefore, the mortality amongst the general population, would be found to fall well below that of the Mother Country, it did not appear to him that he had produced sufficient evidence to warrant that conclusion. The author seemed to rely mainly on the fact that the actual number of deaths in the Canada Life experience amounted to only about 71 per-cent of the number expected according to the  $H^M$  Table; but he (the speaker) had before him the figures of the British business of a prominent office for the last sixteen years, showing the number of actual deaths year by year, and the number expected according to the  $H^M$  Table. The office in question might be taken as a fairly representative one, seeing that it had been established nearly a hundred years, and had been doing a constant average new business, neither too large nor too small to vitiate the results. He found that on four occasions the actual deaths were less than 70 per-cent of the expected, and for the whole period the ratio of the actual to the expected worked out at just over 73 per-cent. In the light of those figures, and bearing in mind the immature nature of the Canada Life experience, as shown by the meagre proportion of observations in the truncated table, excluding the first five years, the ratio of 71 per-cent referred to by Mr. Grant did not seem in any way remarkable. With regard to the general causes which the author referred to as indicating or promoting a more favourable rate of mortality, he could not of course speak. Mr. Grant had provided an interesting and picturesque account of the geographical and climatic conditions under which Canadians lived, but he then propounded a scientific dogma, the bearing of which he (Mr. Hemming) confessed himself unable to follow. Mr. Grant stated with some confidence that no race had ever yet been able to maintain itself for long when removed from its ancient habitat to a country which differed widely from its own in the matter of climate. He could scarcely think that by that the author intended to suggest that eventually the vast territory which was now inhabited by English and French-speaking people would be over-run entirely by North American Indians, but if that was his meaning such a

disappointing conclusion could scarcely be accepted without considerable further investigation.

In the second section of the paper they were provided with a mortality table, which, although undoubtedly of interest, suffered rather severely, as the author himself admitted, from the nature of the material from which it had been derived. In the first place, it dealt exclusively with the Province of Ontario, which took away from its interest, when one was reviewing the vital statistics of Canada, especially as it appeared that Quebec and other provinces presented quite different features. It had been compiled from the census figures of 1901, and the death figures obtained from the Registrar-General of Ontario for that single year. With regard to the census figures, they were told that Canada was perhaps not worse off than England in respect of such statistics—to his mind a very poor commendation. Considerable pains appeared to have been taken to make the death returns as accurate as possible. The drawback, however, of taking a single year's figures could not be disposed of in that way, and the curtailment of the table by including under one head all deaths over age 75 deprived it of much of its usefulness. The principal feature of Mr. Grant's mortality table was the favourable infantile mortality. In the late Mr. Burridge's paper of 1882, to which Mr. Grant referred, the same feature was observed in Victoria (*J.I.A.*, vol. xxiii, pp. 318, 319), and in a later paper by the same writer it was again observed in the other Australian Colonies (vol. xxiv, p. 348). That was due presumably to better surroundings and the absence of slums. Since Mr. Burridge's paper was written, it appeared that, at all events in some of the Australasian Colonies, there had been an increase in the infantile mortality, coupled in one case with an alarming decrease in the birth rate. In the case of New South Wales, these abnormalities had become so pronounced that a Royal Commission was appointed to enquire into the causes. Their report, which was published last year, dealt exhaustively with the subject, and it showed, he thought conclusively, that the decrease in the birth rate was not caused, as Mr. Grant suggested might be the case in Canada, by the postponement of the marriage age, but was due to the self-imposed restriction deliberately placed by parents on the number of children born. Mr. Coghlan, in his statistical essay on New South Wales, showed that over a period of twenty years the marriage age was only postponed some sixteen months, from 22.64 to 23.95. With regard to the actual construction of the table, the methods employed by Mr. Grant appeared to him satisfactory, Milne's graphic method of graduation being particularly suitable for a table of that nature. What Mr. Grant had to say about the unequal distribution of births was interesting, and his diagram comparing the birth and marriage curves was sufficiently striking, though he did not say how those curves were graduated. It was curious that the six months ending 30 September should show a larger number of births than the six months ending 31 March; for although the author stated that the same feature, in a less marked degree, was seen in the corresponding figures for England and Wales, he found, looking at the figures of some other countries, an exactly opposite result was obtained. He had before him a record of the births



month by month in Massachusetts, Sweden and Finland, Montpellier in France, and Kentucky. In all those cases it appeared that the larger number of births came in the winter months, the figures per 1,000 births being, respectively, 501·7, 519·3, 539·4, and 520. Seeing that those figures had not been selected by him, but were merely taken because he could conveniently refer to the observations, he could only imagine that although in Ontario it appeared to be the case that the six months April to September showed a higher birth-rate than the six months October to March, that must be due to purely accidental causes, and not to climatic conditions, for the climates of the four places referred to showed a considerable range of variation, and that of Massachusetts would not, he imagined, differ materially from that of Canada. The table on page 137, showing the distribution of the causes of death for the whole of Canada, and for the Provinces of Quebec and Ontario separately, seemed to indicate that Ontario was not representative of the general population in regard to the causes of death. The table, he gathered, was compiled from the statistics of one year only, which might have accentuated the discrepancies. Quebec and Ontario in that table gave very varied results, for it appeared that, for every 1,000 deaths in Quebec, 74 were attributable to diphtheria and croup, as compared with 28 per 1,000 in Ontario. That might be the cause of the high infantile mortality which Mr. Grant said prevailed in the Province of Quebec. Quebec, however, seemed fortunate in having comparatively few deaths attributable to cancer, 18 per 1,000 deaths amongst male lives, being really a very small proportion, as Dr. Muirhead, in his report on the Scottish Widows' Fund for the twenty-one years ending 1894, attributed 59 out of every 1,000 deaths to that cause, the proportion being raised to 69 if the seven years 1887-94 were taken separately. He would have liked to see a table showing the causes of death per 1,000, or per 10,000, of the population, instead of per 1,000 deaths; but he hoped they might look forward to another paper from Mr. Grant, in which perhaps he would be able to give them that information, and further investigate the several points which arose out of his interesting paper.

MR. RALPH TODHUNTER said that the general lines upon which a mortality table should be constructed from population statistics were laid down so many years ago, and had been followed in so many instances, that, in the ordinary course, it might safely be assumed that the results of such an investigation as this fairly represented the mortality of the community to which it related. He thought, however, that that would hardly be a safe assumption in the present case; in fact, he rather gathered from the paper that the author himself felt that too much reliance could not safely be placed upon the mortality table which he had evolved from the data. Mr. Grant expressed the opinion that the registered deaths formed a satisfactory basis, although they related to only a single year, but he did not give a similar testimony to the census results themselves, or to the registered births; in fact, he admitted that the registered births were very defective. It appeared, for example, from Tables I and III, that on the 31 March, 1901, there were 22,813 female children aged 0, whereas in the previous year only 22,257 female

children had been born; so that there were in existence, according to census, 556 more female children than were born in the whole of the preceding year. Again, there were 23,600 male children aged 0 at the date of the census, and in the previous year 23,774 had been born, *i.e.*, only 174 more. It was obvious that an average of six months' mortality must have reduced the number born to a much greater extent than that. There was also the curious feature that, according to the census return, there were 113,812 male children, and 110,770 female children aged 0-4, as compared with 116,678 male children and 113,651 female children, aged 5-9. That was a conceivable distribution of population, but he did not think he had seen anything like it in the results of any other census. It was certainly most unusual to find the population at ages 5-9, 10-14 and 15-19 considerably in excess of the population at ages 0-4, and he doubted whether it could be sufficiently accounted for in this case by the falling birth-rate. The conclusion was rather important, because it threw some doubt on the correctness not only of the births but also of the census statistics. Assuming, however, that the figures were sufficiently reliable to admit of the construction of a mortality table, a point of some little practical interest arose in connection with the treatment of the first five years of age. The author seemed to him to be under the impression that the basis of the well-established rule for constructing rates of mortality at infantile ages was that the average date of birth of persons born in two successive years might be taken as being at the end of one year and the beginning of the other, *i.e.*, in the middle of the period, and he based his correction on that assumption. He thought it would be found that, although that was an incidental result of the assumptions upon which the usual rule was based, it had in itself no theoretical foundation, and that it was not a necessary assumption. The usual rule was based on two assumptions—first, that the births were uniformly distributed over the year, and secondly that the deaths in the first year of age were uniformly distributed over the year. Those assumptions, however, were not of equal importance. It would be found that any irregularity such as would actually occur in the distribution of births over successive years would make little or no difference. For example, if the births in two successive periods increased or decreased in arithmetical progression, the ordinary rule would still give an absolutely correct result, supposing the deaths to be uniformly distributed over the first year of age; and if the births increased or decreased in geometrical progression the error would be so small as to be negligible. That was not the case, however, as regarded the distribution of deaths over the first year of age. A much larger proportion of the deaths in a given year obviously occurred from the infants born in that year than from the infants born in the previous year, because the rate of mortality was very much heavier immediately after birth than it was at the end of the first year of life, and it would be found, he thought, that taking such a distribution of the first year's mortality month by month, as was shown in the Registrar-General's Returns, it would be more accurate to take about one-third of the births of the first year of the period, and two-thirds of the births of the final year, instead of one-half of



each. With regard to the results of the investigation, he questioned whether the infantile mortality was so remarkably favourable. Of course it compared favourably with the infantile mortality of such a densely populated country as England, but it appeared to be much heavier than that of New Zealand.

MR. GEORGE KING wished to express satisfaction at the Institute receiving a paper from one of its Colonial Fellows. It was quite true that, in some respects, the paper was defective, and its use would in that way be limited. But he did not think that was the fault of the author, because he had made very excellent use of the material that he had at his command. It was a first paper, and it might lead the way to others on similar subjects later on, when statistics might have more matured. He joined with Mr. Hemming in doubting very much the advantage of comparing the mortality of different countries by means of aggregate mortality tables, as was shown in the early part of the paper, and as was very common, for instance, in America. He did not think it was possible to make a proper comparison by using aggregate tables. To obtain the true expected deaths, for instance, in the American and Canadian companies, they must employ select tables, because the business had been growing so rapidly that to compare the deaths by aggregate tables constructed upon old business in this country was most fallacious. He, therefore, did not accept the statement that the deaths in Canada and America were only 70 to 75 per-cent of those in this country, simply from the comparisons made by aggregate tables. He could not help thinking that if the deaths were compared by means of select tables, which allowed for the actual duration of the business, the results would be different; at any rate they would be very much more satisfactory and trustworthy. Turning to Tables V and VI, there were certain anomalies which he thought vitiated the conclusions derived from that kind of statistics. He did not quite see the use of giving the number of deaths, taking as a total a thousand deaths, and then a proportion of that 1,000 as due to different diseases, unless the populations compared were very closely alike in their age distribution. It was quite clear that the populations of Quebec and Ontario differed very much in their age distribution. It was only necessary to look at the heading of Table VI to find that in Quebec the birth rate was 30.6 per 1,000, and in Ontario 21.1 per 1,000. Therefore there must be a very much larger infantile population in Quebec than in Ontario, as compared with the total population. That seemed to him in a large measure to account for the large proportion of deaths from measles, scarlet fever and diphtheria in Quebec, as compared with the same diseases in Ontario. Then, having obtained for these diseases that very large number of deaths in the 1,000 deaths, it necessarily caused a reduction in the relative number of deaths from other diseases. They had not obtained the total deaths, but only a comparison of deaths in 1,000 deaths. That, again, would account for the great difference in the proportion of deaths from cancer, which was a disease of old age. In Quebec there were among males only 18.17 cancer deaths per 1,000, while in Ontario there were 39.07. What was required, in order to compare the healthiness of two districts, and to get a really useful comparison, was the number

of deaths per 1,000 living in each age group, otherwise figures were obtained which were of no use whatever. With regard to the method of constructing the table, he was very pleased to see that the author had not adopted the analytical method, but had gone back to Milne's graphic method, which he believed was much more accurate. Careful investigation showed that the construction, for instance, of the English Life Table No. 3 was radically defective, through the assumption that what Farr called the rate of mortality for an age group was the force of mortality for the exact middle point of that age group. It was not so. The result of that assumption at the older ages over 60 was materially to over-state the vitality, so that in the English Life Table No. 3 the rate of mortality at the older ages was very much under-stated, and an entirely erroneous view was given of the true death rate in this country. The English Life Table No. 5 had been correctly constructed, and it showed a lighter death rate in the earlier years of life, and a heavier death rate later on, than the English Table No. 3. That was not due to any change in the relative death rates in the country ; it was simply due to a change in the method of constructing the table ; and yet it had given rise to the heresy which was very prevalent that, at the older ages of life, the vitality of the country had deteriorated rather than improved. As a matter of fact it had improved all along, and it was only the method of constructing the table that gave a different appearance. The General Register Office had adopted a completely new system of constructing mortality tables, which in some senses might be called analytical, but which, by means of an algebraical formula, practically applied Milne's curves, and that was a very great improvement. Dr. Tatham had been good enough to give him a full explanation of it. It was very similar to the method that was used in constructing the London Life Table, which was very fully explained in a pamphlet issued by the London County Council. The methods were similar, but they differed in details, and both were very laborious, and from the point of view of the arithmetical work they no doubt could be very much simplified. So far as he could see, the improvement was very great over Dr. Farr's methods, and left very little to be desired. He hoped, therefore, that in future the newer methods, which were very much better than the old, would be adopted. In conclusion, he wished to express the hope that other Colonial graduates would submit papers to the Institute.

MR. ERNEST WOODS said that, with reference to the cancer statistics, he would call Mr. King's attention to the fact that lower down in Table V there was a greater number of deaths from affections of the intestines in Quebec as compared with Ontario. Was it not probable that the mere question of certification would account for the difference?

MR. KING remarked that that was so, but the age distribution must also be considered. It was impossible to compare the tables in a general population with those published in the report of a British Life Office.

MR. J. D. WATSON remarked that a very interesting field of discussion was opened up by the question as to whether it was possible to acclimatize nationalities on strange soils. Beyond

an *ex cathedra* statement on the subject, he did not think the author had carried the question very far. In that connection, however, the last two paragraphs of the paper, where the author said that the death rate among Indians and half-breeds was very much heavier than among whites, opened an interesting vista. Presumably that subject had been investigated, and the assertion pointed to a set of statistics which he (Mr. Watson) had unsuccessfully tried to find. He would, therefore, be very much obliged if the author, when he replied to the discussion, would indicate what statistics were referred to, and give a short account of them.

MR. S. G. WARNER said that, in the first place, he wished warmly to endorse what had already been said as to the pleasure with which the Institute received a contribution from one of their Colonial members, who now constituted an important part of the Institute, on a Colonial subject, about which he naturally had a certain amount of expert information, and access to statistics not readily available to English Actuaries. With regard to the important point referred to by two previous speakers, the possible acclimatization of races in a habitat alien to their native one, he rather gathered that the author's intention was not to suggest that the Anglo-Saxon races would not continue to flourish and prosper in Canada, but that nations accustomed to a more tropical habitat would not easily be acclimatized there. Mr. Grant made a very definite statement to the effect that the negro races, apart from immigration, could not survive for many generations north of 45 degrees N. lat. Of course, that statement required verification. In that connection, he was reminded of Professor Charles Pearson's very suggestive book, "National Life and Character," which attracted a good deal of attention a few years ago, the burden of which was that the view expressed as to the Anglo-Saxon race ultimately dominating the world might possibly be founded on a fallacy; that the Anglo-Saxon race had probably at present occupied as much of the world as, in view of climatic conditions, it could hope permanently to occupy; and that the future in regard to tropical regions lay rather with the two great races who could flourish there, the yellow man and the negro. That line of thought was based on the same assumption that, broadly speaking, the two great types of humanity which were associated with Northern and Southern latitudes could not change places permanently, and continue to flourish and prosper. He thought previous speakers had fairly well demonstrated that the figures brought forward in such detail in Table V were really of little value. That would be fairly obvious in any case from the comparatively small number of deaths available, and the fact that the experience related to a single year only. Certainly it was stated to be a fairly typical and normal year, but in any case it seemed somewhat futile upon such a basis to compare in such detail Quebec and Ontario as regarded the experience of death from specific diseases, on the strength of figures which another year's record might exhibit in quite different relations. The permanent value of such an investigation as the author's was a question of some interest. It represented, of course, meritorious work, but work



based inevitably on rather scanty materials, and one wondered what the exact place of such investigations would be in the permanent future of actuarial science. They might be looked at from two points of view; they might be regarded as contributions to the general science of vital statistics, or as contributions to the practical actuarial side of life assurance. From the first standpoint they might certainly have their use. In the paper, for instance, it appeared to him that there was a study of a small and exceptionally constituted community of more than average vigour and vitality, where an automatic process of self-selection was a very potent force. Those were conditions which could not be expected to last for a very great length of time. From time to time new communities, such as were seen now in Canada, would arise, and while they were continually reinforced by fresh streams of emigration, they would exhibit a condition of prosperity and immunity from certain evils to which people in older communities were subject, which would cause them to experience a lighter rate of mortality than older countries and civilizations. But the time would certainly come when population would increase, and would press upon the limits of subsistence, when many of the problems which faced people in older civilizations would face Canada also, and when the mortality would inevitably approximate to that of those older communities. Nevertheless, an investigation of such a kind seemed to him to have an interest as a part of general vital statistics, as showing what happened when many certain causes of deterioration were eliminated. It showed practically their effect, by exhibiting a mortality experience which was free from them. But, coming to the other standpoint, the practical value to the actuary in his life assurance calculations, he thought they must say that this was inappreciable, because such calculations had to look far into the future. In computing premiums, actuaries had to consider, not what was happening to their contemporaries, but what would happen to the next generation, or the generation after that; and it would be unsafe and unwise to base such computations upon what could only be a temporarily favourable mortality.

THE PRESIDENT, in asking the members to join with him in according their best thanks to the Author, said he was sorry that Mr. Grant was not present to read his paper in person, and to make their acquaintance: but later on he hoped the Author would be able to give them another paper, and come over and read it himself. The author lived in the great Dominion of Canada, whose possibilities were far from being exhausted, and were now being rapidly developed in many directions; and it was only to be expected that great changes, both in mortality and in many other ways, would take place before many years had passed away. The Institute was always glad to receive a paper from a new contributor, especially from one over the seas.

The resolution was carried with acclamation.

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MR. GRANT sends us, for publication, the following rejoinder on the above discussion :

I thank the President and members most heartily for the reception extended to my Paper. The Institute has always received contributors in a spirit of just criticism, tempered by a fine courtesy.

Mr. Warner appears to be the only one who interpreted aright my remarks upon the acclimatization of races. I now regret having referred to the subject; not from change of conviction, but because the reference was in no way vital to the context, and has given rise to a discussion which I cannot adequately notice here. I have therefore chosen another place for my rejoinder under this head (see *Insurance Record*, 26 January 1906), and shall here say merely that I consider the Teutonic and Celtic races, and in particular the Anglo-Saxon peoples, eminently fitted for colonising Canada. Mr. Hemming, indeed, tried to apply the *reductio ad absurdum* method to my argument. Granting that the white and Indian population of Canada were, in the beginning, equally well adapted, physically, to the country, and I shall not deny it, there still remained between them the gulf that separates a high civilization from barbarism. Historians have noted, too, that all the races of American Indians have something peculiarly sensitive in their nature that makes them shrink instinctively from contact with foreigners. The question of racial fitness or unfitness, considered with reference to external nature, must surely be kept distinct from that of a high civilization meeting a low.

Mr. Warner desires verification of my statement that the negro races are unable to maintain themselves in cold climates. I have, myself, made no investigations of the mortality of negroes, but it is a matter of common knowledge that they suffer severely north of "Mason and Dixon's line."\* Dr. Nott, in his day a noted ethnologist, says of the negro: "The negro, too, obeys the law of climate. . . . Like the quadrumana of the tropics, he is inevitably killed by cold." "The negro races are peculiarly liable to consumption out of the tropics, or even within them." "He (the negro) is the native of the hottest region on the globe, where he goes naked in the scorching rays of the sun, and can lie down and sleep on the ground in a temperature of at least 150° F., where the white man would die in a few hours. And while the degenerate tropical descendants of the whites are regenerated by transportation to cold parallels of the temperate zone, experience abundantly proves that, in America, the negro steadily deteriorates, and becomes exterminated north of about 40° north latitude." (5°, or, say, 350 miles south of the limit assigned by me on p. 129 of the Paper.) "The statistics of New England, New York, and Philadelphia, abundantly prove this. The mortality of blacks in our northern States averages about double that of the whites; and . . . no one conversant with the facts will deny the baneful influence of cold upon the race." "In Boston, the most northern point, the mortality (of negroes) is highest; and rather less in New York and Philadelphia. I can procure no statistics from Canada, where the blacks must suffer terribly from that climate."

\* A line in the United States, between Pennsylvania on the north, and Delaware, Maryland, and North Virginia on the south, coinciding with 39° 43' 26.3 N. latitude.—ED. J.I.A.



Dr. Dowler, once editor of the *New Orleans Medical Journal*, put on record his conviction that "The blacks imported from Africa, everywhere beyond the limits of the slave States of North America, tend to extinction. . . . The black race is doomed to extinction in the West Indies, as well as in the Northern States of this Republic." At Gibraltar, in 1817, a negro regiment was almost annihilated by pulmonary diseases. Yet, under favourable conditions, the negro shows a superb vitality. The request of Mr. Watson for statistical evidence that the death rate among Indians is heavier than among the whites is natural. Mr. Watson must have looked in vain for such statistics in the Canadian Census reports. A branch of the Federal Government, however, (the Department of Indian Affairs), devotes its energies to these "wards of the nation." From the official reports of this Department, supplemented by other figures compiled from the certified returns, the following short table has been prepared.

*Indians of Ontario.*

| Year ended 30 June | Population | Birth-rate for year,<br>per 1,000 | Death-rate for year,<br>per 1,000 |
|--------------------|------------|-----------------------------------|-----------------------------------|
| 1901               | 20,763     | 29.2                              | 28.0                              |
| 1902               | 20,983     | 28.4                              | 28.3                              |
| 1903               | 21,093     | 32.0                              | 26.9                              |
| 1904               | 21,191     | 30.3                              | 24.7                              |
| 1905               | 20,850     | 32.2                              | 26.4                              |

From this table it will be seen that along with a birth-rate about the same as that of Quebec is coupled a death-rate very much greater than that of Quebec, and still greater than that of Ontario. The best that the officials will say is that the Indians of the Province are *holding their own*. What this amounts to will appear from the following figures, which, making allowance for changes of boundaries of the Province, give the Indian population at quinquennial intervals: 1880, 18,514; 1885, 19,624; 1890, 20,606; 1895, 20,835; 1900, 20,411; 1905, 20,850.

Commenting upon the decrease in the birth-rate of New South Wales, Mr. Hemming states that the report of the Royal Commission seemed to show conclusively that such decrease was due, not to postponement of the marriage age, but to restriction deliberately self-imposed. I have not seen the report, but a searching analysis to which the figures were subjected by Mr. Richard Teece (*Transactions, Actuarial Society of America*, vol. viii, p. 43) does not appear to support this view. Mr. Teece's conclusion is: "The general teaching of the figures given above is that the decreased birth-rate is largely due to the fact that the proportion both of married men and of married women at the ages at which generation is most active has largely decreased." The curves representing, in my diagram, the average marriages and births for a series of years, were derived by setting up rectangles to represent the monthly figures, and drawing a freehand curve through their tops. Each unit length of the abscissa represents a month; and of the ordinate, 200 marriages or births, as the case may be. For any short period of

time, as, for example, one-third of a month, the corresponding ordinate gives the value approximately; for longer periods areas must be substituted.

The excess of Ontario population in each of the 5-9, 10-14, 15-19 age groups over that in the 0-4 group, pointed out by Mr. Todhunter, also attracted my attention, not without misgiving. The only credible explanation of such a distribution that suggests itself to me is that of large numbers of children having been brought in by immigrant families; but this condition cannot be pleaded as obtaining in the present case. If it be an error, then, curiously enough, the 1891 census is also in error, as the figures for Ontario show a similar distribution. Of no other division of Canada does this seem to be true in either census. My belief at the time was, and still is, that the figures of the 0-4 group in both the 1891 and 1901 census are understated, and, as the population figures in that first stage of life were not employed in deriving rates of mortality, nothing appeared to hinge upon the error. I wholly agree, of course, with Mr. Todhunter's remarks, as to the basis upon which rests the method of assigning the births of two consecutive years to the mid-point of time. It is an empirical rule, based upon the condition that fluctuations in the birth-rate are not of such a nature as to disturb its operation, and it neglects unequal incidence of the death-rate. If Mr. Todhunter's proposal to take the births in the proportion of one-third and two-thirds, respectively, had been given effect to, the rates would not have differed materially from those which would have resulted, had my correction of the usual rule by five days not been made.

Mr. King's objection to comparing the actual with the expected deaths, as calculated by an aggregate table, is in general of course well based, and I did not wish it to be understood that I endorsed the practice, or relied implicitly upon results so obtained. There must be instances, however, where the expected deaths by an aggregate table would differ little from the "select" expected. Such an instance would be furnished by a comparatively young office, where the business in force for the shorter durations was relatively heavy. If, for example, the true mortality were exactly that of the  $O^{(M)}$  Table, and the mass of business were centred between durations 0-10 years, the  $O^M$  expected deaths might very easily, for that section, be in excess of the true, and might, for the total business, exceed the  $O^{(M)}$ . The business of a young country, as a whole, might conform to that type, and the lighter mortality endowment-assurance element, so large in modern business, would contribute its share towards equalization.

It is a matter of regret that the raw material underlying my calculations was not more extensive. As to its trustworthiness, I tried to avoid claiming more for it than I thought it justly entitled to. I am hopeful that the results may serve at least as a land-mark in future work. Unless the proportion of urban population increase very rapidly in Canada, the mortality will probably continue light for years to come. A heavy immigration is likely to be a feature of at least the first half of the present century.

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*The Variations in Masculinity under different conditions.\** By JOHN NORMAN LEWIS, F.I.A., F.F.A., and CHARLES JAMES LEWIS, D.Sc., M.D., F.R.C.P. (Edinburgh).

[Read before the Institute, 29 January 1906.]

IN submitting to the Institute the results of our investigation into the subject of masculinity, we are aware that this theme is somewhat outside the ordinary discussions of the Institute, and we only propose to deal with certain aspects of the question. It may therefore be well to begin by defining the term masculinity, which is simply a convenient expression for stating in one word the proportion of the sexes at birth. The choice of this word has been determined by the fact that male births generally predominate in number over female births. According to the extent of this predominance, masculinity may be described as high or low. The usual practice is to state the number of male births to 1,000 females. When the male births are fewer than the female births the masculinity is said to be negative, the representative figure being then below 1,000.

The subject is not a fresh one in the annals of the Institute, for so long ago as 1853 there was published in the *Journal* (vol. iii, p. 17) a paper by the late Samuel Brown, dealing with the proportions in which male and female children are born, and discussing the theory, which had been advanced by Hofacker and Sadler, that these proportions are entirely dependent upon the relative ages of the parents. At that time the statistics which were available for an investigation of this subject were very sparse, and in general were limited to the experience of special classes, such as the peerage. The volume of the observations was small, and the deductions made from them consequently unreliable.

The proportions of masculine and feminine births must be the result of definite causes, and dependent upon laws which are not yet known. Though we are not able to state these laws, it may nevertheless be useful to gather together the large volume of statistics which are now available from many sources. We shall thus be able to observe the influence which various conditions appear to exert upon the proportions of the sexes at birth.

\* In submitting this Paper to the Council, the joint Authors desired permission to include any of the Tables, &c., contained therein, in a separate treatise to be subsequently published by them, and which has since appeared. The expression "Masculinity" is employed by the Authors, upon their sole responsibility, in a special sense, as defined in the first paragraph of the Paper.—ED. *J.I.A.*

In England, the details recorded in the Birth Registers are not so minute as in some other countries. In particular it has never been the practice in this country to record the ages of the parents of the children born, so that it is not possible to analyze the English births with regard to this feature.

In Scotland, however, in the first year of registration (1855), the particulars obtained at the registration of each birth included many details of great value to the student of demography. The registers show the ages of the parents, the order of birth of the registered child, the duration of the marriage, the issue of the parents, and the occupation of the father, in addition to the sex of the registered child. Unfortunately the schedule was altered in the following year, and many of these invaluable facts were omitted in subsequent years.

The Scottish Registers of 1855, therefore, are the only source in Great Britain from which a large volume of statistics is obtainable for the study of this subject. The figures required, however, are not given in the Registrar-General's reports, and it has, therefore, been necessary for us to go direct to the Registers and extract the particulars of each birth for ourselves. Each extract was carefully made upon a separate card and independently checked. The total number of births thus dealt with was 93,369, of which 85,964 were legitimate and 7,405 illegitimate. The total number according to the Registrar's reports was 93,363, the difference of 6 being due to a few errors in registration. The results of our research as dealing with masculinity are shown in this paper, together with those obtained by others in similar investigations abroad.

Under present conditions the possession of a positive masculinity appears to be an integral necessity of a vigorous nationality. The reason for this lies in the heavier mortality which the male suffers as compared with the female in the early years of life. Not only do male children perish in early years, but even in early months, at a greater rate than their sisters. The result of this higher death-rate is to neutralize positive masculinity and to equalize the distribution of the sexes. Aided by emigration, it may even produce a sex-distribution of the population which entirely reverses the proportion which the sexes held at birth, the result being an excess of females in the adult population. This result is more probable if the masculinity is only feebly positive. A negative masculinity is quite exceptional among civilized nations.



*Magnitude of Masculinity.*—The magnitude of masculinity varies within certain defined limits. The masculinity of a people rarely exceeds 1,100 or falls below 900. Therefore, we may say broadly that a 10 per-cent variation in either direction covers the range of masculinity in aggregate populations. We shall see that when smaller groups are considered the range of masculinity is beyond this limit, but at present we are dealing with it only in broad outline. Further, this 10 per-cent variation includes exceptional conditions both of positive and negative masculinity. The common range of masculinity is much less than this. Its representative figures are usually between 1,020 and 1,070. Its variation therefore does not usually exceed 5 per-cent.

*Variations of Masculinity.*—The conditions under which variations of masculinity are observed may conveniently be discussed under the following headings: (1) Nationality, (2) Birth-rate, (3) Legitimacy and Illegitimacy, (4) Still-birth, (5) Primogeniture, (6) Order of Birth, (7) Multiple Births, (8) Urban or Rural Residence, (9) Social Condition, (10) Season of the Year, (11) Parental Age.

(1) *Nationality.*—The magnitude of masculinity varies greatly in different nations. Amongst European countries, Greece and Roumania take the leading places, their average masculinity for the years 1865-90 being 1,138 and 1,108 respectively. Though scattered in different countries, it has frequently been observed that the Jewish race shows an exceptionally high masculinity. The Latin races, with the exception of France—that is to say, Italy, Portugal, and Spain—all show a high masculinity. Russia, Germany, and the United Kingdom represent a medium masculinity, while that of France is lower than in any of the preceding nations. A negative masculinity is said to occur among certain uncivilized races.

Masculinity does not always maintain the same level. Its movement may be either in the direction of increase or of diminution. Such variations may occur in nations with either high or low masculinity. Thus Greece and Roumania both show a high present masculinity; in the former the variation is in the direction of increase, while in the latter there has been a tendency to diminution. Other nations which show an increasing masculinity are Spain and Norway; while Russia, Austria, and Italy maintain a stationary position. A decrease in masculinity is a prominent feature in the statistics of Great Britain, France, and Belgium.



Table I shows the average masculinity in England and Scotland separately for each decade since the registration of births began, and, for comparison, the corresponding figures relating to France from 1811 to 1900 :

TABLE I.—*Birth-rate and Masculinity.*

| Period    | ENGLAND AND WALES |             | SCOTLAND   |             | FRANCE     |             |
|-----------|-------------------|-------------|------------|-------------|------------|-------------|
|           | Birth-rate        | Masculinity | Birth-rate | Masculinity | Birth-rate | Masculinity |
| 1811-1820 | ...               | ...         | ...        | ...         | 31·8       | 1,064       |
| 1821-1830 | ...               | ...         | ...        | ...         | 30·9       | 1,060       |
| 1831-1840 | ...               | ...         | ...        | ...         | 29·0       | 1,060       |
| 1841-1850 | 32·6              | 1,049       | ...        | ...         | 27·4       | 1,056       |
| 1851-1860 | 34·1              | 1,046       | 34·1*      | 1,057*      | 26·7       | 1,056       |
| 1861-1870 | 35·2              | 1,042       | 34·9       | 1,054       | 26·2       | 1,049       |
| 1871-1880 | 35·4              | 1,038       | 34·8       | 1,056       | 25·4       | 1,047       |
| 1881-1890 | 32·5              | 1,037       | 32·3       | 1,054       | 23·9       | 1,046       |
| 1891-1900 | 29·9              | 1,036       | 30·2       | 1,050       | 22·2       | 1,041       |

\* Six years only.

It will be observed that in all three countries the masculinity has steadily decreased from the commencement, with the exception of a slight temporary reaction in the case of Scotland in the decade 1871-80. This irregularity is attributable to an abnormally high masculinity in 1871 and 1872. In the case of England the figures show a decrease in six decades, from 1,049 to 1,036 ; while in the case of France, during a period of nine decades, there is a decrease from 1,064 to 1,041. It is notable that in each decade the lowest masculinity is that of England, and this is approximately as much below that of France as the latter is below that of Scotland. The mean masculinity of Scotland for the 47 years, 1855-1901, was 1,053, a moderate but not very high figure. The maximum in this period was 1,070 attained in the year 1871, and the minimum was 1,042 in the year 1861.

In connection with the influence of race on masculinity, it is essential to enquire whether cross-breeding is associated with a high or a low masculinity, or has any marked effect in increasing or diminishing its magnitude. The researches of Monsieur S. Gache throw some light on this question, at all events as it is observed in Buenos Ayres. Cross-breeding has in this town invariably had the effect of raising the masculinity. The births, as the result of unions of Italian, Spanish, and French male emigrants with the Argentine female natives, show a higher

masculinity than the births produced either by pure Argentine alliances, or by pure alliances of any of these nations in Buenos Ayres. The figures on which this statement is founded cover a period of ten years from 1884 to 1893. Further, the unions of Argentine males with females of foreign nationality provide a higher masculinity than is common among Argentines themselves.

Although, perhaps, we should hardly expect emigration to affect masculinity among the emigrants in so marked a degree as to be capable of demonstration in figures, there is some evidence to this effect. In forming any conclusion on this point it is necessary to eliminate carefully the influence of cross-breeding. The latter increases masculinity, while emigration appears to have the opposite effect. It is necessary, therefore, to be careful that the one influence does not obscure the other. For this purpose the only statistics that are reliable must refer solely to marriages of persons of one nationality but resident in another country. Fulfilling these conditions are the figures as to the natality and masculinity in Buenos Ayres during the years 1884-93. It appears that the Italian, Spanish, and French nationalities furnish a large proportion of the emigrants in Buenos Ayres, more especially the Italian. The English population is small, and provides too small a proportion of the total births to be quoted as to masculinity. The Italians married to Italians and resident in Buenos Ayres have a lower masculinity than Italians in their home country. Whereas the masculinity in Italy is 1,060, in Buenos Ayres Italians have only a masculinity of 1,042. Spaniards married to Spaniards and resident in Buenos Ayres have a masculinity of 1,028, whereas in Spain the masculinity is 1,083. French married to French and resident in Buenos Ayres show a masculinity of 1,036, while the masculinity of France at the same period was about 1,046. These three Latin races, therefore, appear to lose somewhat in masculinity by emigration. This rule does not invariably apply to the Anglo-Saxon race. The general masculinity of the British Colonies is quite as high as that of the home country. Canada, Australia, and New Zealand all possess a masculinity of 1,050 or upwards.

(2) *Birth-rate*.—Does the rise and fall of masculinity vary directly or inversely with the birth-rate, or does it appear that the latter has no influence on masculinity? For the discussion of this question we are able to draw upon a complete series of figures for Scotland, England and France. The birth-rate is

stated per thousand of the whole population living at all ages. The relative figures are found in Table I, which shows that in all three countries the fall in both masculinity and the birth-rate has been very great. In the case of France in particular the regularity of the fall during so extended a period of years is very striking. The English birth-rate, however, reached a high level between 1860 and 1880, and since the latter date its decline has been very pronounced. On the other hand, the masculinity has progressively decreased throughout the sixty years. In Scotland, as in England, the birth-rate attained its highest point between 1860 and 1880 and has since declined. The masculinity of Scotland has notably diminished, but not with that regularity which characterizes the French figures. In Ireland during thirty years the masculinity has fallen from 1,058 to 1,056, while the birth-rate declined from 26 to 23.

It is evident, therefore, that a falling birth-rate may be accompanied by a falling masculinity. The English figures show, however, that a rise in the birth-rate does not necessarily involve a rise in the masculinity. In further confirmation of the fact that the birth-rate and masculinity may vary in different directions during the same period, it is sufficient to quote the experience of Buenos Ayres during the decade 1884-93. In the first five years the birth-rate was 35·97 and the masculinity 1051·6, while in the second five years the birth-rate rose to 43·29 and the masculinity fell to 1033·5. We are therefore justified in concluding that variations in masculinity may be coincident with, but do not depend upon, similar variations in the birth-rate.

(3) *Legitimacy and Illegitimacy*.—Masculinity is seldom of the same magnitude in the legitimate and illegitimate births. It is, however, generally positive in both classes, but in some countries the legitimate, and in others the illegitimate, births show the higher masculinity. In the Continental countries of Europe the illegitimate births almost invariably exhibit a much lower masculinity than the legitimate births. Various explanations have from time to time been suggested to account for this, *e.g.* (1) the probability of a large number of illegitimate births being concealed; (2) the non-registration of still-births, and the likelihood of a larger proportion of illegitimate males being still-born than in the case of the legitimate males; and (3) the more youthful age of the fathers of illegitimate children as contrasted with that of the fathers of legitimate offspring.

TABLE II.—*Mean Masculinity.*

| Period    | FRANCE            |                     | BELGIUM           |                     |
|-----------|-------------------|---------------------|-------------------|---------------------|
|           | Legitimate Births | Illegitimate Births | Legitimate Births | Illegitimate Births |
| 1811-1820 | 1,068             | 1,044               | ...               | ...                 |
| 1821-1830 | 1,064             | 1,041               | ...               | ...                 |
| 1831-1840 | 1,064             | 1,039               | ...               | ...                 |
| 1841-1850 | 1,058             | 1,032               | 1,055             | 1,025               |
| 1851-1860 | 1,059             | 1,036               | 1,054             | 1,025               |
| 1861-1870 | 1,050             | 1,034               | 1,054             | 1,030               |
| 1871-1880 | 1,049             | 1,028               | 1,049             | 1,024               |
| 1881-1890 | 1,047             | 1,034               | ...               | ...                 |
| 1891-1900 | 1,043             | 1,031               | ...               | ...                 |

A comparison of the masculinity in legitimate and illegitimate births in France in each decade from 1811 to 1900 is shown in Table II, from which it will be seen that the difference in masculinity is both considerable and consistent. It may be noted also that the illegitimate masculinity shows the same feature of a steady and large decline which characterizes the legitimate births, and which has been already referred to in the case of the total births shown in Table I. Table II also shows similar figures relating to Belgium during a period of four decades. Here also the legitimate masculinity is the greater. Düsing gives the legitimate masculinity of Prussia in the years 1875-87 as 1,064, while the illegitimate masculinity was only 1,055. In Great Britain the masculinity of the illegitimate births appears to follow a reverse rule to that on the Continent. Both in England and Scotland the illegitimate births show consistently a masculinity exceeding that of the legitimate. In Table III is collected the masculinity of the legitimate and illegitimate births in each of the countries composing the United Kingdom.

TABLE III.—*United Kingdom. Mean Masculinity.*

| Period    | ENGLAND AND WALES |              | SCOTLAND   |              | IRELAND    |              |
|-----------|-------------------|--------------|------------|--------------|------------|--------------|
|           | Legitimate        | Illegitimate | Legitimate | Illegitimate | Legitimate | Illegitimate |
| 1852-1861 | 1,046             | 1,048        | 1,052*     | 1,073*       | ...        | ...          |
| 1862-1871 | 1,041             | 1,046        | 1,056      | 1,062        | 1,056†     | 1,043†       |
| 1872-1881 | 1,038             | 1,040        | 1,054      | 1,058        | 1,058      | 1,062        |
| 1882-1891 | 1,037             | 1,043        | 1,054      | 1,059        | 1,057      | 1,047        |
| 1892-1901 | 1,036             | 1,044        | 1,049      | 1,055        | 1,056      | 1,051        |
| Average   | 1,039             | 1,044        | 1,053      | 1,061        | 1,057      | 1,051        |

\* 1855-61.

† 1864-71.



The figures given are those of the mean masculinity for each decade, and cover the period since registration commenced in Scotland and Ireland and the years 1852-1901 for England and Wales. It is noticeable that whereas in Scotland after the first period of seven years the difference between the legitimate and illegitimate masculinity remains virtually constant, in England this has somewhat increased, and in the last decade amounted to 8. The figures for Ireland show considerable fluctuations, but it must be borne in mind that the illegitimate births in that country are very few, and they therefore hardly form a basis sufficiently secure for argument. The experience of New South Wales appears to coincide with that of the Continent of Europe. Its masculinity is stated by Coghlan as 1058·5 in the case of legitimate, and 1034·1 in the case of illegitimate, births (1882-1901).

In Scotland the masculinity during a period of 47 years (1855-1901) has varied in individual years from 1,067 to 1,037 in the legitimate births, and from 1,103 to 1,002 in the illegitimate births. The mean legitimate masculinity is 1,053, while that of the illegitimate births is 1,061. The difference between French and Scottish masculinity may be explained by the radical difference between the illegitimacy of the two countries. In Scotland, a large proportion of the illegitimate births are the product of conditions akin to ordinary marriage, and are indeed rendered legitimate by subsequent marriage of the parents. It is frequently the custom in Scotland to postpone marriage until the birth of one child, which is registered at first as illegitimate. It is therefore reasonable to expect that in Scotland the illegitimate births should correspond more closely to the legitimate births in masculinity than is the case in France and other countries. In France, the difference between the mean masculinity of legitimate and illegitimate births is 20 over a period of 90 years, whereas in Scotland the corresponding difference over a period of 47 years is only 8. Table III shows that the same difference in the case of England and Wales only amounts to 5 in the mean masculinity of 50 years. Ireland corresponds more closely with France in that the mean legitimate masculinity exceeds the illegitimate figure, but the difference is not nearly so high as in the latter country.

(4) *Still-births*.—It is worth noting that the excess of male births is really appreciably greater than is apparent from the birth registers. The reason is that in the United Kingdom no registration is made of still-births, among which the proportion



of males is very high. In illustration of this phase of masculinity we give in Table IV figures showing, for a variety of places, and for different periods, the masculinity of still-births. The table includes, in respect of several of the places, for comparative purposes, the masculinity among live births, which is commonly about 1,050. The masculinity of still-births is never lower than 1,200, and rises in one instance to 1,700, though it is generally about 1,300. As still-births seem to average about 40 to 1,000 living births, it appears that a masculinity of 1,050 among living births represents a masculinity of 1,060 or more among viable births. Rosenfeld shows that in Vienna, in the years 1895 and 1896, the inclusion of the still-births raises the masculinity from 1,061 to 1,070. Corbaux presents figures relating to Paris, and covering a period of eight years, which show a similar increase from 1,039 to 1,047. In Sweden, the analogous figures as quoted by Wargentín are 1,037 and 1,044.

TABLE IV.—*Masculinity of Still-births.*

| Place                | Observer    | Period    | Masculinity<br>of Still Births | Masculinity<br>of Live Births |
|----------------------|-------------|-----------|--------------------------------|-------------------------------|
| Amsterdam . . .      | Quetelet .  | 1821-32   | 1,312                          | 1,057                         |
| Paris . . . . .      | „ .         | 1823-32   | 1,226                          | 1,040                         |
| Dublin . . . . .     | Clarke .    | ...       | 1,701                          | 1,101                         |
| Westminster . . .    | Bland .     | ...       | 1,400                          | 1,008                         |
| Sweden and Finland . | ...         | 1755-63   | 1,356                          | 1,044                         |
| Alsace-Lorraine . .  | Stieda .    | 1872-73   | 1,279                          | 1,059                         |
| Italy . . . . .      | Bodio .     | 1887-91   | 1,311                          | 1,058                         |
| France . . . . .     | „ .         | 1887-91   | 1,422                          | 1,046                         |
| Germany . . . . .    | „ .         | 1887-91   | 1,283                          | 1,052                         |
| Austria . . . . .    | „ .         | 1887-91   | 1,321                          | 1,058                         |
| Hungary . . . . .    | „ .         | 1887-91   | 1,300                          | 1,050                         |
| Switzerland . . . .  | „ .         | 1887-91   | 1,350                          | 1,045                         |
| Belgium . . . . .    | „ .         | 1887-91   | 1,321                          | 1,045                         |
| Netherlands . . . .  | „ .         | 1887-91   | 1,277                          | 1,055                         |
| Prussia . . . . .    | Düsing .    | 1872-81   | 1,291                          | 1,054                         |
| Paris . . . . .      | Corbaux .   | (8 years) | 1,202                          | 1,039                         |
| Montpellier . . . .  | Mourgue .   | ...       | 1,286                          | 1,065                         |
| Sweden . . . . .     | Wargentín . | ...       | 1,348                          | 1,037                         |
| Norway . . . . .     | Ploss .     | 1865-82   | 1,291                          | ...                           |
| Spain . . . . .      | „ .         | 1865-70   | 1,503                          | ...                           |
| European Russia . .  | „ .         | 1875-78   | 1,282                          | ...                           |
| Massachusetts . . .  | „ .         | 1870-81   | 1,481                          | ...                           |
| Rhode Island . . . . | „ .         | 1875-83   | 1,595                          | ...                           |
| Livonia . . . . .    | Carlberg .  | 1873-82   | 1,269                          | 1,053                         |

(5) *Primogeniture.*—It may naturally be expected that on such a subject as masculinity some light may be thrown by a consideration of its phenomena as they occur in first births. Here there are no

precedent procreations to alter the succeeding births, either by influencing the fecundity of the parents, or by introducing modifications of probability as to the sex of the issue. Accordingly it is to the first births that we look for a clue to some at least of the causes that influence sex and modify masculinity. It is a common belief that the masculinity of first-born children exceeds that of later born children. This means that, on the whole, there is a greater probability of a first birth being a boy than of subsequent births proving male. We find that in Scotland in 1855 the masculinity of the legitimate first births was 1,054, whereas the masculinity of the total legitimate births was 1,049. There is thus an excess of masculinity of 5 in favour of the legitimate first births as compared with the total legitimate births. If we compare the former, not with the total births but with subsequent births, this excess is raised to 6, the masculinity among the latter being 1,048. This is a feature which is not constant at all ages of the parents, but prevails in the case of fathers up to the age of 49. From the age of 50 onwards the masculinity of first births is lower than that of the total births at these ages. With regard to mothers, the first births exceed the total births in masculinity at the early ages, but after the age of 25 years the proportion of males among the first births is lower than that in the total births. The actual figures are shown in Table V.

TABLE V.—*Masculinity.*

| Age   | FATHERS      |              | MOTHERS      |              |
|-------|--------------|--------------|--------------|--------------|
|       | Total Births | First Births | Total Births | First Births |
| 15-19 | 1,150        | 1,203        | 996          | 1,017        |
| 20-24 | 1,019        | 1,037        | 1,038        | 1,073        |
| 25-29 | 1,027        | 1,032        | 1,046        | 1,039        |
| 30-34 | 1,077        | 1,105        | 1,066        | 1,048        |
| 35-39 | 1,066        | 1,112        | 1,041        | 1,039        |
| 40-44 | 1,033        | 1,084        | 1,065        | 1,000        |
| 45-49 | 1,049        | 1,136        | ...          | ...          |
| 50-54 | 1,045        | 852          | ...          | ...          |
| 55-59 | 1,165        | 889          | ...          | ...          |
| 60-64 | 940          | 765          | ...          | ...          |

The general bias of masculinity among the first births is brought out very strongly in Austrian figures, dealing with 1,140,860 living births, and submitted by Bertillon. The masculinity of the first and subsequent births was respectively 1,086 and 1,054. The same feature was noticed by the late W. A. Bowser in his

investigations into the families of Baptist Ministers (*J.I.A.*, vol. xvii, p. 26). Out of 1,653 children, 314 were first births, and these exhibited a masculinity of 1,220 in contrast to a masculinity of 1,008 among the subsequent births. The same result appears conspicuously in the figures given by the late Archibald Day with regard to the issue of second marriages in the British Peerage (*J.I.A.*, vol. xii, p. 185). There was a majority of female births as a whole, yet the first births of 307 fruitful marriages showed a masculinity so high as 1,302. In both of these latter investigations, however, the number of births under observation was small. In an enquiry by Stieda into the births in Alsace-Lorraine in 1872-1873, although the numbers dealt with were large, the first births were found to display a slightly lower masculinity than the subsequent births. It appears, therefore, that the rule as to a higher masculinity in first births is not invariable.

TABLE VI.—*Masculinity and Primogeniture.*

| Source                                          | Observer            | Period  | Total Births | MASCULINITY  |                   |
|-------------------------------------------------|---------------------|---------|--------------|--------------|-------------------|
|                                                 |                     |         |              | First Births | Subsequent Births |
| British Peerage . . . . .<br>(Second Marriages) | Day . . . . .       | 1856-64 | 1,046        | 1,302        | 849               |
| Ministers' Families. . . . .                    | Bowser . . . . .    | 1871    | 1,653        | 1,220        | 1,008             |
| Austria . . . . .                               | Bertillon . . . . . | ...     | 1,140,860    | 1,086        | 1,054             |
| Alsace-Lorraine . . . . .                       | Stieda . . . . .    | 1872-3  | 47,198       | 1,058        | 1,059             |
| Scotland . . . . .                              | Lewis . . . . .     | 1855    | 85,964       | 1,054        | 1,048             |
| Denmark, Norway and<br>Austria                  | Bertillon . . . . . | ...     | ...          | 1,100        | 1,050             |
| Saxony . . . . .                                | Geissler . . . . .  | 1876-85 | 4,794,304    | 1,054        | (1,000)           |

(6) *Order of Birth.*—The masculinity of the first births has just been described, and contrasted with subsequent births as a whole. So far as Scotland is concerned, it is possible for us to differentiate between the subsequent births, and to assign a masculinity to each individual order of birth for the year 1855. This is done in Table VII for all births up to the twelfth, and it is found that the results are extremely irregular. No doubt this is due to the limitation of the data both as to time and numbers. The second and third births present low masculinities, whereas the fourth and fifth births are substantially above the average. There is a fall in the masculinity of the sixth births, but thereafter

up to the tenth births the masculinity rises. Later the births are too small in number for reliable proportion in their masculinities.

TABLE VII.—*Masculinity and Order of Birth.*  
*Scotland, 1855.*

| Order of Birth | Masculinity | Order of Birth | Masculinity |
|----------------|-------------|----------------|-------------|
| 1              | 1,054       | 7              | 1,058       |
| 2              | 1,036       | 8              | 1,073       |
| 3              | 1,005       | 9              | 1,085       |
| 4              | 1,071       | 10             | 1,102       |
| 5              | 1,078       | 11             | 947         |
| 6              | 1,047       | 12             | 1,005       |

It is interesting to note that Geissler, assorting the families of Saxony in the years 1876–85, found that the masculinity of marriages with 2 to 7 children was 1,058, while that of marriages having over 7 children was 1,068.

(7) *Multiple Births.*—During the year 1855 there were 11 triplet births in Scotland. Of these one was illegitimate and may be dismissed with the note that the mother's age was 23 and that the birth consisted of three males and was stated to be the first pregnancy of the mother. In the 10 legitimate triplet births the ages of the mothers varied from 30 to 42, and those of the fathers from 30 to 51. It is commonly stated that triplet births rarely occur in women under 30 years of age, and this is confirmed by the incidence of triplet births in Scotland in 1855. The only case in which the mother's age was below 30 was the illegitimate triplet referred to above.

TABLE VIII.—*Legitimate Triplet Births.*

| Father's Age | Mother's Age | Sex of Triplets       |
|--------------|--------------|-----------------------|
| 43           | 40           | 3 Females             |
| 40           | 30           | 3 Females             |
| 30           | 32           | 3 Females             |
| 51           | 42           | 3 Males               |
| 42           | 32           | 3 Males               |
| 30           | 32           | 3 Males               |
| 31           | 38           | 3 Males               |
| 42           | 39           | { 2 Males<br>1 Female |
| 42           | 35           | { 2 Males<br>1 Female |
| 45           | 38           | { 2 Females<br>1 Male |

Table VIII shows the combinations of the ages of the parents of legitimate triplets in 1855. It also shows the distribution of the sexes among the individual components of the triplets. In 4 cases there were 3 males, in 3 cases 3 females, in 2 cases 2 males and 1 female, and in 1 case 2 females and 1 male. If the illegitimate case be included, it appears that in 8 out of 11 cases of triplets in the same year and nation the children were not diverse as to sex. There appears, therefore, to be a strong probability that in any given occurrence of triplets the children will all be of the same sex, either all males or all females. The masculinity of the triplets was high, probably abnormally so, as it reached 1,538. The masculinity of triplets is, however, often high, for an investigation including France, Germany and Austria gave a masculinity of 1,080. Düsing gives the masculinity of triplets in Prussia in the years 1824-87 as 1,051. On the other hand, Carlberg found that in Livonia triplets showed a masculinity of only 781.

The 1,050 cases of twin births which occurred in Scotland in 1855 comprised 321 cases of two males, 320 cases of two females, and 409 cases in which the sexes were diverse. It is remarkable that the first two of these numbers should be so nearly identical. The sex distribution of the 976 legitimate twins gives a masculinity of 1,002, there having been 977 males to 975 females. The inclusion of the 74 illegitimate twins does not alter the masculinity, as they were equally divided as to sex.

The sex-distribution of the twin births in Paris during the 4 years 1892-95 showed that their masculinity was negative when the mother was under the age of 25 years and positive when she was over that age. In France as a whole it is stated that the masculinity of twin births is only 1,020, while in Prussia it is 1,056, and in Saxony 1,078; and Carlberg found that in Livonia the masculinity of twin births was 1,066, or slightly higher than the total births.

(8) *Urban and Rural Residence*.—Masculinity varies very decidedly under urban and rural conditions. Attention has been drawn to this circumstance by Continental statisticians, and the figures given in the Scottish reports enable us to ascertain whether the same variation exists in Scotland. Considering first the total births, we find that there is in rural districts a higher masculinity than in towns. At first, *i.e.*, from 1855 to 1870, Scotland was divided into Town Districts, Mainland Rural Districts, and Insular Rural Districts. In 1871 the Town



Districts were sub-divided into Principal Towns, Large Towns, and Small Towns, and the figures were no longer given in one group for all towns. We therefore submit the figures for the different groups of Town Districts from 1872-1901, and for Rural Districts from 1855-1901. We should premise that a principal town is a town with a population of over 25,000, a large town one containing from 10,000 to 25,000, and a small town from 2,000 to 10,000 persons. In 1901 the population required to confer the dignity of a principal town was raised to 30,000.

TABLE IX.—*Masculinity. Urban and Rural Residence. Scotland. Total Births.*

| Period    | Principal Towns | Large Towns | Small Towns | Mainland Rural Districts | Insular Rural Districts |
|-----------|-----------------|-------------|-------------|--------------------------|-------------------------|
| 1855-61   | ...             | ...         | ...         | 1,056                    | 1,066                   |
| 1862-71   | ...             | ...         | ...         | 1,059                    | 1,056                   |
| 1872-81   | 1,050           | 1,056       | 1,061       | 1,053                    | 1,080                   |
| 1882-91   | 1,051           | 1,056       | 1,055       | 1,055                    | 1,087                   |
| 1892-1901 | 1,047           | 1,046       | 1,049       | 1,052                    | 1,071                   |
| Average   | 1,049           | 1,053       | 1,055       | 1,055                    | 1,072                   |

Table IX shows the mean masculinity of the total births in Scotland, according to the urban or rural character of the districts in which the births occurred. It is clear from this table that in towns with populations higher than 10,000 the masculinity is lower than in rural districts. Further, the island population shows a higher masculinity than the mainland rural districts, and there is little difference between the smaller towns and the country districts. The latter feature is quite consistent, in view of the small population required to rank a place as a town. As Scotland possesses few really large towns, the comparison must be mainly between these and the rest of the country. The conclusion is, undoubtedly, in favour of rural life as a stimulus to masculinity in the population as a whole. Our statistics do not enable us to explain the manner in which the influence of rural life is exercised upon individual births.

To show that the variation in masculinity in town and country is not influenced by legitimacy and illegitimacy, we give similar Tables (X and XI) for legitimate and illegitimate births separately.

TABLE X.—*Masculinity. Urban and Rural Residence.  
Scotland. Legitimate Births.*

| Period    | Principal Towns | Large Towns | Small Towns | Mainland Rural Districts | Insular Rural Districts |
|-----------|-----------------|-------------|-------------|--------------------------|-------------------------|
| 1855-61   | ...             | ...         | ...         | 1,051                    | 1,064                   |
| 1862-71   | ...             | ...         | ...         | 1,058                    | 1,055                   |
| 1872-81   | 1,049           | 1,056       | 1,058       | 1,054                    | 1,083                   |
| 1882-91   | 1,051           | 1,056       | 1,054       | 1,055                    | 1,085                   |
| 1892-1901 | 1,048           | 1,044       | 1,050       | 1,052                    | 1,062                   |
| Average   | 1,049           | 1,052       | 1,054       | 1,055                    | 1,070                   |

TABLE XI.—*Masculinity. Urban and Rural Residence.  
Scotland. Illegitimate Births.*

| Period    | Principal Towns | Large Towns | Small Towns | Mainland Rural Districts | Insular Rural Districts |
|-----------|-----------------|-------------|-------------|--------------------------|-------------------------|
| 1855-61   | ...             | ...         | ...         | 1,081                    | 1,101                   |
| 1862-71   | ...             | ...         | ...         | 1,074                    | 1,072                   |
| 1872-81   | 1,056           | 1,058       | 1,073       | 1,047                    | 1,049                   |
| 1882-91   | 1,054           | 1,060       | 1,075       | 1,055                    | 1,140                   |
| 1892-1901 | 1,045           | 1,085       | 1,047       | 1,061                    | 1,126                   |
| Average   | 1,052           | 1,068       | 1,065       | 1,064                    | 1,098                   |

The legitimate births show very much the same incidence of masculinity as the total births. The insular districts exhibit the highest masculinity, and the principal towns the lowest masculinity.

In the illegitimate births, the numbers dealt with are smaller and more erratic, but the main fact that masculinity is higher in rural districts than in towns is plain, whether the births be lawful or not.

A similar experience is manifest in France, as shown by Maurel, and reproduced in Table XII. The masculinity in rural districts is higher than in towns, or in the Department of the Seine. In these figures the legitimate and illegitimate births are taken together, and still-births are excluded. The figures are, therefore, entirely comparable with those for the total births of Scotland (Table IX).

TABLE XII.—*Masculinity. Urban and Rural Residence.  
France. Total Births.*

| Period    | Seine<br>Department | Towns | Rural<br>Districts |
|-----------|---------------------|-------|--------------------|
| 1853-60   | 1,039               | 1,046 | 1,056              |
| 1861-70   | 1,033               | 1,041 | 1,050              |
| 1871-80   | 1,033               | 1,039 | 1,047              |
| 1881-90   | 1,028               | 1,038 | 1,054              |
| 1891-1900 | 1,038               | 1,037 | 1,046              |
| Average   | 1,034               | 1,040 | 1,050              |

An even more complete parallel is found in the following table given by Düsing, and relating to a period of 13 years in Prussia. The districts are divided in a very similar manner to those of Scotland.

TABLE XIII.—*Masculinity. Urban and Rural Residence.  
Prussia. Total Births.*

| Period  | Berlin | Large<br>Towns | Middle<br>Towns | Small<br>Towns | Rural<br>Districts |
|---------|--------|----------------|-----------------|----------------|--------------------|
| 1876-87 | 1,052  | 1,053          | 1,056           | 1,062          | 1,066              |

In other countries broader divisions are commonly made, all towns being included in one group and compared with rural districts. The following figures indicate that the same feature of a higher masculinity in rural districts is found in other civilized countries.

TABLE XIV.—*Masculinity. Urban and Rural Residence.  
Total Births.*

| Place           | Period  | MASCULINITY |       |
|-----------------|---------|-------------|-------|
|                 |         | Urban       | Rural |
| Belgium . .     | 1815-24 | 1,067       | 1,070 |
| Belgium . .     | 1825-29 | 1,053       | 1,061 |
| Massachusetts . | 1849    | 1,047       | 1,114 |
| Sweden . .      | 1861-70 | 1,046       | 1,052 |
| Sweden . .      | 1871-75 | 1,042       | 1,055 |
| Norway . .      | 1851-70 | 1,045       | 1,060 |

In rare instances the masculinity of towns may exceed that of rural districts. Carlberg states that the masculinity of towns in Livonia is 1,078, as compared with 1,053 in the rural districts. He attributes this to the fact that the town populations contain more Jews, Poles and Russians, who all have a high masculinity.

It is clear that there is something in the conditions of country life which tends to the birth of a higher proportion of male children there than in towns.

(9) *Social Condition*.—Though no statistics on this subject have been published in Great Britain, we are able to refer to evidence from other countries, and to add the record of Scotland for 1855.

Swedish statistics referring to the years 1851-60 state the masculinity as follows in various classes of the population :

TABLE XV.—*Masculinity. Social Condition.*  
*Sweden, 1851-60. Total Births.*

| Class                  | Masculinity |
|------------------------|-------------|
| Nobles . . . .         | 983         |
| Clergy . . . .         | 1,086       |
| Officials . . . .      | 1,057       |
| Commercial . . . .     | 1,050       |
| Agriculturists . . . . | 1,057       |

Maurel, investigating the population of two rich streets in Toulouse, found that, of 225 couples, 31, or 14 per-cent, were sterile; and that of 194 fruitful couples there was issue 147 male and 155 female—*i.e.*, a masculinity, in a presumably high social state, of only 950. In the families where children were numerous, the later born children showed the lowest masculinity; and in the families with only one child, the masculinity attained only the extremely low figure of 760. So far it would appear that the richer classes have a feeble masculinity. A confirmation of this is afforded by Maurel's details relative to a large town, in which he obtained figures as to 186 couples belonging to two of the liberal professions. Of these 35, or 18·8 per-cent, were sterile, and 151 fecund. The issue of the latter numbered 126 boys and 149 girls, showing a masculinity of 845.

We have analyzed the births of Scotland in 1855 according to the social condition of the fathers. The results are shown in Table XVI.

TABLE XVI.—*Masculinity. Social Condition.*  
*Scotland, 1855. Total Legitimate Births.*

| Class               | Number of Births | Masculinity |
|---------------------|------------------|-------------|
| Professional . . .  | 2,877            | 1,032       |
| Commercial . . .    | 9,667            | 1,070       |
| Agricultural . . .  | 18,409           | 1,068       |
| Seafaring . . .     | 5,461            | 1,066       |
| Working Classes . . | 49,427           | 1,037       |
| Not stated . . .    | 123              | ...         |
| Total . . . . .     | 85,964           | 1,049       |

The professional class includes the nobility and persons of independent means, as well as the professions. The commercial group includes the merchants and shopkeepers. On comparing Tables XV and XVI, it will be seen that in each case the lowest masculinity figure appertains to the highest social class. It is not possible to say whether this feature would have remained constant in the Swedish figures if the nobles and clergy had been amalgamated in one class. In both tables the masculinity of the agricultural class is high, and it is noteworthy that the seafaring population of Scotland also shows a high masculinity.

As further illustrative of the masculinity found in special classes of the population we may mention the work of other observers. Kisch analyzed the figures relating to 556 marriages in regnant, or aristocratic, European families. There were 1,972 births, of which 1,023 were boys and 949 girls. This gives a masculinity of 1,077, which is distinctly high for the social rank of the parents. Wall made a composite investigation in which he included 1,200 families, selected from the Royal Houses of Europe (120), families of princely rank (27), the English Peerage (890), and the German Peerage (163). There were 6,529 children in these families, and of these 3,381 were males and 3,148 females. This gives a masculinity of 1,074. Orschansky publishes the masculinity of 2,441 families having 13,277 children. He found that in the German Peerage the masculinity was 1,051, in the German colonists 851, in the Russian peasantry 1,147, in various Russian families 1,020, and among the Jews 1,000.

Düsing contrasts the high masculinity of the agricultural



class with the low masculinity manifested by industrial occupations, and both he and Ploss note that those residing in mountainous regions have a high masculinity. The latter indeed states that at 500 feet above the sea-level the masculinity is 1,059, at 1,000-1,500 feet it is 1,073, and at 1,500-2,000 feet it becomes 1,078.

The agricultural class appears to have a high masculinity in the New World as well as in the Old, for it is specially noted by Wynne that in Kentucky, in 1852, and in Virginia, in 1853, the masculinity of this class was 1,120 and 1,140 respectively.

In conclusion, it is advisable to point out that the conflict of evidence as to the masculinity of the births belonging to the higher social classes is largely dependent upon the relatively small numbers of such births. The universal agreement as to the high masculinity found among the agricultural population confirms the evidence given in a previous section as to the superiority in masculinity of rural over urban residents.

(10) *Season of the year.*—The suggestion has sometimes been made that the season of the year has an influence in determining the proportion of male and female births. With a view to investigating the value of this suggestion, we have calculated and brought together in Table XVII the masculinity observed among the births which occurred at different periods in Montpellier, Sweden and Finland, Prussia, Massachusetts and Kentucky respectively.

These figures relate to places widely separate from one another, and differing considerably as to the number of births under observation. It will be observed that the maximum masculinity is found in each column in connection with a different month. The incidence of the minimum masculinity is likewise widely divergent. In none of the columns is there such regularity of progression as to indicate that any special season is particularly associated with an excess of male births. It is true that the Prussian figures suggest that midsummer and midwinter coincide with the highest masculinity. There is no confirmation of this in the other columns of the table.

If we compare the births of winter—say November to February—with those of summer—say May to August—we find that the winter months show a higher masculinity than the summer months in three of the columns. In one column the summer masculinity is somewhat greater than that of the winter, and in the remaining case the figures are equal.

TABLE XVII.—*Masculinity. Season of the Year.*

| Month of Birth | Montpellier.<br>1772-1792,<br>25,064 Births | Sweden and<br>Finland,<br>1776-1795,<br>98,571 Births | Prussia,<br>1872-1881,<br>10,674,254 Births | Massachusetts,<br>1845-1849,<br>92,272 Births | Kentucky,<br>1853,<br>21,498 Births |
|----------------|---------------------------------------------|-------------------------------------------------------|---------------------------------------------|-----------------------------------------------|-------------------------------------|
| January .      | 1,057                                       | 1,041                                                 | 1,063                                       | 1,073                                         | 1,080                               |
| February .     | 1,085                                       | 1,039                                                 | 1,063                                       | 1,049                                         | 1,071                               |
| March .        | 1,068                                       | 1,034                                                 | 1,059                                       | 1,077                                         | 1,233                               |
| April .        | 1,002                                       | 1,043                                                 | 1,060                                       | 1,069                                         | 1,130                               |
| May .          | 1,082                                       | 1,034                                                 | 1,061                                       | 1,112                                         | 973                                 |
| June .         | 1,050                                       | 1,042                                                 | 1,068                                       | 1,078                                         | 1,037                               |
| July .         | 1,056                                       | 1,035                                                 | 1,068                                       | 1,022                                         | 1,144                               |
| August .       | 1,079                                       | 1,054                                                 | 1,063                                       | 1,058                                         | 1,146                               |
| September .    | 1,046                                       | 1,050                                                 | 1,060                                       | 1,021                                         | 1,173                               |
| October .      | 1,107                                       | 1,041                                                 | 1,063                                       | 1,086                                         | 1,073                               |
| November .     | 1,080                                       | 1,047                                                 | 1,062                                       | 1,114                                         | 1,120                               |
| December .     | 1,099                                       | 1,054                                                 | 1,066                                       | 1,075                                         | 1,099                               |
| Total .        | 1,064                                       | 1,043                                                 | 1,063                                       | 1,068                                         | 1,106                               |

TABLE XVIII.—*Masculinity. Season of the Year.  
Births.*

|              | Montpellier | Sweden and<br>Finland | Prussia | Massachusetts | Kentucky |
|--------------|-------------|-----------------------|---------|---------------|----------|
| Nov. to Feb. | 1,067       | 1,045                 | 1,063   | 1,077         | 1,094    |
| May to Aug.  | 1,067       | 1,041                 | 1,065   | 1,065         | 1,074    |

Here we have considered the months in which the births took place, but it is more important to compare the months in which conception occurred, seeing that the sex is undoubtedly determined a considerable time prior to birth.

Table XIX has been prepared with the object of showing the masculinity among winter and summer conceptions respectively.

TABLE XIX.—*Masculinity. Season of the Year.  
Winter and Summer Conceptions.*

|              | Montpellier | Sweden and<br>Finland | Prussia | Massachusetts | Kentucky |
|--------------|-------------|-----------------------|---------|---------------|----------|
| Nov. to Feb. | 1,064       | 1,048                 | 1,062   | 1,068         | 1,127    |
| May to Aug.  | 1,060       | 1,037                 | 1,061   | 1,075         | 1,099    |

Here, in four out of the five cases, a somewhat higher masculinity is shown in respect of the winter conceptions. On the other hand, the remaining case shows precisely the opposite. The evidence is conflicting, and we may be justified in concluding that the season exercises no definite influence on the sex of



Although the number of births upon which these figures are based is in each case considerable, the masculinity frequently fluctuates, and shows no sign of any regular variation in magnitude in relation to the age of the father.

Similarly, in Table XXI, are given the figures for the masculinity according to the age of the mother. Those relating to Scotland show an ascending masculinity up to the age-group 30-34, followed by a brief decline. Mothers aged 40 and upwards exhibit a recrudescence of masculinity, a feature which is also manifest in certain of the other localities included in the table. These places, however, do not indicate a comparatively regular ascent of masculinity among the younger mothers, such as is shown in the case of Scotland.

TABLE XXI.—*Masculinity according to the Age of the Mother.*

| Ages  | Scotland,<br>1855 | Alsace-<br>Lorraine,<br>1872-3 | Norway,<br>1871-5 | Berlin | Oldenburg | Vienna,<br>1895-6 | France,<br>1861-70 | France,<br>1892 |
|-------|-------------------|--------------------------------|-------------------|--------|-----------|-------------------|--------------------|-----------------|
| 15-19 | 996               | 1,124                          | 1,062             | 1,049  | 1,079     | 1,063             | ( 1,090            | 1,070           |
| 20-24 | 1,038             | 1,061                          |                   |        |           |                   | ( 1,058            |                 |
| 25-29 | 1,046             | 1,054                          | 1,045             | 1,062  | 1,058     | 1,073             | 1,080              | 1,050           |
| 30-34 | 1,066             | 1,067                          | 1,066             | 1,053  | 1,075     | 1,057             | ( 1,070            |                 |
| 35-39 | 1,041             | 1,061                          | 1,045             | 1,041  | 1,070     |                   | ( 1,057            | 1,040           |
| 40-44 | 1,065             | 1,078                          | 1,079             | 1,062  | 1,049     | 982               | ( 1,066            |                 |
| 45-49 | 1,055             |                                |                   |        |           |                   | ( 1,055            | 1,060           |

The general trend of experience, as shown in these two tables, completely confirms the conclusion arrived at by Sadler, that “no law of the proportion of the sexes at birth can be traced by depending merely upon the ages of the parents separately.”

Next we may consider the ages of the parents in combination, and observe the masculinity of the offspring.

TABLE XXII.—*Masculinity according to the Combined Ages of the Parents. Scotland, 1855.*

| Ages of<br>Mothers | AGES OF FATHERS |       |       |         |       |         |       |         |         |
|--------------------|-----------------|-------|-------|---------|-------|---------|-------|---------|---------|
|                    | 15-19           | 20-24 | 25-29 | 30-34   | 35-39 | 40-44   | 45-49 | 50-54   | 55-59   |
| 15-19              | (1,024)         | 1,084 | 825   | (689)   | ...   | ...     | ...   | ...     | ...     |
| 20-24              | ...             | 1,028 | 1,022 | 1,093   | 1,055 | 1,121   | ...   | ...     | ...     |
| 25-29              | ...             | 986   | 1,037 | 1,050   | 1,083 | 1,044   | 1,084 | (1,524) | ...     |
| 30-34              | ...             | 841   | 1,037 | 1,092   | 1,052 | 1,082   | 1,054 | 908     | (1,319) |
| 35-39              | ...             | ...   | 983   | 1,117   | 1,065 | 978     | 1,049 | 997     | 1,250   |
| 40-44              | ...             | ...   | ...   | (1,138) | 1,118 | 1,060   | 1,071 | 1,016   | 1,052   |
| 45-49              | ...             | ...   | ...   | ...     | ...   | (1,122) | 994   | (1,200) | (1,050) |

NOTE.—No figure given when less than 100 couples in the group, and figures bracketed when less than 200 couples in the group.

Table XXII shows the masculinity for the legitimate births which occurred in Scotland in 1855, when grouped with reference to the ages of both parents. Tables XXIII, XXIV, XXV, and XXVI show corresponding figures for Alsace-Lorraine, Vienna, France and New South Wales respectively.

TABLE XXIII.—*Masculinity according to the Combined Ages of the Parents. Alsace-Lorraine, 1872-3. (STIEDA).*

| Ages of Mothers | AGES OF FATHERS |       |       |       |       |       |       |           |
|-----------------|-----------------|-------|-------|-------|-------|-------|-------|-----------|
|                 | 15-19           | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 | 50 & over |
| 15-19           | ...             | 1,085 | 1,041 | 1,313 | ...   | ...   | ...   | ...       |
| 20-24           | ...             | 1,075 | 1,070 | 1,051 | 1,024 | 908   | ...   | ...       |
| 25-29           | ...             | 1,082 | 1,091 | 1,025 | 1,025 | 1,049 | 1,175 | 1,122     |
| 30-34           | ...             | 1,222 | 1,076 | 1,083 | 1,057 | 1,032 | 1,072 | 1,000     |
| 35-39           | ...             | ...   | 1,035 | 1,086 | 1,039 | 1,081 | 1,081 | 1,043     |
| 40 and over     | ...             | ...   | ...   | 1,082 | 1,101 | 1,077 | 1,059 | 1,086     |

TABLE XXIV.—*Masculinity according to the Combined Ages of the Parents. Vienna, 1896. (ROSENFELD).*

| Ages of Mothers | AGES OF FATHERS |       |       |       |         |       |
|-----------------|-----------------|-------|-------|-------|---------|-------|
|                 | Under 25        | 25-30 | 30-40 | 40-50 | Over 50 | All   |
| Under 20        | ...             | 1,036 | ...   | ...   | ...     | 1,161 |
| 20-25           | 1,133           | 1,038 | 1,079 | ...   | ...     | 1,075 |
| 25-30           | 1,084           | 1,047 | 1,064 | 1,196 | ...     | 1,065 |
| 30-40           | ...             | 995   | 1,085 | 956   | 1,808   | 1,067 |
| Over 40         | ...             | ...   | 817   | 1,121 | 559     | 954   |
| All . .         | 1,127           | 1,037 | 1,070 | 1,040 | 1,255   | 1,062 |

TABLE XXV.—*Masculinity according to the Combined Ages of the Parents. France, 1892. (TURQUAN).*

| Ages of Mothers | AGES OF FATHERS |       |       |           |
|-----------------|-----------------|-------|-------|-----------|
|                 | Under 25        | 25-35 | 35-45 | 45 & over |
| Under 20        | 1,000           | 1,100 | 1,070 | 1,320     |
| 20-30           | 1,040           | 1,060 | 1,050 | 1,040     |
| 30-40           | 1,150           | 1,040 | 1,050 | 1,050     |
| 40-50           | 880             | 1,060 | 1,070 | 1,060     |
| 50 and over     | 750             | 640   | 870   | 670       |



TABLE XXVI.—*Masculinity according to the Combined Ages of the Parents. New South Wales, 1891–1900. (COGHLAN).*

| Age of Mother | AGE OF FATHER |       |       |       |       |       |       |
|---------------|---------------|-------|-------|-------|-------|-------|-------|
|               | 20            | 25    | 30    | 35    | 40    | 45    | 50    |
| 20            | 949           | 1,041 | 1,110 | 1,174 | 1,232 | ...   | ...   |
| 25            | ...           | 1,070 | 1,088 | 1,092 | 1,088 | 1,033 | 898   |
| 30            | ...           | 1,058 | ...   | 1,083 | ...   | 1,066 | ...   |
| 35            | ...           | ...   | 1,037 | 1,058 | 1,075 | 1,088 | 1,092 |
| 40            | ...           | ...   | ...   | 1,058 | ...   | 1,079 | ...   |
| 45            | ...           | ...   | ...   | 681   | 818   | 923   | 1,008 |

In the French table the fathers are gathered into decennial groups of age, which differ from those used for the mothers. The Viennese figures present another method of grouping; while the New South Wales table gives the masculinity for individual ages and not for groups of ages, and is understood to have undergone some process of graduation.

Consideration of these five tables, both separately and in conjunction with one another, does not clearly show that the proportion of the sexes at birth follows any law based upon the absolute ages of the parents.

Finally, let us examine the masculinity when the births are grouped according to the number of years of difference between the ages of the parents. It was in this direction that Hofacker and Sadler believed that they found the main law of the proportion of the sexes at birth.

Sadler's figures are given in Table XXVII, and, side by side with them, the corresponding figures from our Scottish investigation, and those obtained by Orschansky, Kisch, Schumann, and Rosenfeld. The Scottish and Norwegian figures are based upon much the largest number of births, and deal with a whole nation in each instance, in contrast to special classes of the population to which most of the other figures refer.

The Scottish births, when grouped in this manner, do appear to some extent, though not without irregularity, to show the same tendency as was found by Sadler—namely, to an increase in the masculinity according to the excess of the age of the husband over that of the wife. A closer agreement with Sadler's conclusions is found in the observations of Kisch, but it should be noted that the total births analyzed are small in number. No

confirmation is found in the more numerous data of Orschansky, Schumann, and Rosenfeld; and the same is true of the following table given by Berner, based upon no fewer than 213,224 Scandinavian births. Indeed, in the last case precisely the opposite feature appears. The highest masculinity occurs when

TABLE XXVII.—*Masculinity according to the Relative Ages of the Parents.*

| The Father being<br>as regards the Mother | English<br>Peerage<br>(Sadler)<br>2,068 Births | Scotland,<br>1850,<br>85,964 Births | German<br>Peerage, &c.<br>(Orschansky)<br>13,277 Births | European<br>Aristocracy<br>(Kisch)<br>1,972 Births | Norway<br>(Schumann),<br>198,764 Births | Vienna<br>(Rosenfeld) |
|-------------------------------------------|------------------------------------------------|-------------------------------------|---------------------------------------------------------|----------------------------------------------------|-----------------------------------------|-----------------------|
| Younger                                   | 865                                            | 1,035                               | 1,000                                                   | 1,052                                              | ...                                     | 1,151                 |
| Same Age                                  | 947                                            | 1,049                               | 1,000                                                   | 809                                                | ...                                     | 1,065                 |
| Older 1-5 years                           | 1,037                                          | 1,048                               | 970                                                     | 1,038                                              | 1,039                                   | 988                   |
| „ 6-10 „                                  | 1,287                                          | 1,051                               | 940                                                     | 1,068                                              | 1,055                                   | 1,061                 |
| „ 11-15 „                                 | 1,475                                          | 1,067                               | 1,100                                                   | 1,137                                              | 1,023                                   |                       |
| „ 16-20 „                                 | 1,600                                          | 1,058                               | 1,180                                                   | 1,221                                              | 1,074                                   |                       |
| „ Over 20 „                               | 1,667                                          | 1,118                               |                                                         |                                                    | 1,026                                   |                       |

the mother is the elder by 1-10 years: there is a decrease when the parents are of equal age, and a progressive decrease as the age of the father exceeds that of the mother—

|                            |   |   |       |
|----------------------------|---|---|-------|
| Father over 10 years older | . | . | 1,035 |
| „ 1-10 „ „                 | . | . | 1,046 |
| „ equal in age .           | . | . | 1,062 |
| Mother 1-10 years older    | . | . | 1,075 |
| „ over 10 „ „              | . | . | 1,041 |

If the births are divided into only three large groups—namely, (1) father older, (2) equal in age, (3) mother older, figures may be quoted from many sources, as shown in Table XXVIII.

Such a table ought to afford definite confirmation of the Hofacker-Sadler law if this law expressed an invariable relation between masculinity and the relative age of the parents. Examination of the table shows, however, that no unanimity exists in the experience of the different localities. While the observations of Goehlert, Legoyt, Boulenger, Boeckh, Boudin, as well as our own, may be taken as confirming those of Hofacker and Sadler; the investigations of Stieda, Berner, Rosenfeld, Schumann and Noirot not only do not agree with the former, but, on the contrary, suggest a law precisely the reverse. Additional weight is to be attached to the latter researches on account of the larger numbers of births under consideration. In these circumstances we cannot

TABLE XXVIII.—*Masculinity according to the Relative Ages of the Parents.*

| Observer   | Number of Births | Source of Data or Locality | Date    | RELATIVE AGE OF PARENTS |                      |              | Total Masculinity |
|------------|------------------|----------------------------|---------|-------------------------|----------------------|--------------|-------------------|
|            |                  |                            |         | Father older            | Parents equal in age | Mother older |                   |
| Hofacker . | 1,996            | Tubingen .                 | 1829    | 1,178                   | 920                  | 906          | 1,075             |
| Sadler .   | 2,068            | English Peerage            | 1830    | 1,214                   | 948                  | 865          | 1,147             |
| Goehlert . | 4,584            | (Vienna<br>(Aristocracy))  | ...     | 1,082                   | 933                  | 826          | 1,053             |
| Legoyt .   | 52,311           | Paris .                    | 1854-5  | 1,045                   | 1,021                | 975          | 1,030             |
| Boulenger  | 6,006            | Calais .                   | 1833-52 | 1,100                   | 1,079                | 1,016        | 1,079             |
| Noirot .   | 4,000            | Dijon .                    | ...     | 997                     | ...                  | 1,160        | 1,035             |
| Breslau .  | 8,084            | Zurich .                   | ...     | 1,039                   | 1,031                | 1,176        | 1,066             |
| Stieda .   | 100,590          | Alsace-Lorraine.           | 1872-3  | 1,050                   | 1,095                | 1,084        | 1,063             |
| Berner .   | 267,946          | Sweden .                   | ...     | 1,046                   | 1,062                | 1,075        | 1,060             |
| Rosenfeld  | ...              | Vienna .                   | 1895    | 1,022                   | 1,065                | 1,151        | 1,062             |
| Oehrle .   | 14,818           | Livonia .                  | 1834-81 | 1,033                   | 941                  | 1,008        | 1,019             |
| Boeckh .   | 156,761          | Berlin .                   | 1878-81 | 1,052                   | 1,046                | 1,026        | 1,047             |
| Boudin .   | ...              | (Paris) .                  | 1863    | 1,092                   | 945                  | 910          | ...               |
| Schumann   | 198,764          | Norway .                   | 1871-5  | 1,045                   | 1,059                | 1,069        | 1,051             |
| Lewis .    | 85,964           | Scotland .                 | 1855    | 1,052                   | 1,049                | 1,035        | 1,049             |

regard the Hofacker-Sadler law as proved; nay, more, notwithstanding that the Scottish figures do support the existence of such a law, the balance of evidence tends to suggest that there is no such law of universal application.

*General conclusions.*—Having referred to so many conditions as characterized by variations in masculinity, it may be advisable to sum up briefly the conclusions to which these variations lead. These conclusions are:

- (1) That certain nations and races exhibit a high, and others a low, masculinity.
- (2) That masculinity does not remain stationary in each nation or race, but may rise or fall in the course of years.
- (3) That cross-breeding raises masculinity.
- (4) That emigration lowers the masculinity of the Latin races, but does not so definitely affect the Anglo-Saxon colonist.
- (5) That masculinity does not vary directly with the birth-rate, though its movements are often coincident with the fluctuations of the latter. The recent decline in the birth-rate has been accompanied in some cases by a fall in masculinity.

- (6) That the legitimate and illegitimate masculinities bear different relations to each other in different countries. Legitimacy or illegitimacy has little influence on masculinity, for in some countries the legitimate, and in others the illegitimate, masculinity is the higher. On the whole, the legitimate masculinity is generally superior.
- (7) That still-births exhibit a very high masculinity, in accordance with the higher mortality affecting male births during parturition.
- (8) That masculinity is higher in first than in subsequent births.
- (9) That masculinity shows no constant high level in multiple births as compared with single issue.
- (10) That masculinity in urban districts is always less than it is in rural districts. Either country life increases masculinity or town life diminishes it.
- (11) That masculinity is not constant in all social ranks, and is always high in populations which live an open-air life. The agricultural and sea-faring classes have a high masculinity, but the upper classes of society frequently show a low masculinity.
- (12) That the season of the year at which conception takes place does not affect masculinity.
- (13) That masculinity is not directly dependent upon either the absolute or the relative ages of the parents, though these may be significant of physiological vigour, and hence account for the correlation which has sometimes been observed between the ages of the parents and the masculinity of their issue.

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#### ABSTRACT OF THE DISCUSSION.

MR. T. E. YOUNG thought the paper was very fittingly included within the range of the Institute's discussions. Part of its title to that position lay, he conceived, in its excellent example of the study of the modes in which statistics should be analyzed and scrutinized, with a view to ascertaining any natural uniformities they might involve; and further, if the paper had not failed (through no default in industry and ability on the part of the authors) in reaching some reasonably approximate conclusion, they might have possessed the means of deducing the probabilities of sex at birth in connection with the valuation of certain reversionary interests. Being somewhat of a purist in technical speech, he must most distinctly demur to the title, although naturally he sympathized with the authors in their desire to devise a compact and at the same time a descriptive term. The works of Whewell and Mill formed the practically final tribunal



of technical language; but so difficult, he admitted, was it to compress within a succinct and happy verbal phrase the essential nature and scope of any science or enquiry, that the dicta of those writers, though ideally complete, must, he feared, largely form a mere counsel of perfection. Still, the most fitting phrase should be sought. Masculinity simply meant the condition or state of being masculine in general, and hence possessed no distinctive application in the paper. The word *Natality*, however, signified the rate of birth, and had, he found, been sanctioned technically by Mr. Havelock Ellis in his extremely interesting work upon "*Man and Woman*", and was, moreover, the exact correlative of the current phrase of the rate of mortality. Consequently, for the specific purpose to which the paper had been addressed, he should distinctly prefer the compound term "*Sex-Natality*" as a title of much more pertinent significance. The obvious objection that sex-natality was an inclusive term was not in his judgment valid, because the word "*Masculinity*" itself equally involved what might perhaps be termed *Femininity*. With that criticism he passed to the paper itself, which was, in his judgment, a judicious and laborious investigation. The authors had surveyed the subject practically from every relevant point of view, and were fully cognizant of the limitations of the enquiry, especially he observed as to the paucity of data. It would have been helpful, he thought, mainly in connection with foreign statistics, and also in relation to certain home statistics which were not very familiar, if the authors had expressed an approximate judgment upon the reliance to be placed upon the compilation of the facts, and thus indirectly upon the validity of those facts, and upon the methods employed in their construction. As shown in the paper, and in many separate treatises, there was a plethora of statistics, but the difficulty invariably was to measure and assign the confidence with which they might be utilised as the basis of investigation. Looking to the paper as a whole, the interpretation of statistics demanded the acutest and most varied survey, and, as the authors themselves had perceived, it was of the rarest occurrence that that interpretation, especially in complex anthropological problems, lay close to the surface. Statistics simply expressed the relations by which certain events were connected with certain other events; but that relationship was simply one of figures, and in no way necessarily implied that the cause resided in the relations themselves. There was a very striking illustration of that fact in the persistent notion that marriage aided longevity, which superficially appeared to represent the meaning of the numerical ratios, until Mr. Spencer and Mr. Proctor conclusively proved that the relation exhibited in the tabulated figures was attributable to a different cause altogether. The subject of the paper had been persistently and almost continuously discussed, since human reason began and human curiosity was aroused, but no definite or sustainable result had yet been attained. Hence the paper was in no degree singular in failing to present an adequate criterion, but he wished to add, with all sincerity, that both in method and judgment it was certainly not inferior to the other able treatises on the same subject which it had been his fortune to read. On that point he noticed that

Messrs. Geddes and Thomson, in their volume upon the "Evolution of Sex", stated, in 1889, that up to the beginning of the last century no less than 500 distinct hypotheses had been propounded upon the subject, and that the accumulation of similar hypotheses had not abated.

As bearing upon the subject of the paper, he had referred to Charles Darwin's "Descent of Man," and, while Darwin was a man who always expressed himself with the humility which is native to a genuine man of science, in chapter 8 he spoke upon this question in the most hesitating and tentative terms, simply affirming that the evidence he possessed was extremely meagre, but that analogy suggested that a tendency to produce either sex would, in all probability, be inherited, like almost every other peculiarity. He noticed from the literature upon the subject that the most generally received determining factor appeared to have been that which was denominated the "comparative vigour" of constitution, but it seemed to him that, apart from the ambiguity and the necessity of strict definition of the term "vigour," it was perfectly impossible to express the notion with any numerical precision. He had himself been usually in favour—not through any strict enquiry, however, nor from patient or extended thought—of what the authors had rightly termed the Hofacker-Sadler law, or (more accurately) the uniformity which referred to the relative ages of the parents; but he thought the results shown in Tables XXVII and XXVIII sufficiently indicated that that notion was no nearer the truth than any other. The truth was that in that question, as in all questions concerning human nature, whether mental or physical, it was imperative to assume that the causes were not single but in combination, and included both hereditary capacity and direction of transmission, personal qualities, environment and other factors, each exhibiting, moreover, the complication of double functions—both manifold functions and many-valued functions. Hence, the question ultimately belonged to that obscure class of phenomena which comprised, as John Stuart Mill had said, a plurality of causes and an intermixture of effects. Each cause, or more scientifically speaking, each force, like all the forces concerned in the production of physical phenomena, produced its full effect, modifying by inter-mixture the effects of each other force, and hence a complex result was produced which apparently could not be analyzed with any serviceable approach to precision, and where that obscure complexity existed, the established rules of inductive inference were incompetent of application. For, from the nature of the case, it was impossible to pursue the usual scientific methods and segregate the phenomena, so as to contemplate them at one time under one cause and at another time under another. Actuaries must therefore be guided in their judgment exclusively by the balance of probabilities, and of the direction in which that balance inclined in the present instance he had not the remotest notion. Though he cordially welcomed the authors' able and interesting paper, and though he knew from physical history that every genuine enquiry like the present conducted them more and more closely to the source of explanation, he was afraid the quantitative and qualitative detection of the forces involved in the problem of the prior determination of sex was as distant as the advent of the philosopher's magical stone.

MR. W. P. ELDERTON remarked, with regard to the statistical value of the paper, that in some of the tables, for instance Table III (Scotland), a small probable error would be enough to upset any conclusions as to the sequence of the values. In Table VI, where fuller particulars were given, it was impossible to make any estimate of the probable deviation from the results obtained, because the number of cases that referred to first births, and to subsequent births, did not seem to be stated. Then in Table VII the series was extremely rough, and, as had been pointed out, it was impossible to come to any very definite conclusions; but if the number of cases had been given, or, still better, if some estimate of the probable error involved in the figures had been stated, he thought the scientific value of the work would have been considerably increased. He had attempted to deduce very roughly an approximate value for the probable errors in some of the figures given in Table XVI. He estimated in Table XVI that the probable error in the commercial class would be 16, and in the working classes, which was the largest class shown, about 5. If there were one million cases he thought it would be just about  $1\frac{1}{2}$ . He did not know how far these deviations would affect any conclusions drawn in the paper, but, as a large number of the conclusions seemed to be negative, this part of the work would not be affected. On this last point he sympathized with the authors, for, as he himself knew, negative conclusions were not altogether satisfactory to an author, and sometimes—although valuable—were very disappointing. He thought that in almost all statistical work it would be a good thing if authors made it a rule to give the probable errors of the functions with which they were dealing, or failing that, the number of cases which had been investigated.

MR. C. D. HIGHAM said that he was a little surprised that the authors had made no mention of the younger Charles Ansell's "Statistics of Families in the upper professional classes," because they would have found there some paragraphs to the point, with also a reference to Mr. Frederick Hendriks' paper on the Vital Statistics of Sweden, read before the Statistical Society in 1862. Though Mr. Ansell did not give a table of "masculinity" as children were born, he did furnish the figures for the first three children, and then the next three, and the rest, which perhaps roughly bore out the conclusions of the authors. He also gave another table, showing that the masculinity in large families is greater than in small, and that was curious, bearing in mind the greater masculinity of the first birth. Mr. Young had said there was a plethora of statistics on the subject, but he had rather thought that one of the defects of the paper, although it was not the fault of the authors, was that the statistics were rather antiquated and very sporadic; for instance, a good many of the figures were based on the births in Scotland for one year only, 1855. The only rule that could be arrived at seemed to be that very much depended on vigour—no doubt, as Mr. Young said, in conjunction with a variety of other causes,—but the question of vigour seemed all-important. On page 159, the authors gave, as one of the explanations of the lower masculinity of illegitimate births on the Continent, that a good many illegitimate births were concealed. He did not understand



why concealment of birth should increase masculinity, because, he presumed, the parents would not conceal the birth of one sex more than the other. The third reason there given came back again to vigour. Could the lower masculinity on the Continent be partly due to the influence of the Church, as in Ireland, in checking illegitimate births among the healthy peasant classes? That was borne out by the masculinity of illegitimate births in Scotland being higher than the legitimate, because, as the authors said, a good deal of the illegitimacy there was, sad to say, almost like regular marriage. The very strong masculinity of the Peerage, shown on page 164, brought back to the mind a suggestion which Dr. Sprague had thrown out in that room, depending on the great desire of the parents for a son. The curious point that mothers of triplets were commonly over thirty years of age was new to him and interesting though unimportant. Passing on to the general conclusions, if emigration lowered the masculinity of the Latin races, but did not definitely affect Anglo-Saxon colonists, was not that due to the fact that in Anglo-Saxon emigration the people did not mostly go away to foreign countries among foreigners, but lived among their own people, as much at home in Australia or Canada as in England? They were leading an English life amongst English people, and most of them in the open air; which could hardly be called "emigration" in questions of varying masculinity. Vigour also seemed to be indicated in the authors' eighth, tenth and eleventh conclusions, but why vigour should tend to produce the male sex rather than the other he did not think anybody knew or could know. He hoped the authors would be able to give references to the various publications to which they had referred.

MR. A. W. WATSON, in concluding the discussion, complimented the authors upon the enthusiasm which had led them to make an investigation into something over ninety thousand separate cases of registration of birth. If he might be permitted to express some little criticism, it was that the results of that stupendous work had not at all times received proportionate weight, when brought into comparison with the deductions of some enquirers whose credentials were not quite apparent to the ordinary reader, and whose fields of observations appeared, in notable instances, to be very circumscribed. It was not going too far to say that the importance of the theories put forward by some investigators would seem to vary inversely with the extent of the observations upon which they rested, and that the present authors had done good service, if in nothing else, in gathering evidence to show that the conclusions of those investigators were not entitled to be accepted as the discovery of natural laws. It had been forcibly remarked, he thought by Mr. R. P. Hardy, that the discovery of the law of mortality was the last secret that Nature would yield to man, and one's impression, after the perusal of the present paper, was that the law of natality was to be placed in the same category of humanity's remote acquisitions. If they were to agree with the authors that, under present conditions, the possession of a positive masculinity appeared to be an integral necessity of a vigorous nationality, a somewhat pessimistic view of the future of humanity would be

forced upon them by Table I, which showed that during fifty years in England and ninety years in France—periods which counted but as moments in the infinity of time—the excess of male births termed “masculinity” had lost from a quarter to a third of its whole bulk. The figures were too startling, however. He preferred to ground his faith upon the more encouraging assumption that the race would adapt itself to its environments. Accepting as established the decline in masculinity in this and other countries during recent years, might it not be traced to the migration of their populations from the agricultural districts, where the masculinity was shown to be high, to the urban localities, where it was shown to be lower? If so, they saw one result of the changing conditions through which many nations were passing, and the effects of which presented larger problems than could be included in the discussion. The general conclusion of the authors seemed for the most part to be fairly supported by the evidence tendered, though one was tempted to relegate to the fantastic such investigations as those which purported to disclose the degree of masculinity with reference to relative height above sea-level; whilst, for the formation of any definite opinion on such questions as the effects of cross-breeding or emigration, the evidence was, in his opinion, far too slight. Passing to a practical question arising, not only out of that paper but from statistical discussions elsewhere, he would venture to deprecate the desire for more elaborate registration of births which was evidenced in the authors’ reference to the “unfortunate” alteration of the Scottish schedule made after the one year’s experience of a more elaborate system. He noted that at a recent statistical congress the following moderate list of desiderata in registration was submitted—Age of husband and wife at marriage; age at birth of each child; number of births, and interval between each; survival of the offspring; and occupation of the bread-winner;—and the Scottish Register of 1855 appeared to be much in that form. Could any list of enquiries be more irrelevant to the limited purposes for which the State was justified in the expenditure of public moneys, more forbidding to the populace, and more prolific in errors? Speaking simply for himself, he thought not; and feeling, as all the members must, the imperfections of the records, censal and otherwise, which the State vouchsafed to them, he was strongly of opinion that no expression of implied authority should emanate from the Institute from which it might be inferred that they made common cause with those whose statistical demands upon the State would seem to have no superior limit.

THE PRESIDENT said it was his pleasant duty to ask the members to pass a hearty vote of thanks to the authors for the paper they had so laboriously and carefully prepared. The Institute was always glad to welcome new contributors, and it did so sincerely in the case of Mr. Lewis, who had collaborated with his brother, Dr. Lewis. There was no doubt that such a patient, careful and lengthy enquiry very often led, in the long run, to rich results, and the work would not be thrown away. He was glad to see a Fellow of the Institute collaborating with a Physician, and in other cases perhaps with statisticians, to whom statistics were of interest or



necessary, even if the actuary did not in all cases take the primary part in such work. As time went on, he thought there was a good deal more of such work which could be engaged in when opportunities occurred.

The resolution was carried with acclamation.

Dr. LEWIS, in response, said that in the discussion the paper had been taken in certain senses to relate to the determination of sex, or to deal with the sex question in some way. He wished to make it perfectly clear that, in his mind, the paper had not dealt in any way whatever with the determination of sex. That was a subject which must be left to the biologist to solve; it was not a problem which could be investigated by means of statistics. The authors had endeavoured in the paper to deal with the subject of the sex distribution of the population at the hour of birth; they had not attempted to deal with the causes of sex. If he had gone fully into the subject of the determination of sex, he thought the members would have been astonished at the number of theories and the different suggestions which had been put forward. From the point of view of the statistician, he thought the most interesting part of the paper was the connection between the decline of the birth-rate and the fall in masculinity. That was not a feature confined to this country at all; it affected other countries, and he thought it would be found to be a subject in which politicians and statesmen would have to take some interest before many years had gone by, because the birth-rate had been falling so fast.

MR. J. NORMAN LEWIS, in reply to Mr. Young's remarks in regard to the title of the paper, said that his brother and he had found it difficult to devise a title altogether applicable, and they adopted the word masculinity, although it was not quite the word they wished, because it was brief and convenient, and because they found it was a term which had commonly been used in a great many books dealing with the subject, more particularly on the Continent. Reference had been made to the unreliability of many of the statistics which were quoted from other authors. In every case they gave, where they could, the numbers of the births dealt with, which in many cases were very large; but it was not always possible, especially in figures from abroad, to form any better idea than that as to the reliability of the results published. A large number of different statistics were given in the paper, and by comparing one with the other he thought it was possible to form some idea as to the importance to be attached to each. With regard to the criticism of Table III, that table simply dealt with the whole of the population of the country; the figures were derived from the Registrar-General's reports, from which the number of legitimate and illegitimate children could be ascertained if desired. Table VI did not actually state the number of first births, and the number of subsequent births, but the authors had given the number of total births, and he thought it was generally the case that the first births were roughly about 20 per-cent, one in five, of the total births. It might perhaps have been better to give the actual figures there. With reference to Mr. Higham's remarks, the authors did not overlook the papers by Mr. Ansell and Mr. Hendriks, but had found it quite impossible to quote all the authorities on the question.

The reason why the figures for Scotland were given for one year only was that that was the only year possible. The suggestion that the difference between the legitimate and illegitimate masculinity on the Continent might have, as one cause, the concealment of births in some cases, was merely quoted as having been made by somebody else, and the authors were not responsible for it. He took the statement to be intended merely to account for incorrect figures, and not as producing any particular definite result. Certainly, the remarks with regard to emigration were, as Mr. Watson said, based on very slight evidence, and the authors did not attach any great importance to them; but, so far as they were aware, they were the only figures at all obtainable on the subject, and therefore they thought it well to quote them. If there were any other figures available which could be produced they would be very pleased to see them. It was unfortunate that the results of the investigation had proved to be of such a negative character. When the authors undertook the task, they were in hopes of being able to produce something a good deal more positive; but, in spite of the popular impression to the contrary, it was not always possible to make statistics prove anything that might be desired, and they had merely been able to set forth the figures obtainable, and let them tell their own tale. If the authors had not been able to prove anything definite in regard to masculinity, perhaps their research might, at any rate, clear the way and smooth the path for anyone who might, on a future occasion, pursue the subject further, and if they had been able to do that they would consider their labours had not been wasted.

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*On some Special Features of Widows' and Orphans' Funds.*  
By ERNEST CHARLES THOMAS, F.I.A., of the Gresham Life Assurance Society.

HAVING been associated with Mr. George King in the valuation of a Bank Widows' Fund, possessing features of more than usual interest, I have undertaken at his suggestion to write a short account of the principles and methods used in estimating the liability.

The Fund is closely connected with a Guarantee Fund, and, from the following brief abstract of the rules, it will be seen that the annuities and contributions depend upon the amount for which each member is guaranteed.

Every officer of the Bank is required to join the Guarantee Fund. Managers, and officers with salaries of £300 per annum and upwards, must enter into a guarantee for £3,000; officers with salaries of £180 and under £300 per annum, for £2,000; officers with salaries of £120 and under £180, for £1,500, and officers with salaries below £120 for £1,000.

The officers pay in respect of these guarantees a contribution at the rate of 8s. per-cent for the first 5 years, and 3s. per-cent thereafter ; and, should an officer be promoted to a class requiring a higher amount of guarantee than that which he previously gave, the contributions on the increase of guarantee are at the rate of 8s. per-cent for the first 5 years from date of increase, and 3s. per-cent thereafter. There are certain rules under which additional contributions may be demanded, but for present purposes these may be disregarded.

Three-fourths of the amount of the above-mentioned contributions are placed to the credit of the Guarantee Fund, and the remaining one-fourth to the credit of the Widows' and Orphans' Fund ; but whenever and so long as the balance at the credit of the Guarantee Fund amounts to £10,000, then the whole of the contributions of the officers, as well as the whole of the interest on the Guarantee Fund, is credited to the Widows' and Orphans' Fund. For many years past the Guarantee Fund has not fallen below £10,000, and therefore the whole of the contributions, and the whole of the interest, have been credited to the Widows' and Orphans' Fund.

In consideration of the foregoing contributions, annuities are paid to the widows and orphans of officers or members on the following scale :—

£60 per annum in respect of those who contribute for a guarantee of £3,000 ;

£50 per annum in respect of those who contribute for a guarantee of £2,000 ; and

£40 per annum in respect of those who contribute for a guarantee of less than £2,000.

The annuities are payable to the widows for life during widowhood, and are continued to the children until the youngest attains the age of 21 or marries.

It is hardly necessary to discuss the precise relations between the two Funds. It will be sufficient for present purposes if we consider that the whole of the contributions are carried direct to the Widows' Fund, and we are left with the problems of a varying annuity to widows with continuation to youngest child, and a varying contribution, both regulated, though in slightly different ways, by a guarantee increasing by sudden movements according to the salary or position of the member.

An illustration of the card used is here appended.

*Front of Card.*

| BANK WIDOWS', &c., FUND.                  |   |                |                    |
|-------------------------------------------|---|----------------|--------------------|
| Member.....                               |   | No. ....       |                    |
| Born.....                                 |   |                |                    |
| Entered<br>the Fund                       | } | .....          | Age at Entry.....  |
| Married<br>1st time                       |   | .....          | Age at<br>Marriage |
| Qy. on Active List.....                   |   |                |                    |
| Date and<br>Cause<br>of Exit<br>from Fund | } | [      ] ..... | Age at<br>Exit     |
| 1st Wife.....                             |   |                | Born.....          |
| Exit<br>from Fund                         | } | [      ] ..... | Age at Exit.....   |
| 2nd Wife.....                             |   |                | Born.....          |
| Married                                   | } | .....          | Age at<br>Marriage |
| Exit<br>from Fund                         |   | }              | [      ] .....     |

It was found impossible to obtain any statistics upon which to form a mortality or marriage table from past experience, but the cards were filled up for existing members, showing their present status and their past history as regards marriages and guarantees.

It should be mentioned that after ten years' service, any member retiring or withdrawing voluntarily from the service may continue his membership of the Fund.

It is obvious that the complications inherent in the Fund rendered the use of existing tables (such as Huie's) out of the question, and from the fact that the rate of withdrawal is probably so small as to be negligible, Mr. Manly's tables were in like manner inapplicable.

*Back of Card.*

## GUARANTEES.

| DATE  | AMOUNT  | AGE   |
|-------|---------|-------|
| ..... | £ ..... | ..... |
| ..... | .....   | ..... |
| ..... | .....   | ..... |
| ..... | .....   | ..... |

## CHILDREN.

| SEX   | INITIALS | BORN  | DIED  |
|-------|----------|-------|-------|
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |
| ..... | .....    | ..... | ..... |

Interest is allowed at 5 per-cent per annum upon the Guarantee Fund, and upon the Widows' Fund up to an amount of £10,000. The surplus of the latter fund is, however, invested in securities returning an average rate of three per-cent. The latter rate was therefore used in the valuation; the additional two per-cent upon the fixed sum of £10,000 being valued as a separate asset.

The general scheme followed was the application of the collective method to the statistics drawn from living members of the Fund at the moment of valuation, already brought to public notice by Messrs. King, Manly, Allin and other writers.

In default of any mortality statistics of the Fund itself, the



mortality of the members was taken as that of the  $OM^{(5)}$  Table, and there seems no reason to suppose that the experience of such a Fund, after the original benefit of selection on entry into service has run off, would be materially different from that of an average assurance company.

It should be remarked that the case now being considered is essentially different from that of a staff pension Fund. In the latter case, it is the almost invariable experience that the mortality among continuing active members is much lighter than that of any standard table, owing to the fact that there is a constant process of selection going on by the retirement of the invalid lives upon pension. Moreover, as this light mortality increases the liability in respect of continuing members, it is eminently desirable that the table used should be based upon the Fund's own experience. These considerations do not affect the present case, there being no reason to anticipate an exceptionally light mortality, and if such should be experienced in the future, the value of the benefits, so far from being increased, would be diminished thereby.

The Widows' Annuities were valued by the Government Annuitants', 1884 (Females) Ultimate Table, and the re-marriage element was ignored.

#### METHODS.

*Guarantees.*—We might have obtained an average age at which these amounts increased from 1,000 to 1,500, 2,000, &c., and assumed that every member in each group of ages was subject to the same guarantee. It was considered more scientific, however, to obtain a graduated scale, increasing slowly from age to age, the underlying assumption being that a certain number of members increased at each age sufficient to produce an average for all members of the amount fixed by the scale.

Accordingly the guarantees were scheduled according to *nearest ages* of members at the date of the valuation, and an average amount of guarantee therefrom obtained for each age, the resulting scale being graduated by a summation formula.

From this graduated scale of guarantees we are enabled to assess the value of the future contributions. These consist of a uniform proportion of 3s. per-cent of the guarantees, payable throughout membership, plus, for each age attained by the member, the present value of a term-annuity for 5 years of 5s. per-cent upon the increment for that year. When the value of this annuity is added to the 3s. per-cent of guarantee at each age,

we have the factor to be multiplied at each age into the column  $D_x$ .

The work arranges itself as follows :

| Age<br>( $x$ ) | Amount<br>of<br>Guarantee | Col. (2)<br>$\times \cdot 0015$ | $\Delta(2)$ | $\Delta(2)$<br>$\times \cdot 0025$ | Col. (5)<br>$\times (1 + a_{x+1})$ | (6) + (3) | $\bar{D}_x^c$<br>$= (7) \times$<br>$\bar{D}_x(O^M(5))$ | $\bar{N}_x^c$<br>$= \Sigma \bar{D}_x^c$ | $\frac{\bar{N}_x^c}{D_x(O^M(5))}$ | (10) $\times$<br>No. of<br>Members |
|----------------|---------------------------|---------------------------------|-------------|------------------------------------|------------------------------------|-----------|--------------------------------------------------------|-----------------------------------------|-----------------------------------|------------------------------------|
| (1)            | (2)                       | (3)                             | (4)         | (5)                                | (6)                                | (7)       | (8)                                                    | (9)                                     | (10)                              | (11)                               |

The symbol  $\bar{D}_x$  in column (8) has the same significance as is given to it in Mr. King's paper on Staff Pension Funds, namely:  $\frac{D_x + D_{x+1}}{2}$ , the contributions being spread evenly over the year, and the payment ceasing absolutely at the death of the member. The whole contribution being considered as payable practically in the middle of the year, and that fact being allowed for in the function  $\bar{D}_x$ , the factor for the increment is therefore an annuity-due (*vide* column 6).

#### BENEFITS.

*Average amount of Annuity.*—This varies with the guarantee, but not directly as in the case of the contribution, and accordingly a new scale is required. This is obtained by scheduling, in respect of every member at each present age, the annuity allowed by the rules for the guarantee then in force, and taking the average. The table thus deduced of course required graduating as before.

*Value of Annuity of 1 per annum to Widow as at death of Husband.*—For each age of the husband, a corresponding figure representing the nearest age of the wife was set down for all husbands on the Fund. The appropriate annuity-value by the Government Annuitants' (F) 1884, 3 per-cent Ultimate Table for the said age of wife was then scheduled and an average annuity-value obtained for each age of the husband.

The following are the graduated results compared with Mr. Manly's and Mr. Allin's :

*Value of Continuous Annuity of 1 per annum to Wife at the moment of Death of Husband.*

| Age of Husband | MANLY 3% | ALLIN |      | BANK FUND | Age of Husband | MANLY 3% | ALLIN |      | BANK FUND |
|----------------|----------|-------|------|-----------|----------------|----------|-------|------|-----------|
|                |          | 1881  | 1902 |           |                |          | 1881  | 1902 |           |
| 25             | 21.3     | 21.3  | ...  | 22.1      | 64             | 14.3     | 12.5  | 12.8 | 13.6      |
| 26             | 21.2     | 21.2  | ...  | 21.9      | 65             | 14.0     | ...   | 12.5 | 13.2      |
| 27             | 21.1     | 21.1  | ...  | 21.7      | 66             | 13.7     | ...   | 12.2 | 12.8      |
| 28             | 21.0     | 21.0  | ...  | 21.5      | 67             | 13.4     | ...   | 12.0 | 12.4      |
| 29             | 20.8     | 20.8  | ...  | 21.3      | 68             | 13.0     | ...   | 11.7 | 12.1      |
| 30             | 20.7     | 20.7  | 19.6 | 21.1      | 69             | 12.7     | ...   | 11.5 | 11.8      |
| 31             | 20.6     | 20.5  | 19.5 | 20.9      | 70             | 12.3     | ...   | 11.2 | 11.5      |
| 32             | 20.5     | 20.2  | 19.4 | 20.7      | 71             | 11.9     | ...   | 10.9 | 11.2      |
| 33             | 20.4     | 20.0  | 19.2 | 20.5      | 72             | 11.6     | ...   | 10.5 | 10.9      |
| 34             | 20.3     | 19.9  | 19.1 | 20.3      | 73             | 11.2     | ...   | 10.1 | 10.6      |
| 35             | 20.1     | 19.7  | 19.0 | 20.1      | 74             | 10.8     | ...   | 9.8  | 10.3      |
| 36             | 20.0     | 19.5  | 18.9 | 19.9      | 75             | 10.4     | ...   | 9.4  | 9.9       |
| 37             | 19.9     | 19.3  | 18.8 | 19.7      | 76             | 10.0     | ...   | ...  | 9.5       |
| 38             | 19.8     | 19.0  | 18.6 | 19.5      | 77             | 9.5      | ...   | ...  | 9.2       |
| 39             | 19.7     | 18.8  | 18.5 | 19.3      | 78             | 9.1      | ...   | ...  | 8.9       |
| 40             | 19.6     | 18.6  | 18.3 | 19.1      | 79             | 8.8      | ...   | ...  | 8.5       |
| 41             | 19.5     | 18.3  | 18.1 | 18.9      | 80             | 8.4      | ...   | ...  | 8.2       |
| 42             | 19.3     | 18.1  | 17.8 | 18.7      | 81             | 8.0      | ...   | ...  | 7.8       |
| 43             | 19.2     | 17.9  | 17.5 | 18.5      | 82             | 7.7      | ...   | ...  | 7.4       |
| 44             | 19.0     | 17.7  | 17.2 | 18.3      | 83             | 7.4      | ...   | ...  | 7.0       |
| 45             | 18.9     | 17.4  | 17.0 | 18.1      | 84             | 7.1      | ...   | ...  | 6.7       |
| 46             | 18.7     | 17.1  | 16.8 | 17.8      | 85             | 6.7      | ...   | ...  | 6.4       |
| 47             | 18.5     | 16.9  | 16.6 | 17.5      | 86             | 6.4      | ...   | ...  | 6.1       |
| 48             | 18.3     | 16.7  | 16.3 | 17.2      | 87             | 6.1      | ...   | ...  | 5.9       |
| 49             | 18.2     | 16.4  | 16.1 | 17.0      | 88             | 5.8      | ...   | ...  | 5.7       |
| 50             | 18.0     | 16.1  | 15.9 | 16.8      | 89             | 5.5      | ...   | ...  | 5.4       |
| 51             | 17.7     | 15.9  | 15.6 | 16.6      | 90             | 5.2      | ...   | ...  | 5.1       |
| 52             | 17.5     | 15.6  | 15.3 | 16.4      | 91             | 4.8      | ...   | ...  | 4.8       |
| 53             | 17.3     | 15.4  | 15.1 | 16.2      | 92             | 4.5      | ...   | ...  | 4.6       |
| 54             | 17.1     | 15.1  | 14.9 | 16.0      | 93             | 4.2      | ...   | ...  | 4.4       |
| 55             | 16.8     | 15.0  | 14.6 | 15.8      | 94             | 3.8      | ...   | ...  | 4.1       |
| 56             | 16.6     | 14.8  | 14.4 | 15.6      | 95             | 3.5      | ...   | ...  | 3.9       |
| 57             | 16.3     | 14.5  | 14.2 | 15.4      | 96             | 3.1      | ...   | ...  | 3.7       |
| 58             | 16.0     | 14.3  | 14.0 | 15.2      | 97             | 2.8      | ...   | ...  | 3.5       |
| 59             | 15.8     | 14.0  | 13.8 | 15.0      | 98             | ...      | ...   | ...  | 3.3       |
| 60             | 15.5     | 13.8  | 13.6 | 14.8      | 99             | ...      | ...   | ...  | 3.1       |
| 61             | 15.2     | 13.5  | 13.4 | 14.6      | 100            | ...      | ...   | ...  | 2.9       |
| 62             | 14.9     | 13.1  | 13.2 | 14.3      | 101            | ...      | ...   | ...  | 2.7       |
| 63             | 14.6     | 12.9  | 13.0 | 14.0      |                |          |       |      |           |

By the methods followed it was necessary to have annuity-values for all ages of husbands up to the limit of the  $O^{M,5)}$  Table, age 101. In the absence of any statistics for ages over 70, the table of annuity-values was arbitrarily continued by gradually increasing the seniority of husbands over wives by half-years from  $8\frac{1}{2}$  years at age 70 to a maximum at the highest age of 12 years. At this end of the table these arbitrary figures agree very closely with Mr. Manly's. At the important ages in middle life it will be noticed that the values now published seem to strike the happy mean between those of Mr. Allin and Mr. Manly respectively.

*Ratio of Husbands, Bachelors and Widowers respectively to total.*—The number in each class was set down for each age as at the valuation date, and the ratios obtained and graduated graphically. Curves were drawn for husbands and bachelors, and by the aid of these, together with a somewhat arbitrary spreading of widowers (who were very few in number), a complete table of ratios for all ages was obtained. As there were practically no figures obtainable after age 80, and as all the members under observation over 64 were husbands, it was assumed that in the future all over 80 would be husbands. This assumption is on the safe side, and the over-estimate of the liability cannot be very great, because the annuity-values are correspondingly small.

The probability is that this assumption is strictly in accordance with the facts, because the *option* of withdrawal only comes with retirement from service, and the widowers of long standing who had no intention of re-marrying, and the confirmed bachelors, would have no inducement to continue after superannuation. A scientific treatment of this question would appear to be possible only by means of a secession rate solely affecting bachelors and chronic widowers.

*Value of continuation of a Widow's Annuity to Children as at Death of Husband.*—The age of the youngest child of each father was set down for each age of the father. Corresponding term-certain annuities  $a_{\overline{21-z-\frac{1}{2}}}$  scheduled—also term-annuities for wives (obtained from average age of wife)  $a_{y.\overline{21-z-\frac{1}{2}}}$  where  $z$  represents the age of the child.

The differences between the annuities-certain and the widows' term annuities were divided by the *total number of husbands at each age*. The resulting figures were graduated by the graphic method. The final values were compared with those given by Mr. Manly in his paper *J.I.A.*, xxxviii, p. 101. His figures commence with a maximum at age 25, and steadily decrease to the end of the table. Our figures increase from age 25 up to a maximum at age 47 (at that age being approximately equal to Mr. Manly's values), thence decreasing to the end of the table. This feature of the curve was so marked that we felt bound to retain it. It does not appear to be necessarily an unreasonable result, although perhaps hardly an expected one, the probable explanation being that the age of the youngest child per father would not vary much between those limits, the principal factor being, therefore, the age of the wife, by the increase in which the liability now considered would also be increased. On the whole, however, our figures are considerably

lower than Mr. Manly's. There are two apparent reasons for this :

- 1st. That Mr. Manly's figures are based on the statistics relating to the probably more prolific families of New Zealand.
- 2nd. That, in the absence of analyzed statistics, he credited each one of  $n$  fathers with one of the  $n$  youngest children, thereby over-estimating the *liability per husband*. In other words, he included some children who were not the youngest of their family, in place of older ones who were (some of the latter being over the age). Consequently the number of families included is too high and the average age of the youngest child too low.

In our case the ratio of the total number of youngest children under the age to the number of husbands was 67·6 per-cent, whereas in Mr. Manly's tables the ratio was 85·1 per cent.

A table is appended showing the values of this function, as deduced from the Bank's Experience, and Mr. Manly's figures are added for purposes of comparison.

*Value of Annuity to youngest Child of Widower at the moment of the Death of the latter.*—The number of widowers was too small to give more than a rough idea of this function. The values of term-certain annuities were tabulated as before and averaged. The ages of actual widowers ranged from 47–61, and the average annuity-value for these ages, deduced from the limited facts available, was 6·084. In Mr. Manly's tables the average of the same group was 9·83. For the reasons already stated, and also because Mr. Manly did not distinguish between the two classes, husbands and widowers, it is reasonable to think that our figures should be considerably lighter than his. In the absence of full statistics, we adopted Mr. Manly's figures reduced by one-third.

Having now deduced the elementary functions, it is a simple matter to combine them to obtain valuation multipliers, but before doing so it will be convenient, as some of these particular functions have not been included in any published system of notation, to give each one a definite and appropriate symbol.

$\bar{D}_x^c$  and  $\bar{N}_x^c$  have already been used to symbolize the necessary commutation columns for obtaining the value of the variable contribution.



*Value, on the Death of a Husband, of a continuous Reversionary Annuity of 1, to commence on the death of the Widow, and continue until the youngest surviving Child reaches the age of 21.*

| Age of<br>Husband<br>at death<br>$x$ | Bank Fund | Mr. Manly's<br>$3\frac{1}{2}\%$ Table | Age of<br>Husband<br>at death<br>$x$ | Bank Fund | Mr. Manly's<br>$3\frac{1}{2}\%$ Table |
|--------------------------------------|-----------|---------------------------------------|--------------------------------------|-----------|---------------------------------------|
| 25                                   | ·460      | 1·235                                 | 60                                   | ·290      | ·699                                  |
| 26                                   | ·474      | 1·234                                 | 61                                   | ·272      | ·615                                  |
| 27                                   | ·488      | 1·232                                 | 62                                   | ·256      | ·531                                  |
| 28                                   | ·503      | 1·230                                 | 63                                   | ·246      | ·447                                  |
| 29                                   | ·518      | 1·228                                 | 64                                   | ·236      | ·370                                  |
| 30                                   | ·533      | 1·225                                 | 65                                   | ·226      | ·296                                  |
| 31                                   | ·548      | 1·222                                 | 66                                   | ·220      | ·241                                  |
| 32                                   | ·563      | 1·218                                 | 67                                   | ·214      | ·200                                  |
| 33                                   | ·578      | 1·213                                 | 68                                   | ·204      | ·168                                  |
| 34                                   | ·594      | 1·207                                 | 69                                   | ·196      | ·145                                  |
| 35                                   | ·612      | 1·201                                 | 70                                   | ·190      | ·128                                  |
| 36                                   | ·631      | 1·194                                 | 71                                   | ·185      | ·115                                  |
| 37                                   | ·651      | 1·187                                 | 72                                   | ·180      | ·105                                  |
| 38                                   | ·673      | 1·179                                 | 73                                   | ·174      | ·096                                  |
| 39                                   | ·697      | 1·171                                 | 74                                   | ·168      | ·086                                  |
| 40                                   | ·724      | 1·162                                 | 75                                   | ·162      | ·078                                  |
| 41                                   | ·758      | 1·153                                 | 76                                   | ·156      | ·071                                  |
| 42                                   | ·794      | 1·144                                 | 77                                   | ·150      | ·064                                  |
| 43                                   | ·838      | 1·134                                 | 78                                   | ·144      | ·058                                  |
| 44                                   | ·882      | 1·123                                 | 79                                   | ·138      | ·052                                  |
| 45                                   | ·932      | 1·112                                 | 80                                   | ·132      | ·047                                  |
| 46                                   | ·982      | 1·101                                 | 81                                   | ·126      | ·042                                  |
| 47                                   | 1·012     | 1·090                                 | 82                                   | ·118      | ·036                                  |
| 48                                   | 1·012     | 1·078                                 | 83                                   | ·104      | ·031                                  |
| 49                                   | ·952      | 1·066                                 | 84                                   | ·090      | ·026                                  |
| 50                                   | ·884      | 1·051                                 | 85                                   | ·070      | ·021                                  |
| 51                                   | ·796      | 1·035                                 | 86                                   | ·040      | ·017                                  |
| 52                                   | ·700      | 1·016                                 | 87                                   | ·010      | ·013                                  |
| 53                                   | ·610      | ·993                                  | 88                                   | ·000      | ·009                                  |
| 54                                   | ·500      | ·966                                  | 89                                   | ·000      | ·006                                  |
| 55                                   | ·434      | ·934                                  | 90                                   | ·000      | ·004                                  |
| 56                                   | ·386      | ·898                                  | 91                                   | ·000      | ·002                                  |
| 57                                   | ·354      | ·857                                  | 92                                   | ...       | ...                                   |
| 58                                   | ·332      | ·811                                  | 93                                   | ...       | ...                                   |
| 59                                   | ·310      | ·759                                  |                                      |           |                                       |

Coming now to the benefits, let

${}^h k_x, {}^w k_x, {}^b k_x$  represent respectively the ratios of the numbers living in the three classes (husbands, widowers and bachelors), to the total at each age.

$\bar{a}_y$  the value, at the moment of death of a husband aged ( $x$ ), of a continuous annuity of 1 per annum to the widow.

\*  $E_{21}$  the addition to the above annuity-value required to continue the payment until the youngest child attains age 21.

${}^w a_{z(21)}$  the value, at the moment of death of a widower, of an annuity of 1 per annum payable until the youngest child attains age 21.

P the amount of pension or annuity secured to the widow or children of a member who dies at age ( $x$ ).

Let

$$\bar{a}_y^E = \bar{a}_y + E_{21}$$

$${}^h P \bar{C}_x^a = \bar{C}_x \times {}^h k_x \times \bar{a}_y^E \times P$$

$${}^w P \bar{C}_x^a = \bar{C}_x \times {}^w k_x \times {}^w a_{z(21)} \times P$$

$${}^{wh} P \bar{C}_x^a = {}^h P \bar{C}_x^a + {}^w P \bar{C}_x^a$$

$${}^{wh} P \bar{M}_x^a = \sum {}^h P \bar{C}_x^a$$

$B_x$  = total number of members of all classes aged ( $x$ )

Then  $B_x \times \frac{{}^{wh} P \bar{M}_x^a}{D_x}$  = total value of all benefits in respect of members aged ( $x$ ).

The following extract from the valuation schedule containing the figures for selected ages shows all the working columns that are necessary to obtain the gross liability in respect of the whole of the benefits. Column (17) multiplied by the number of members at each age gives the required results.

There were certain fines on marriage, the value of which was quite insignificant, and these were valued by Mr. Hewat's tables.

The methods adopted may be open to some question and in the discussion on Mr. Allin's paper I indicated what I thought were some possible objections. The point at issue is really whether the mortality experience of bachelors, husbands and widowers respectively is substantially the same, and, in regard to husbands, whether the size of the family has any bearing on the question. Mr. Allin's suggestion that the material possessed by the various life offices should be drawn upon, is a practical

\* This is Mr. Manly's symbol for the same function.

## Bank Widows' Fund.

Commutation Columns for obtaining the Value of the Benefits.

| $x$ | $OM^{(3)} \frac{3}{\log} \bar{C}_x$ | $\log h k_x$ | $\log d^E_y$ | $\log P$ | $\log h^a \bar{C}_x$<br>(2) + (3)<br>+ (4) + (5) | $h^a \bar{C}_x$ | $\log h^a k_x$ | $\log a_{z(21)}$ | $\log w^b \bar{C}_x$<br>(2) + (5)<br>+ (8) + (9) | $w^b \bar{C}_x$<br>(7) + (11) | $w^b P^a \bar{M}_x$ | $\log w^b M_x^a$ | $OM^{(3)} \frac{3}{\log} D_x$ | $\log w^b M_x^a$<br>$D_x$<br>(14) + (15) | $w^b P^a \bar{M}_x$<br>$D_x$ |      |
|-----|-------------------------------------|--------------|--------------|----------|--------------------------------------------------|-----------------|----------------|------------------|--------------------------------------------------|-------------------------------|---------------------|------------------|-------------------------------|------------------------------------------|------------------------------|------|
| (1) | (2)                                 | (3)          | (4)          | (5)      | (6)                                              | (7)             | (8)            | (9)              | (10)                                             | (11)                          | (12)                | (13)             | (14)                          | (15)                                     | (16)                         | (17) |
| 20  | 2.55440                             | ...          | ...          | ...      | ...                                              | ...             | ...            | ...              | ...                                              | ...                           | ...                 | ...              | 5.25341                       | 2.19598                                  | 157.0                        |      |
| 30  | 4.5512                              | 1.27184      | 1.33512      | 1.60423  | 4.66661                                          | 46,410          | ...            | ...              | ...                                              | ...                           | 46,410              | 8,761,397        | 6.94257                       | 3.4943                                   | 223.6                        |      |
| 40  | 4.0746                              | .67943       | .29719       | .65031   | 5.03439                                          | 108,240         | 2.41497        | 0.90200          | 3.37474                                          | 2370.0                        | 7,887,264           | 8,662,969        | .93767                        | .47344                                   | 297.5                        |      |
| 50  | 4.2544                              | .84261       | .24758       | .71265   | .22828                                           | 169,150         | .73239         | .85854           | .72902                                           | 5358.2                        | 6,502,043           | 8,1305           | .75697                        | .57002                                   | 371.6                        |      |
| 60  | .48318                              | .93247       | .17869       | .76418   | .35852                                           | 228,310         | .64345         | .72916           | .61997                                           | 4168.4                        | 4,482,882           | .65156           | .97599                        | .62755                                   | 424.2                        |      |
| 70  | .49758                              | .97727       | .06781       | .77815   | .32081                                           | 299,320         | .34242         | .12057           | 2.73872                                          | 547.9                         | 209,868             | .34001           | .428969                       | .62970                                   | 426.3                        |      |
| 80  | .29925                              | .99826       | .0.92075     | .77815   | 4.99641                                          | 99,177          | 3.30103        | 2.84510          | 0.22353                                          | 1.7                           | 99,179              | 528,371          | .83578                        | .55872                                   | 362.0                        |      |
| 90  | 1.51947                             | 0.00000      | .70757       | .77815   | .00519                                           | 10,120          | ...            | ...              | ...                                              | ...                           | 10,120              | 29,031           | .3.95232                      | .41518                                   | 260.1                        |      |
| 100 | 1.31192                             | 0.00000      | .46240       | .77815   | 1.55247                                          | 36              | ...            | ...              | ...                                              | ...                           | 36                  | 52               | 0.43862                       | .15303                                   | 142.2                        |      |

one, but whether sufficient data would be forthcoming to form a number of classified mortality tables is open to doubt.

Against these possible objections, which may deserve further consideration, we can set the following very definite advantages:—

1. The method is simple, effective, and adapted to meet almost all complications.

2. The experience obtained is more up to date, which, as Mr. Allin pointed out, is a most important point, particularly in reference to the number and ages of children per marriage.

3. If the method is consistently used for a number of consecutive valuations, the basis of the valuation automatically adjusts itself to the changing circumstances. In effect, therefore, the assumptions made are for a definite limited period and not for the whole duration of membership.

*On the Calculation of the Contributions to be made to an Annuity Fund for Widows and Children, by the members of a Society all of whom, whether married or single, are obliged to contribute. By OSCAR SCHJOLL, formerly Manager (Kontorchef) of the "Idun" Life Insurance Company, Christiania; Manager of the Royal Office for Workmen's Assurances (Rigsforsikringsanstalten), Christiania.*

[The following is an abridged translation of an essay written in Norwegian by Mr. Oscar Schjoll, and published at Christiania, in 1892.\* The Author's translation was revised, some time since, by Dr. T. B. Sprague, with a view to insertion in the *Journal*, but publication had to be deferred, owing to the exceptional pressure on our space. The abridged translation (somewhat further condensed) with illustrative tables and diagrams, is now published, as the methods adopted by the Author, some years ago, in dealing with Widows' and Orphans' Pension Funds, will, it is thought, be of general interest, as illustrating an early application of the "collective method" which has been under recent discussion in the *Journal*. The Author's notation has been somewhat varied, to avoid confusion with recognized symbols employed by English writers.—ED. J.I.A.]

THESE notes were prepared at the request of the General Director of the Norwegian Railways (Mr. L. Segelcke), in consequence of a proposed reconstruction of the Benevolent

\* *Om en beregningsmetode for enke-og barnepensioner i en forening, hvori alle medlemmer, gifte såvelsom ugifte, er indskudspligtige: Af Oscar Schjoll. (Kristiania: Det Steenske bogtrykkeri, 1892.)*

Funds of the railways. Each traffic district has, besides the Pension Fund for its functionaries, a Benevolent Fund; and these latter funds have gradually accumulated considerable sums, out of which they grant allowances to widows of the functionaries; also, in exceptional cases, to retired functionaries in needy circumstances; but always in the form of gratuities given from year to year at the discretion of the Directors. As, however, there exists a strong feeling in favour of the conversion of the Benevolent Funds into funds securing a fixed annual provision for the widow, and for the children under a certain age (for instance, 18), I was requested to prepare a report on the subject.

All the Norwegian railways belong to the State, with a single exception, "the Norwegian Main Railway"; and, as the Government is a large shareholder in this railway, and has considerable influence in its management, it might easily be arranged that there should be a single Widows' Fund for all the railways, which would of course be a great advantage.

In calculating the contributions to be made by the members of a fund which is to provide benefits for their widows and their children under a certain age, it is obvious that we must take into account, not only the rate of mortality, but also the probabilities of (1) being married, (2) having children under the stipulated age. These probabilities I will denote by  $m$  and  $c$  respectively. Furthermore, we must know the average number of children chargeable on the fund for each father of a given age. Mr. G. Behm has calculated these probabilities by means of statistics furnished by the "Bergische Märkische Railway",\* and his figures are reproduced for purposes of comparison. As the limiting age for children in his investigation was 15, instead of 18 as in the Norwegian railways, the comparison is to a certain extent invalidated.

It was important for the purpose in view to obtain statistics from which to calculate the above-mentioned probabilities (or, at all events, to determine whether the German values employed by Behm might be safely adopted in Norway); and I have been furnished with cards containing the necessary particulars as to all the functionaries of the Norwegian railways and their families, as at 30 June 1891. From these materials I have obtained the

\* "*Denkschrift betreffend die Gefahrenklassen, &c.*"; printed as an Appendix to "*Entwurf eines Gesetzes betreffend die Unfallversicherung der Arbeiter*" (Berlin, 1882).



figures which are given in Table III, and are represented graphically in the Diagrams I and II.

The ungraduated probability of being married at any age is obtained by dividing the number of married men by the total number of functionaries at that age. For example, the proportion of married men at age 30 is  $47 \div 59 = .7966$ .

As trustworthy results cannot be derived from the statistics for the higher ages, I have taken the probabilities at those ages from the Norwegian census returns for 1875. These figures are indicated in Diagram I, for every fifth year from 20–95, by the dotted squares. It will be seen that the probabilities, according to the Norwegian population statistics, are less than those obtained from the German railway statistics, up to about the age of 62; but the contrary is the case at higher ages. For the purpose of comparison, Behm's figures are printed in column (3) of Table III, and graphically represented in Diagram I by the dotted curve. As far as comparison is possible, we see that the former probabilities are also less than those obtained from the statistics as to the Norwegian railway functionaries. This is what might be expected, as the proportion of married men is likely to be greater among a group of men in fixed employment than among the whole population. At higher ages the figures appear to approach each other, so that it seems quite legitimate to supplement our statistics as above mentioned.

Comparing Behm's figures with my graduated figures, it will be seen that the differences between the two are, upon the whole, not great; and although the Norwegian statistics are more limited in extent than the others, I think that, as our object is to form a Widows' Fund for the very class of men who furnished these statistics, they are greatly preferable to the others.

In column (4) of Table III, I give the graduated probabilities of having children under age 18, and in column (5) the corresponding figures deduced by Mr. Behm, the limiting age of the children being in this case 15. Allowing for this difference in the limiting age, Mr. Behm's figures are a confirmation of the Norwegian experience, the figures being practically identical at the younger ages (up to about age 35), at which ages it is extremely improbable that there should be any children over 15; the German figures thereafter being consistently lower than the Norwegian. We conclude from what has been said that the German statistics are unsuitable, if the intended fund is to have 18 as the limiting age for children who are to receive benefits.

The first process is to tabulate the ages of the wives in groups according to the age of the husband, and thence to deduce a mean age ( $u_x$ ) of the wives of husbands aged  $x$ , calculated in such a manner that the annuity for this mean age is equal to the average of the annuities on the T wives of various ages whose husbands are aged  $x$ . For this purpose I have used the 4 per-cent annuities based on the experience of the Norwegian General Widows' Fund, 1814-1885. The following Table I shows these annuity-values.

TABLE I.

*Present value of an Annuity of 1. Norwegian Widows' Fund Experience. Females 4 per-cent.*

| $y$ | $a_y$  | $y$ | $a_y$  | $y$ | $a_y$ |
|-----|--------|-----|--------|-----|-------|
| 10  | 19.606 | 40  | 16.380 | 70  | 7.466 |
| 1   | 19.460 | 1   | 16.190 | 1   | 7.124 |
| 2   | 19.309 | 2   | 15.983 | 2   | 6.783 |
| 3   | 19.154 | 3   | 15.760 | 3   | 6.438 |
| 4   | 19.004 | 4   | 15.519 | 4   | 6.090 |
| 5   | 18.872 | 5   | 15.269 | 5   | 5.752 |
| 6   | 18.755 | 6   | 15.009 | 6   | 5.425 |
| 7   | 18.653 | 7   | 14.740 | 7   | 5.113 |
| 8   | 18.565 | 8   | 14.461 | 8   | 4.818 |
| 9   | 18.489 | 9   | 14.175 | 9   | 4.523 |
| 20  | 18.423 | 50  | 13.882 | 80  | 4.285 |
| 1   | 18.361 | 1   | 13.583 | 1   | 4.035 |
| 2   | 18.302 | 2   | 13.289 | 2   | 3.785 |
| 3   | 18.244 | 3   | 12.998 | 3   | 3.540 |
| 4   | 18.183 | 4   | 12.709 | 4   | 3.301 |
| 5   | 18.118 | 5   | 12.419 | 5   | 3.067 |
| 6   | 18.048 | 6   | 12.127 | 6   | 2.843 |
| 7   | 17.973 | 7   | 11.831 | 7   | 2.633 |
| 8   | 17.890 | 8   | 11.530 | 8   | 2.440 |
| 9   | 17.801 | 9   | 11.222 | 9   | 2.262 |
| 30  | 17.706 | 60  | 10.907 | 90  | 2.095 |
| 1   | 17.604 | 1   | 10.584 | 1   | 1.937 |
| 2   | 17.496 | 2   | 10.253 | 2   | 1.790 |
| 3   | 17.382 | 3   | 9.913  | 3   | 1.653 |
| 4   | 17.262 | 4   | 9.566  | 4   | 1.520 |
| 5   | 17.135 | 5   | 9.215  | 5   | 1.396 |
| 6   | 17.002 | 6   | 8.860  | 6   | 1.273 |
| 7   | 16.861 | 7   | 8.506  | 7   | 1.155 |
| 8   | 16.712 | 8   | 8.155  | 8   | 1.042 |
| 9   | 16.553 | 9   | 7.810  | 9   | 0.914 |

As the above-mentioned experience relates to married women, it seems suitable for our present purpose; and, as it can hardly be supposed that the wives of the railway functionaries have greater vitality than those of the Government officials, I think that in making this choice we shall be on the safe side.

In order to ascertain the above-mentioned average annuity, we have

$$a_{n,x} = \frac{w_1 a_{y_1} + w_2 a_{y_2} + \dots + w_n a_{y_n}}{T} \quad (1)$$

where  $w_1, w_2, \dots, w_n$  are the numbers of the  $T$  wives of the ages  $y_1, y_2, \dots, y_n$ , so that  $w_1 + w_2 + \dots + w_n = T$ .

*Example:* For age  $x=24$ , we have

$$a_{n,24} = \frac{a_{18} + a_{22} + a_{23} + 3a_{25} + a_{26} + a_{27} + a_{28} + a_{37}}{10} = 18.024;$$

whence, by Table I,  $u_{24}=26.3$ .

In columns 6 and 7 of Table III the averages ages of wives, deduced as above, are compared with those given by Mr. Behm. As the latter's figures are based on observations of 24,480 married couples, *i.e.*, 12 times as many as in the Norwegian experience, it might be thought that they were a more reliable guide. It is only at the lowest and highest ages, however, that the Norwegian experience is deficient in numbers, and at these ages the German statistics are equally scanty. I have therefore seen no reason to substitute the German figures in any part of the table. At age 20 it will be noticed that Mr. Behm's graduation has led to the absurd result that the probability of having children is greater than the probability of being married. At the higher ages Mr. Behm's figures appear to have been obtained by a wholly arbitrary increase in the difference of age. I have been unable to obtain any reliable statistics regarding the experience in this respect of railway functionaries at advanced ages, and although general considerations point to a difference constantly increasing with the age of the husband, I think Mr. Behm makes this increase too rapid, and I have therefore been content with a slower rate. Fortunately the figures at these ages are of comparatively little importance.

In addition to the foregoing tables, it is necessary, for the purpose we have in view, to have a mortality table suitable for railway functionaries. In the absence of special mortality statistics of Norwegian Railway functionaries, I have thought it

better not to make use of any Norwegian Mortality Table, but to adopt a table based on the extensive statistics collected during a long series of years by the German Railway Officials Union. (Dr. Herman Zimmermann: Ueber Dienstunfähigkeits und Sterbensverhältnisse. Berlin, 1866; pages 108 and 109).

Lastly, we must have a mortality table from which to calculate the values of annuities on the lives of children. For this purpose I have selected the table calculated by the Norwegian Central Statistical Office, which shows the survivors at each age, out of 100,000 children born alive; of whom 51,360 are boys and 48,640 girls.

On the present occasion, I require to make use only of the numbers living at ages 0-18; and these are given in column (2) of the following Table II:—

TABLE II.  
*Norwegian Table, 4 per-cent.*

| $x$ | $l_x$   | $D_x$     | $N_x^{(12)}$ | ${}_{18-x}a_x^{(12)}$ | ${}_{18-x}a_x^{(12)}$ |
|-----|---------|-----------|--------------|-----------------------|-----------------------|
| (1) | (2)     | (3)       | (4)          | (5)                   | (6)                   |
| 0   | 100,000 | 100,000·0 | 1,085,151·7  | 10·6478               | 10·612                |
| 1   | 89,569  | 86,124·0  | 991,585·2    | 11·2769               | 11·239                |
| 2   | 86,402  | 79,883·5  | 908,354·5    | 11·1160               | 11·079                |
| 3   | 84,610  | 75,218·0  | 830,634·1    | 10·7722               | 10·736                |
| 4   | 83,345  | 71,243·7  | 757,258·8    | 10·3432               | 10·309                |
| 5   | 82,365  | 67,698·0  | 687,659·0    | 9·8569                | 9·824                 |
| 6   | 81,627  | 64,511·0  | 621,438·6    | 9·3173                | 9·286                 |
| 7   | 81,038  | 61,582·1  | 558,285·6    | 8·7349                | 8·706                 |
| 8   | 80,545  | 58,853·4  | 497,968·6    | 8·1151                | 8·088                 |
| 9   | 80,123  | 56,293·4  | 440,302·2    | 7·4597                | 7·435                 |
| 10  | 79,752  | 53,877·6  | 385,128·8    | 6·7701                | 6·748                 |
| 11  | 79,410  | 51,583·1  | 332,315·0    | 6·0474                | 6·027                 |
| 12  | 79,090  | 49,399·4  | 281,744·4    | 5·2911                | 5·273                 |
| 13  | 78,784  | 47,315·6  | 233,311·1    | 4·5005                | 4·485                 |
| 14  | 78,473  | 45,316·1  | 186,922·6    | 3·6754                | 3·663                 |
| 15  | 78,142  | 43,389·5  | 142,499·7    | 2·8146                | 2·805                 |
| 16  | 77,780  | 41,527·3  | 99,973·6     | 1·9169                | 1·911                 |
| 17  | 77,377  | 39,723·3  | 59,282·8     | ·9796                 | ·976                  |
| 18  | 76,934  | 37,976·8  | 20,369·2     | ...                   | ...                   |

From these are calculated the values in columns (4) to (6) by the following formulas at 4 per-cent interest

$$N_x^{(12)} = \sum_x^{18} D_x - \frac{11}{24} D_x.$$

$${}_{18-x}a_x^{(12)} = \frac{N_x^{(12)} - N_{18}^{(12)}}{D_x} = \text{the value of a temporary annuity of 1, payable by monthly instalments in advance, and ceasing at age 18.}$$

${}_{18-x}^{(12)}a_x = {}_{18-x}^{(12)}a_x v^{1/2}$  = the value of a temporary annuity payable by monthly instalments, with a payment at the end of the month in which death occurs, and ceasing at age 18.

I now proceed to obtain the necessary formulas for calculation of the contributions for widows' and children's annuities; and for this purpose I employ Behm's method, as modified by Dr. Zillmer.\*

#### A.—WIDOWS' ANNUITIES.

As I assume that the annuity is to be paid by monthly instalments at the end of each month, a final payment being made at the end of the month of death, the annuity-values given in Table I must be replaced by others calculated by the formula

$$a_y = (a_y + \frac{1}{2} \frac{3}{4}) v^{1/2} \dots \dots \dots (2)$$

For a functionary who dies between ages  $x$  and  $(x+1)$ , the value of the widows' annuity is, on an average,

$$W_x = \frac{1}{2} [m_x \cdot a_{u_x}^{(12)} + m_{x+1} a_{u_{x+1}}^{(12)}] \dots \dots \dots (3)$$

If we suppose that  $l_x$  functionaries of the age  $x$  enter as members at the same time, and that the deaths happen on an average in the middle of the year, the value of the annuities which become payable in consequence of the  $d_x$  deaths which occur in the year is  $W_x d_x v^{1/2}$ . For the second year the result is  $W_{x+1} d_{x+1} v^{1/2}$ , &c. Multiplying by  $v^x$ , and dividing by  $l_x v^x$ , the contribution by way of single payment to the Widows' Fund is equal to

$$\frac{C_x W_x + C_{x+1} W_{x+1} + C_{x+2} W_{x+2} \dots}{D_x v^{1/2}}$$

which, for the sake of brevity may be expressed as

$$\frac{v^{-1/2} \Sigma (C_x W_x)}{D_x} \dots \dots \dots (4)$$

#### B.—CHILDREN'S ANNUITIES.

Table II contains the calculation of the value of a temporary annuity to a child, payable until the age of 18 is attained. In order to get a formula analogous to (4), we must first find the average value, according to our statistics, of the children's

\* *Assicuranz-Jahrbuch*. Herausgegeben von A. Ehrenzweig.  
XII Jahrgang. II, S. 60 ff.



annuities which become payable on the death of a father. To  $S$  fathers of the age  $x$  correspond a certain number of children of various ages from 0 to 17. These were tabulated in respect of each age of the father, and are summarized for grouped ages in Table IV. Calling the number of children  $s$ , we get the following table :

| Ages | No. of Children | Value of their Annuities |
|------|-----------------|--------------------------|
| 0    | $s_0$           | $s_0  _{18} a_0$         |
| 1    | $s_1$           | $s_1  _{17} a_1$         |
| 2    | $s_2$           | $s_2  _{16} a_2$         |
| ...  | ...             | ...                      |
| ...  | ...             | ...                      |
| ...  | ...             | ...                      |
| 17   | $s_{17}$        | $s_{17}  _{11} a_{17}$   |

If we put  $a_{cx}$  for the average value of the children's annuities which would become payable on death of a father aged  $x$ , the value for  $S_x$  fathers will be  $S_x a_{cx}$  ;

and  $S_x a_{cx} = s_0 |_{18} a_0 + s_1 |_{17} a_1 + s_2 |_{16} a_2 + \dots + s_{17} |_{11} a_{17}$

$$a_{cx} = \frac{s_0 |_{18} a_0 + s_1 |_{17} a_1 + \dots + s_{17} |_{11} a_{17}}{S_x} \quad \dots \quad (5)$$

As an example of the application of this formula, I give the calculation for age 25.

| Age of Children<br>(1) | No. of Children<br>(2) | VALUE OF ANNUITY      |                      |
|------------------------|------------------------|-----------------------|----------------------|
|                        |                        | For each Child<br>(3) | Total<br>(4)=(2)×(3) |
| 0                      | 9                      | 10·61                 | 95·49                |
| 1                      | 7                      | 11·24                 | 78·68                |
| 2                      | 7                      | 11·08                 | 77·56                |
| 3                      | 3                      | 10·74                 | 32·22                |
| 4                      | 3                      | 10·31                 | 30·93                |
| 5                      | 2                      | 9·82                  | 19·64                |
| Total                  | 31                     | ...                   | 334·52               |

Number of fathers = 20.

Consequently  $S_{25} a_{cx} = 334·52$ , and  $a_{cx} = 16·726$ .

Following the same process as that by which we obtained formula (3), we get the following formula for the value of the children's annuities which become payable at the decease of a functionary of the age  $x$ .

$$\chi_x = \frac{1}{2}(c_x a_{cx} + c_{x+1} a_{c_{x+1}}) \quad . \quad . \quad . \quad . \quad (6)$$

Lastly, the process by which we obtained formula (4) gives us

$$\frac{v^{-\frac{1}{2}} \Sigma (C_x \chi_x)}{D_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

as the single premium which a functionary of age  $x$  must pay, in order to secure, for each of his children who survives him, and is under 18 at his death, an annuity payable by monthly instalments up to that age.

For the annual premium payable in advance during the life of the member, the formulas will be respectively—

$$P_{wx} = \frac{v^{-\frac{1}{2}} \Sigma (C_x W_x)}{N_{x-1}}$$

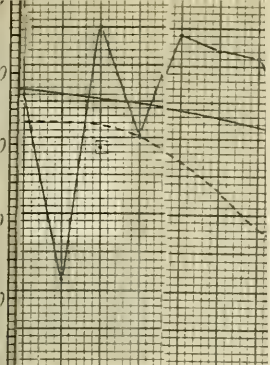
$$P_{cx} = \frac{v^{-\frac{1}{2}} \Sigma (C_x \chi_x)}{N_{x-1}}$$

and for a premium ceasing in  $n$  years

$${}_n P_{wx} = \frac{v^{-\frac{1}{2}} \Sigma (C_x W_x)}{N_{x-1} - N_{x+n-1}}$$

$${}_n P_{cx} = \frac{v^{-\frac{1}{2}} \Sigma (C_x \chi_x)}{N_{x-1} - N_{x+n-1}} .$$

If it should be further required that the premium should cease on the member's invalidity, the annuity-value would have to be calculated by a combined mortality and invalidity table.



n Railways.  
 c- Ungraduated  
 "ürkische Eisenbahn."

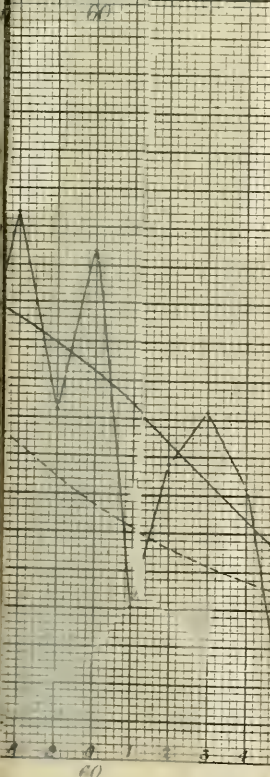
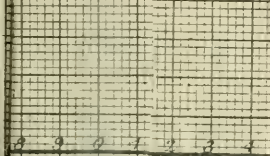


DIAGRAM I

Probability of being married according to Statistics of the Norwegian Railways  
or No 1831 - 1832 - 31 May 1831 ungraduated

|       |                |                |                                                         |                |             |
|-------|----------------|----------------|---------------------------------------------------------|----------------|-------------|
| —     | D <sup>o</sup> | D <sup>o</sup> | according to                                            | D <sup>o</sup> | ungraduated |
| —     | D <sup>o</sup> | D <sup>o</sup> | according to the Norwegian Census 1875                  |                |             |
| - - - | D <sup>o</sup> | D <sup>o</sup> | according to Statistics of Bergisch Märkische Eisenbahn |                |             |

DIAGRAM II

Probability of being married under 15 years according to Statistics of No 1831 ungraduated

|       |                |                |                                            |
|-------|----------------|----------------|--------------------------------------------|
| —     | D <sup>o</sup> | D <sup>o</sup> | ungraduated                                |
| - - - | D <sup>o</sup> | D <sup>o</sup> | under 15 years                             |
| - - - | D <sup>o</sup> | D <sup>o</sup> | Statistics of Bergisch Märkische Eisenbahn |

TABLE III.

*Probabilities of being Married, and of having Children under Age 18; also Average Ages of Wives.*

| Age of Member | PROBABILITY OF BEING MARRIED (m) $\times 100$ ACCORDING TO |      | PROBABILITY OF HAVING CHILDREN UNDER AGE 18 (c) $\times 100$ ACCORDING TO |      | AVERAGE AGE OF WIFE ACCORDING TO |      |
|---------------|------------------------------------------------------------|------|---------------------------------------------------------------------------|------|----------------------------------|------|
|               | Schjoll                                                    | Behm | Schjoll                                                                   | Behm | Schjoll (adjusted)               | Behm |
| (1)           | (2)                                                        | (3)  | (4)                                                                       | (5)  | (6)                              | (7)  |
| 17            | 5                                                          | ...  | 1                                                                         | ...  | 20                               | ...  |
| 18            | 15                                                         | ...  | 4                                                                         | ...  | 21                               | ...  |
| 19            | 30                                                         | ...  | 15                                                                        | ...  | 22                               | ...  |
| 20            | 55                                                         | 41   | 30                                                                        | 78   | 22                               | 28   |
| 21            | 90                                                         | 213  | 55                                                                        | 174  | 23                               | 24   |
| 22            | 135                                                        | 550  | 95                                                                        | 384  | 24                               | 23   |
| 23            | 195                                                        | 1065 | 150                                                                       | 744  | 24                               | 24   |
| 24            | 270                                                        | 1866 | 220                                                                       | 1241 | 25                               | 24   |
| 25            | 360                                                        | 2896 | 300                                                                       | 1932 | 26                               | 25   |
| 26            | 445                                                        | 4002 | 380                                                                       | 2804 | 27                               | 26   |
| 27            | 525                                                        | 5101 | 455                                                                       | 3780 | 27                               | 26   |
| 28            | 600                                                        | 6168 | 525                                                                       | 4789 | 28                               | 27   |
| 29            | 665                                                        | 7065 | 590                                                                       | 5753 | 29                               | 28   |
| 30            | 720                                                        | 8172 | 650                                                                       | 6527 | 29                               | 28   |
| 31            | 765                                                        | 8382 | 704                                                                       | 7200 | 30                               | 29   |
| 32            | 800                                                        | 8707 | 750                                                                       | 7635 | 31                               | 30   |
| 33            | 830                                                        | 8943 | 788                                                                       | 7910 | 32                               | 31   |
| 34            | 855                                                        | 9111 | 820                                                                       | 8117 | 33                               | 32   |
| 35            | 877                                                        | 9225 | 848                                                                       | 8287 | 34                               | 32   |
| 36            | 896                                                        | 9318 | 872                                                                       | 8429 | 35                               | 33   |
| 37            | 912                                                        | 9401 | 892                                                                       | 8565 | 36                               | 34   |
| 38            | 924                                                        | 9471 | 907                                                                       | 8669 | 37                               | 35   |
| 39            | 932                                                        | 9506 | 916                                                                       | 8735 | 38                               | 36   |
| 40            | 936                                                        | 9508 | 918                                                                       | 8729 | 39                               | 37   |
| 41            | 935                                                        | 9481 | 917                                                                       | 8678 | 40                               | 38   |
| 42            | 934                                                        | 9462 | 914                                                                       | 8618 | 41                               | 39   |
| 43            | 933                                                        | 9424 | 908                                                                       | 8548 | 42                               | 40   |
| 44            | 931                                                        | 9392 | 899                                                                       | 8444 | 42                               | 41   |
| 45            | 929                                                        | 9391 | 887                                                                       | 8349 | 43                               | 42   |
| 46            | 927                                                        | 9367 | 872                                                                       | 8243 | 44                               | 43   |
| 47            | 924                                                        | 9399 | 855                                                                       | 8106 | 45                               | 44   |
| 48            | 921                                                        | 9302 | 836                                                                       | 7925 | 45                               | 44   |
| 49            | 917                                                        | 9248 | 815                                                                       | 7729 | 46                               | 45   |
| 50            | 913                                                        | 9143 | 793                                                                       | 7487 | 47                               | 46   |
| 51            | 909                                                        | 9044 | 770                                                                       | 7230 | 48                               | 47   |
| 52            | 904                                                        | 8889 | 746                                                                       | 6872 | 49                               | 48   |
| 53            | 899                                                        | 8722 | 721                                                                       | 6461 | 50                               | 49   |
| 54            | 894                                                        | 8542 | 695                                                                       | 6004 | 50                               | 50   |



TABLE III (continued).

*Probabilities of being Married, and of having Children under Age 18; also Average Ages of Wives.*

| Age of Member | PROBABILITY OF BEING MARRIED (m) × 100<br>ACCORDING TO |       | PROBABILITY OF HAVING CHILDREN UNDER AGE 18<br>(c) × 100<br>ACCORDING TO |       | AVERAGE AGE OF WIFE<br>ACCORDING TO |      |
|---------------|--------------------------------------------------------|-------|--------------------------------------------------------------------------|-------|-------------------------------------|------|
|               | Schjoll                                                | Behm  | Schjoll                                                                  | Behm  | Schjoll (adjusted)                  | Behm |
| (1)           | (2)                                                    | (3)   | (4)                                                                      | (5)   | (6)                                 | (7)  |
| 55            | 88.8                                                   | 83.91 | 66.8                                                                     | 54.81 | 51                                  | 50   |
| 56            | 88.2                                                   | 82.80 | 64.0                                                                     | 50.10 | 52                                  | 51   |
| 57            | 87.6                                                   | 82.33 | 61.1                                                                     | 45.57 | 53                                  | 52   |
| 58            | 86.9                                                   | 82.03 | 58.0                                                                     | 41.43 | 54                                  | 53   |
| 59            | 86.2                                                   | 82.23 | 54.7                                                                     | 37.69 | 55                                  | 53   |
| 60            | 85.4                                                   | 81.88 | 51.0                                                                     | 34.25 | 56                                  | 54   |
| 61            | 84.6                                                   | 80.17 | 46.8                                                                     | 30.79 | 56                                  | 55   |
| 62            | 83.7                                                   | 76.95 | 41.7                                                                     | 27.90 | 57                                  | 56   |
| 63            | 82.7                                                   | 73.04 | 36.6                                                                     | 25.91 | 58                                  | 57   |
| 64            | 81.6                                                   | 67.67 | 31.5                                                                     | 23.98 | 59                                  | 57   |
| 65            | 80.4                                                   | 66.75 | 26.9                                                                     | 22.31 | 60                                  | 58   |
| 66            | 79.0                                                   | 66.21 | 22.7                                                                     | 20.74 | 61                                  | 59   |
| 67            | 77.4                                                   | 65.68 | 18.8                                                                     | 18.94 | 61                                  | 60   |
| 68            | 75.6                                                   | 65.11 | 15.2                                                                     | 15.91 | 62                                  | 60   |
| 69            | 73.6                                                   | 64.53 | 11.9                                                                     | 12.82 | 63                                  | 61   |
| 70            | 71.4                                                   | 64.07 | 8.9                                                                      | 10.01 | 64                                  | 62   |
| 71            | 69.0                                                   | 63.76 | 6.2                                                                      | 6.61  | 64                                  | 62   |
| 72            | 66.5                                                   | 62.54 | 3.9                                                                      | 3.81  | 65                                  | 63   |
| 73            | 63.9                                                   | 62.39 | 2.1                                                                      | ...   | 66                                  | 64   |
| 74            | 61.2                                                   | 58.81 | .8                                                                       | ...   | 67                                  | 64   |
| 75            | 58.5                                                   | 56.73 | ...                                                                      | ...   | 67                                  | 64   |
| 76            | 55.8                                                   | 53.71 | ...                                                                      | ...   | 68                                  | 65   |
| 77            | 53.2                                                   | 50.89 | ...                                                                      | ...   | 69                                  | 65   |
| 78            | 50.6                                                   | 43.94 | ...                                                                      | ...   | 69                                  | 66   |
| 79            | 48.0                                                   | 41.23 | ...                                                                      | ...   | 70                                  | 66   |
| 80            | 45.5                                                   | 34.44 | ...                                                                      | ...   | 71                                  | 67   |
| 81            | 43.1                                                   | 26.12 | ...                                                                      | ...   | 72                                  | 67   |
| 82            | 40.8                                                   | 16.44 | ...                                                                      | ...   | 73                                  | 67   |
| 83            | 38.6                                                   | ...   | ...                                                                      | ...   | 73                                  | ...  |
| 84            | 36.5                                                   | ...   | ...                                                                      | ...   | 74                                  | ...  |
| 85            | 34.5                                                   | ...   | ...                                                                      | ...   | 74                                  | ...  |
| 86            | 32.6                                                   | ...   | ...                                                                      | ...   | 75                                  | ...  |
| 87            | 30.8                                                   | ...   | ...                                                                      | ...   | 76                                  | ...  |
| 88            | 29.2                                                   | ...   | ...                                                                      | ...   | 76                                  | ...  |
| 89            | 27.8                                                   | ...   | ...                                                                      | ...   | 77                                  | ...  |
| 90-95         | 26.6-23.4                                              | ...   | ...                                                                      | ...   | 78                                  | ...  |

TABLE IV.

*Number of Fathers with Children under 18 years of age; and number of Children, according to Age last Birthday, and grouped Ages of Fathers; also Total Number of Functionaries at grouped ages.*

| Father's<br>age last<br>Birthday | Number<br>of Fathers<br>with<br>Children<br>under 18 | NUMBER OF CHILDREN AT AGE LAST BIRTHDAY |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | Total<br>0-17 | Total<br>number of<br>Functionaries | Age last<br>Birthday |       |
|----------------------------------|------------------------------------------------------|-----------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---------------|-------------------------------------|----------------------|-------|
|                                  |                                                      | 0                                       | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  |               |                                     |                      | 17    |
| 14-22                            | 8                                                    | 5                                       | 4   | 1   | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ...           | 10                                  | 205                  | 14-22 |
| 23-27                            | 76                                                   | 37                                      | 28  | 21  | 16  | 12  | 9   | 2   | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ...           | 127                                 | 253                  | 23-27 |
| 28-32                            | 228                                                  | 87                                      | 85  | 74  | 78  | 70  | 54  | 55  | 36  | 23  | 10  | 8   | 5   | 2   | 1   | ... | ... | ... | ...           | 589                                 | 335                  | 28-32 |
| 33-37                            | 439                                                  | 120                                     | 151 | 136 | 153 | 139 | 141 | 116 | 133 | 107 | 96  | 63  | 71  | 38  | 38  | 25  | 9   | 5   | 2             | 1,543                               | 522                  | 33-37 |
| 38-42                            | 396                                                  | 95                                      | 109 | 104 | 108 | 106 | 125 | 113 | 128 | 124 | 118 | 115 | 107 | 103 | 86  | 72  | 69  | 37  | 31            | 1,750                               | 435                  | 38-42 |
| 43-47                            | 314                                                  | 35                                      | 59  | 51  | 65  | 81  | 61  | 77  | 76  | 76  | 87  | 75  | 93  | 88  | 82  | 80  | 82  | 65  | 40            | 1,273                               | 348                  | 43-47 |
| 48-52                            | 169                                                  | 23                                      | 12  | 22  | 29  | 27  | 24  | 34  | 26  | 58  | 30  | 32  | 50  | 45  | 44  | 52  | 37  | 54  | 40            | 639                                 | 219                  | 48-52 |
| 53-57                            | 105                                                  | 7                                       | 8   | 6   | 10  | 11  | 9   | 17  | 11  | 24  | 21  | 19  | 20  | 24  | 29  | 24  | 30  | 30  | 31            | 331                                 | 155                  | 53-57 |
| 58-62                            | 56                                                   | 2                                       | 4   | 2   | 6   | 2   | 7   | 6   | 3   | 3   | 5   | 7   | 10  | 10  | 12  | 11  | 15  | 14  | 10            | 129                                 | 103                  | 58-62 |
| 63-67                            | 14                                                   | 1                                       | ... | ... | 1   | ... | 1   | ... | ... | 2   | 1   | 1   | ... | 2   | 2   | 2   | 3   | 3   | 1             | 20                                  | 58                   | 63-67 |
| 68-72                            | 4                                                    | ...                                     | ... | ... | ... | ... | ... | ... | ... | 1   | 1   | ... | 1   | ... | 1   | ... | 2   | ... | 1             | 9                                   | 21                   | 68-72 |
| 73-80                            | ...                                                  | ...                                     | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ...           | ...                                 | 4                    | 73-80 |
| Total...                         | 1,809                                                | 412                                     | 460 | 417 | 466 | 449 | 431 | 421 | 415 | 418 | 369 | 320 | 357 | 312 | 295 | 267 | 247 | 208 | 156           | 6,420                               | 2,658                | Total |

*Historical Memorandum on Friendly Societies in connection with Actuarial Certificates.*

[The following historical Memorandum as to the course of Legislation in respect of Actuarial Certificates of Tables of Contributions, &c., was prepared by the Chief Registrar of Friendly Societies, in connection with a proposal, recently under discussion, as to compulsory valuation of certain classes of Friendly Societies, which are exempted from valuation under the Friendly Societies Act of 1896. The Memorandum is included as Appendix A (3) in Part A of the Report of the Chief Registrar of Friendly Societies for the year 1904.—  
ED. J.I.A.]

1819.  
Provision for  
actuarial  
approval of  
tables.

The question of legislation for the purpose of compelling societies, registering under the Friendly Societies Acts, to become established on sound actuarial principles has, as will be seen in this memorandum, on several occasions engaged the attention of the Legislature and formed the subject of enquiry by Select Committees.

Passing of Act  
59 Geo. III,  
c. 128.

It was first contemplated prior to the passing of the Act of 1819, 59 Geo. III, c. 128, and as a clause was introduced into the Act under section 2 providing that no Society should be registered under the said Act unless its tables of contributions and benefits had been previously approved by two actuaries "as fit and proper according to the most correct calculation of which the nature of the case will admit" it is evident that the Committee in charge of the Bill were of opinion that the introduction of such a provision would have the effect of safeguarding the interests of the members of Friendly Societies.

For text of section 2, *see* Appendix 1.

The Act of 1819 is said to deserve notice more for what it attempted than for what it actually accomplished. At that date the profession of an actuary was not a defined one, and in many counties it was found that the bench were satisfied with the signatures of "petty schoolmasters and accountants," with the result that the confirmation of the rules of a society in many instances was more of a danger than a safeguard of the interests of its members, no proper consideration being given to the question of its administration.

1825  
Select  
Committee of  
Enquiry as to  
existing law.

In the year 1825 a Select Committee was appointed to inquire into the existing state of the law relating to Friendly Societies, &c., and sat under the presidency of Mr. T. P. Courtney from 8 March to 21 June.

During the deliberations of this Committee, the question arose in connection with the certification of tables of contributions and benefits as to the correct

definition of the term "professional actuaries or persons skilled in calculations", because it was found that signatures of unqualified persons had been accepted by certain justices as satisfying the provisions of section 2 of the Act of 1819.

1825.  
Report of  
Select  
Committee.

The Committee appear to have suggested that the justices of every county should name two or more persons whose certificate only they should sanction to the tables, and that if it were certain that justices would be careful to name only actuaries of acknowledged ability and experience without reference to personal connection, their suggestion might prudently be accepted.

The Committee also suggested that the number of members belonging to a society applying for registration should be not less than 200.

The final findings of this Committee appear to be that in consequence of the alarm that section 2 of the Act of 1819 occasioned to members of Friendly Societies, and of the fact that the provisions of the said section were possibly a deterrent to their taking advantage of registration under the Act, it would be advisable, in their opinion, to amend the Act by repealing the said section.

The opinion of an important witness, examined before the Committee, was to the effect that great difficulty had been experienced since the passing of the Act of 1819 in inducing persons to become members of new friendly societies on account of the fact that the contributions they were required by law to pay were greater than those of members who had joined societies previously to the passing of this Act.

The Committee was re-appointed in the year 1827 to further investigate the matter, and its inquiries turned almost entirely upon the question of drawing up tables principally for use by societies granting superannuation allowances, and they were of opinion that societies should be formed on the largest scale. The use of the Northampton and the Becher tables was recommended.

1827.  
Re appointment  
of  
Committee  
of Enquiry.

No legislation took place as a consequence of the two reports referred to until the year 1829, when the Act of 10 Geo. IV., c. 5, was passed. By this Act the provision of section 2 of the Act of 1819 relating to the certificate of an actuary, or person skilled in calculation, was dispensed with, but the justices were required before confirmation of a society's rules, to satisfy themselves that its tables of contributions and benefits might "be adopted with safety to all parties concerned."

1829.  
Passing of  
Act, 10 Geo.  
IV., c. 5.

In the year 1846 an attempt was once more made, by the passing of the Act 9 & 10 Vict., c. 27, to secure societies being established on sound financial bases. It was required that no new societies should be certified unless the tables had been certified by the actuary to the Commissioners for the Reduction of the National Debt,

1846.  
Passing of  
Act, 9 & 10  
Vict., c. 27.

or by some person who had been at least five years actuary to some life insurance company.

For text of the section of the Act relating to this provision, *see* Appendix 2.

1849.  
Select  
Committee of  
Enquiry.

In the year 1849 a select Committee was appointed to enquire into and report upon the working of the Act of 1846, 9 & 10 Vict. c. 27. The Committee found, from the statements of actuaries and others, that in spite of the provision of Section XIII. of the Act of 1846, 9 & 10 Vict. c. 27, in the great majority of existing Societies the rates of contributions were insufficient. On these grounds they did not feel themselves justified, in spite of the evidence of several witnesses, in recommending that the Government should in any way appear to guarantee any assurance, and they were of opinion that the responsibility should continue to be left to each Society of adopting such tables as should be agreed upon by its own members.

1850.  
Passing of  
Act, 13 & 14  
Vict., c. 115

In consequence of the findings of the Select Committee of 1849, Parliament passed the Friendly Societies Act of 1850, 13 & 14 Vict., c. 115.

Under the provisions of this Act, Section XIII of the Act of 1846, 9 & 10 Vict. c. 27 was repealed, and the question of an actuary's certificate to the tables of contributions and benefits was left entirely to the discretion of the members.

Section VII of the Act of 1850, 13 & 14 Vict., c. 115, enacted that the rules of a Friendly Society should be in accordance with the provisions of the Act, and, if so, would be registered as such; and that if the rules and tables had been certified by some actuary holding the necessary qualifications, the Registrar's certificate should be in the form set forth in Schedule B of this Act; that a certificate in the form set forth in Schedule C of the Act should be furnished, and the rules and the tables be signed by the actuary certifying the same. The Society would then be registered as a "Certified Society;" but if the rules and tables had not been so certified by an actuary, the Registrar's certificate would be in the form of Schedule A, and the Society denominated "Registered Friendly Society." *See* Appendix 3.

Although the Act of 1850 did not enforce the furnishing of an actuary's certificate to the tables and rules, Section VIII provided and enacted that... "it shall not be lawful for the Registrar to grant any certificate to any Society assuring to any member thereof a certain annuity, deferred or immediate, unless the tables of contributions payable for such kind of assurance shall have been certified by an actuary as aforesaid, or furnished by the Registrar."

1854.  
Appointment  
of Select  
Committee of  
Enquiry.

A Friendly Societies Bill was again introduced in the year 1854, and a Select Committee of the House of



Commons was appointed to inquire into, and report upon the same, and following the Committee's Report, an Act was passed in the year 1855:—18 and 19 Vict., c. 63.

1855.  
Passing of  
Act, 18 & 19  
Vict., c. 63.

By this Act the provisions of Section VII. of the Act of 1850 were modified, and the distinction between a registered and a certified Society was done away with, but the Registrar's certificate was to follow the form set forth in the Second Schedule to the Act: the latter paragraph of which provided that "the rates of contributions and payments are stated to have been prepared by *A.B.*, actuary of , or (as the case may be) are not stated to have been prepared by an actuary."

This certificate remained in force until the passing of the Act of 1875 which came into operation on the 1st January, 1876. The certificate introduced was in the same form as that now prescribed. For full text of the Registrar's Certificate, *see* Appendix 4.

The question of the establishment of Friendly Societies on sound financial principles does not appear to have been again touched upon until the appointment of the Royal Commission in the year 1871, for the purpose of inquiring into the state of the law relating to Friendly Societies existing at that date.

1871.  
Appointment  
of Royal  
Commission  
of Enquiry as  
to existing  
law.

A considerable amount of valuable evidence was brought before the Commissioners, and the question of the requirement of an actuary's certificate to the tables of contributions and benefits of Societies was again carefully considered.

In the evidence before the Commissioners it was pointed out that whereas on the one hand the Central Office of the Registry of Friendly Societies could be made more useful and efficient in some respects, by certain additional powers being vested in the hands of the Registrar, on the other hand the less the Government interfered with the management of a Society or the rates they adopt, &c., &c., the better; and that the question to be considered was not only the soundness of the particular rates adopted by a Friendly Society, but as to the general rules and efficiency of its administration.

The Commissioners found that there were serious fundamental differences of opinion (as there are now, and probably always will be) as to the extent to which the Legislature should interfere in matters of the nature with which Friendly Societies have to do; so that, whereas it may appear to some persons that the law is defective because it fails to procure adequate security against mismanagement and error, to others it appears faulty because it attempts to regulate matters that should rather be dealt with on the responsibility of the members themselves; and to another class who maintain that the State should withdraw from any

Report of  
Com-  
missioners.

1871.  
Report of  
Com-  
missioners.

intervention whatever. It was also found that whereas on the one hand it was considered absolutely essential that the Registrar should, before granting registration, satisfy himself that the tables of contributions were sound and suitable, on the other hand it was held that the Registrar should not give any certificate at all: so that the public might not be misled into the erroneous belief that he had investigated and vouched for the soundness of a society.

The Commissioners further pointed out that in theory it is objectionable to assign to the State what are called paternal functions, and that it is not the duty of the Government to look after the affairs of individuals; that by attempting to do so it discourages the exercise of individual prudence and watchfulness, that after all it can only do the work imperfectly, and if it is unsupported by such individual prudence and watchfulness it cannot do it at all, and that the certificate of registration of Friendly Societies does not give any assurance against the principal dangers to which societies are exposed.

The Commissioners, whilst refraining from recommending that the tables of rates of contributions and benefits should be certified before registration, recommended that tables of rates should be prepared by Government for the guidance of Friendly Societies, but that such tables should not be compulsory; and they were of opinion that the value of such tables would be fully recognised by those interested in Friendly Societies, and that comparison of the experience of Societies would be made with the Government standard for the purpose of ascertaining how far the experience coincided.

Sickness and  
Mortality  
Experience of  
Friendly  
Societies  
(1898).

(Owing to this recommendation a special report on the sickness and mortality experience in registered Friendly Societies was prepared by the actuary to the Registry of Friendly Societies, Wm. Sutton, Esq., F.I.A., and issued in the year 1898.)

Even more importance was attached by the Commissioners to the properly-conducted periodical valuations, and to the corrections made from time to time in the tables according to the results of those valuations, than to the original tables themselves, which in the case of small societies could hardly be expected to agree with the Society's experience.

The Commissioners also proposed that whilst it should be the duty of the Registrar to refuse registration to Societies which he might consider to be established for illegal purposes, the certificate or acknowledgment of registration should only purport to testify to the fact of compliance with the statutory requirements, and not to general conformity of the rules with law.

An additional separate report of certain of the Commissioners, however, recommended that the Registrar,

when rules or alterations of rules of Societies were tendered for registration, should not merely satisfy himself that the rules contained the required provisions, but should examine the rates of contributions and benefits, and, where he thought necessary, advise Societies to adopt such rates which could, apart from mismanagement and fraud, secure solvency.

1871.  
Report of  
Com-  
missioners.

It was not proposed to give powers to the Registrar to compel the adoption of certain rates, but that discriminating Certificates of Registration might be given for the purpose of informing would-be members as to whether the Society which they contemplated joining had adopted approved rates. In cases where Societies had adopted approved rates such Societies would be classified under A., and Societies which had merely complied with the requirements should simply be registered.

(At the time this Commission was sitting the Registrar's certificate stated whether the rates of contribution had or had not been prepared by an actuary). (See page 215.)

It was also suggested that it should be made known that the Registrar's certificate implied no guarantee of solvency, as is now done in the case of Building Societies.

After duly considering the reports of the Commissioners it was apparently found impracticable to compel Societies to adopt certain rates which were to be certified by duly qualified actuaries, and further it was not deemed advisable that the certificate of the Registrar should state whether the rates had or had not been approved for fear of misleading would-be members as to a Society's financial position.

The Registrar's acknowledgment of registry under the Act of 1875, which was passed after the report of the Commission had been considered, did not state whether Societies' rates had or had not been certified by an actuary.

1875.  
Acknowledg-  
ment of  
registry under  
Act 88 & 89  
Vict., c. 60.

This brings the history of the various attempts to make actuarial certificates compulsory up to the present time.

#### APPENDIX 1.

##### *The Friendly Societies Act, 1819.*

59 Geo. III, c. 128.

Section (2) of this Act provided that Justices should satisfy themselves that the formation of Societies would be useful and beneficial, regard being had to the existence of any other Society already formed under wholesome rules within the same district for the like purposes. Such Justices shall not confirm and allow any tables of payments or benefits, or any rules dependent upon or connected with the calculations thereof, until it shall have been made to appear to such Justices that the said tables

Appendix 1.  
59 Geo. III,  
c. 128.

## Appendix 1

and rules are such as have been approved by two persons at least known to be professional actuaries, or persons skilled in calculations, as fit and proper according to the most correct calculations of which the nature of the case will admit.

This Act also provides that the Lord High Treasurer shall appoint five or more persons skilled in arithmetical calculations. Any Society submitting their rules for approbation shall pay such Committee such reasonable fee as the said Lord High Treasurer shall appoint.

Repealed by 10 Geo. IV, c. 5.

Provision reintroduced in the Act of 1846—  
9 & 10 Vict. c. 27, Sect. XIII.

Provision modified under the Act of 1850—  
13 & 14 Vict. c. 115, Sect. VII.

Provision repealed under the Act of 1855—  
18 & 19 Vict. c. 63, Sect. XXVI.

## APPENDIX 2.

*The Act of 1846.*

9 & 10 Vict. c. 27.

Appendix 2  
9 & 10 Vict.,  
c. 27, § xiii.

Section XIII.—“And be it enacted that after the passing of this Act the Registrar of Friendly Societies in England, Scotland, or Ireland *shall not certify* the rules of any Friendly Society established after the passing of this Act for the purpose of securing any Benefits depending on the Laws of Sickness or Mortality *unless such Society shall adopt a table which shall have been certified to be a table which may be safely and fairly adopted for such purpose under the hand of the actuary to the Commissioners for the Reduction of the National Debt, or of some person who shall have been for at least five years an actuary to some Life Insurance Company in London, Edinburgh, or Dublin; and the name of the Actuary by whom any such table shall have been certified shall be set forth in the rules, and printed at the foot of all copies of such table printed for the use of the Society.*”

## APPENDIX 3.

*The Friendly Societies Act, 1850.*

13 & 14 Victoria, c. 115.

Appendix 3.  
13 & 14 Vict.,  
c. 115, § vii,  
viii.

Section VII.—Section XIII. of 9 & 10 Victoria, c. 27 was repealed by this Act, and the provision was passed:—That the rules of a Friendly Society should conform to the provision of the Act, and if they did the Registrar to register the same. . . . *And if such rules and tables have been certified by some other actuary who shall have been for more than five years an actuary of some Life Assurance Company established in London, Edinburgh,*



or Dublin, in the form set forth in Schedule (C) to this Act annexed, and signed by him at the foot both of the rules and tables, or shall have been furnished to such Society or Branch by the Registrar, the Registrar shall give a certificate in the form set forth in Schedule (B) to this Act annexed, and such Society or Branch shall be denominated *Certified Friendly Society*; but if such rules and tables shall not have been so certified by an actuary as above mentioned, in such case the Registrar shall give a certificate in the form set forth in Schedule (A) to this Act annexed, and such Society shall be denominated a *Registered Friendly Society*. . . .

Appendix 3.

Under this Act Societies assuring to any member a certain annuity, immediate or deferred, were compelled to furnish an actuary's certificate to the tables of contributions for annuities in compliance with section viii.

Section VIII.—“ Provided always, and be it enacted that it shall not be lawful for the Registrar to grant any certificate to any Society assuring to any member thereof a certain annuity, deferred or immediate, unless the tables of contributions payable for such kind of assurance shall have been certified by an actuary as aforesaid, or furnished by the Registrar.”

#### SCHEDULE (A).

##### *Form of Registrar's Certificate to Rules of Registered Friendly Societies.*

I hereby certify that these rules (or alterations of rules) are in conformity to law, and to the provisions of the statute in force relating to *Registered Friendly Societies*.

Schedule A.

#### SCHEDULE (B).

##### *Form of Registrar's &c. &c.*

I hereby certify that these rules (or alterations of rules) are in conformity to law, and to the provisions of the statute in force relating to *Certified Friendly Societies*.

Schedule B.

#### SCHEDULE (C).

##### *Form of Actuary's Certificate.*

I, \_\_\_\_\_, hereby certify that the preceding table or tables to each of which my name is subscribed, may be fairly and safely adopted for the purposes of the \_\_\_\_\_ Society, subject to the terms, conditions and provisions of clauses Nos. \_\_\_\_\_ of the rules, to each of which my initials are affixed. And I hereby declare that I consider each of the said tables fairly to represent for each year or term of age (which said term does not exceed five years), the rate of contribution necessary to provide the respective benefits intended to be provided

Schedule C.



Appendix 3.

thereby, and that they are in conformity with the provisions of the Act passed in the year of Her present Majesty, chapter , and fairly represent the interest of members entering at those years or terms of age, without prejudice to any; and that the said tables are calculated on the supposition that the capital of the society will realise at least per cent. (compound interest), (if any addition has been made to the rates of contributions for expenses of management, here state the same), and are founded on the rate of sickness and mortality (here describe the same, and if the data on which the calculation has been based is published, state the work, and date of publication) upon the following data:—

| Age | Number out of which One will Die in each Year | Average Amount of Sickness annually experienced in each Year |
|-----|-----------------------------------------------|--------------------------------------------------------------|
| 20  |                                               |                                                              |
| 25  |                                               |                                                              |
| 30  |                                               |                                                              |
| 35  |                                               |                                                              |
| 40  |                                               |                                                              |
| 45  |                                               |                                                              |
| 50  |                                               |                                                              |
| 55  |                                               |                                                              |
| 60  |                                               |                                                              |

And I declare that I have read the rules of the said Society, and find nothing in them inconsistent with the said table, or which is calculated, in my opinion, to endanger the stability of the Society.

(Signed)

Actuary.

## APPENDIX 4.

*The Friendly Societies' Act, 1855.*

18 &amp; 19 Vict. c. 63.

## SECOND SCHEDULE.

*Form of Registrar's Certificate.*

Appendix 4.  
18 & 19 Vict.,  
c. 63,  
Schedule II.

I hereby certify that the foregoing rules (or the alterations or amendments of the rules) of the Society, at , in the county of , are in conformity with law (and, in the case of a new society), and that the Society is duly established from the present date, and is subject to the provisions and entitled to the privileges of the Acts relating to friendly societies.

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## CONTENTS OF THIS NUMBER.

---

- I. Canadian Vital Statistics; with particular reference to the Province of Ontario. By M. D. GRANT, B.A., F.I.A., Assistant Actuary, Government Insurance Department, Ottawa, Canada.

Abstract of the Discussion on the preceding.

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Abstract of the Discussion on the preceding.

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- IV. On the Calculation of the Contributions to be made to an Annuity Fund for Widows and Children by the members of a Society, all of whom, whether married or single, are obliged to contribute. By OSCAR SCHJOLL, formerly Manager (Kontorchef), of the "Idun" Life Insurance Company, Christiania; Manager of the Royal Office for Workmen's Assurances (Rigsforsikringsanstalten), Christiania.

- V. Historical Memorandum on Friendly Societies in connection with Actuarial Certificates.

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VOL. XL. —PART III.

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# CONTENTS OF NO. 225.

|                                                                                                                                                              | PAGE |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| On a form of Spurious Selection which may arise when Mortality Tables are amalgamated. By W. Palin Elderton, F.I.A., of the Guardian Assurance Company ..... | 221  |
| Abstract of the Discussion on the preceding.....                                                                                                             | 234  |
| Some Aspects of Registration of Title to Land. By James Robert Hart, F.I.A., of the Pelican and British Empire Life Office ... ..                            | 246  |
| Abstract of the Discussion on the preceding.....                                                                                                             | 273  |
| ACTUARIAL NOTES:                                                                                                                                             |      |
| I. Interpolation by Finite Differences (Two Independent Variables). By Herbert H. Edwards .....                                                              | 289  |
| II. Some Practical Hints on Two-Variable Interpolation. By John Spencer, F.I.A. ....                                                                         | 293  |
| III. On the Law of Uniform Seniority .....                                                                                                                   | 302  |
| CORRESPONDENCE:                                                                                                                                              |      |
| Letter from Mr. James Bacon on "Spurious Selection" .....                                                                                                    | 304  |
| „ „ Mr. W. Palin Elderton on the same subject .....                                                                                                          | 309  |
| THE INSTITUTE OF ACTUARIES:                                                                                                                                  |      |
| Alteration in Rules of Examination .....                                                                                                                     | 312  |
| The Life Assurance Companies of the United Kingdom. (Extracted from the Parliamentary Returns for 1905.) .....                                               | 313  |

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*On a form of Spurious Selection which may arise when Mortality Tables are amalgamated. By W. PALIN ELDERTON, F.I.A., of the Guardian Assurance Company.*

[Read before the Institute, 26 February 1906.]

IT is probable that some of the points I venture to bring forward have already suggested themselves to other students, but in view of the interest at present attaching to selection, it seems an opportune time for putting forward any considerations that may help to throw light on a somewhat complicated subject.

If a large number of lives were assured at the same moment at exact integral ages, and all these lives were traced throughout their whole durations, the effect of selection on rates of mortality would be measured accurately by variations in the values of  $q_{[x-t]+t}$  corresponding to changes in the values of  $t$ . Spurious Selection\* may be defined as the selection indicated by a difference in  $q$  which has arisen entirely from statistical processes. The causes of these differences can be easily seen as follows:—

Let  $E_{[x]+t}$  be the number exposed to risk by one table A, and  $q_{x+t}$  the rate of mortality at a duration at which selection has disappeared, and let  $E'_{[x]+t}$  and  $q'_{x+t}$  be the same functions by another table B showing a heavier mortality, then the number of

\* The term "Spurious Selection", which is used throughout the present paper in the technical sense indicated, was suggested by the "spurious correlation" of statistical work (See Pearson, *Proc. Roy. Soc.*, lx, pp. 489-498).

deaths by table A is  $E_{[x]+t}q_{x+t}$  and the number by table B is  $E'_{[x]+t}q'_{x+t}$ , so that if we amalgamated the two tables we should have as the rate of mortality

$$\frac{E_{[x]+t}q_{x+t} + E'_{[x]+t}q'_{x+t}}{E_{[x]+t} + E'_{[x]+t}}.$$

Similarly, if we take the same attained age, and duration  $t+1$ , we have as the rate of mortality

$$\frac{E_{[x-1]+t+1}q_{x+t+1} + E'_{[x-1]+t+1}q'_{x+t+1}}{E_{[x-1]+t+1} + E'_{[x-1]+t+1}}.$$

The  $q_{x+t}$  and  $q'_{x+t}$  remain unchanged, because we assume that selection has worn off; but if  $q$  and  $q'$  are unequal, the two expressions given above may be unequal, and, if they prove to be so, selection will appear to be still in operation at duration  $t$  in the amalgamated table, though it has worn off in both A and B.

The condition can be seen easily, for

$$\frac{E_{[x]+t}q + E'_{[x]+t}q'}{E_{[x]+t} + E'_{[x]+t}} > \frac{E_{[x-1]+t+1}q + E'_{[x-1]+t+1}q'}{E_{[x-1]+t+1} + E'_{[x-1]+t+1}}$$

$$\begin{aligned} \text{as } E_{[x-1]+t+1}E'_{[x]+t}q' + E_{[x]+t}E'_{[x-1]+t+1}q \\ > \\ = E'_{[x-1]+t+1}E_{[x]+t}q' + E_{[x-1]+t+1}E'_{[x]+t}q \\ < \end{aligned}$$

by multiplying across and cancelling like products,

$$\text{i.e., as } E_{[x-1]+t+1}E'_{[x]+t}(q' - q) > E_{[x]+t}E'_{[x-1]+t+1}(q' - q)$$

$$\text{or as } \frac{E'_{[x]+t}}{E_{[x]+t}} > \frac{E'_{[x-1]+t+1}}{E_{[x-1]+t+1}}.$$

In other words, if we write  $Q_{[x]+t}$  as the probability of death in the amalgamated table, we have

$$Q_{[x]+t} > Q_{[x-1]+t+1}$$

$$\text{as } \frac{E'_{[x]+t}}{E_{[x]+t}} > \frac{E'_{[x-1]+t+1}}{E_{[x-1]+t+1}}.$$

If the ratio  $\frac{E'}{E}$  is increasing, selection will appear to last longer than it really does, while if the ratio is decreasing it will appear

to have too short a duration. It is only when the ratio is constant that reliance can be placed on the selection shown by a table formed by amalgamating experiences.

The difference between the rates,  $Q_{[x-1]+t+1} - Q_{[x]+t}$ , can be expressed as

$$\frac{(q' - q)(E'_{[x-1]+t+1}E_{[x]+t} - E'_{[x]+t}E_{[x-1]+t+1})}{(E_{[x]+t} + E'_{[x]+t})(E_{[x-1]+t+1} + E'_{[x-1]+t+1})} = (q' - q)K_{xt} \text{ say,}$$

and this expression might be of use for measuring the amount of spurious selection introduced.

A few examples, showing the application of the theory, may be of interest.

The first example assumes a purely hypothetical case in which there is no selection at all, and two tables having different numbers exposed to risk and different rates of mortality are amalgamated, and the appearance of select mortality is introduced simply by this process. I have assumed that  $q_{x+t} = .01$ , and  $q'_{x+t} = .02$ , whatever the duration, and the following table shows the entire work :—

TABLE I.

| Duration | ATTAINED AGE ( $x+t$ ) |           | $\frac{E'}{E} \%$ | Rate of Mortality<br>found by amalgamation<br>of data |
|----------|------------------------|-----------|-------------------|-------------------------------------------------------|
|          | $E'_{x+t}$             | $E_{x+t}$ |                   |                                                       |
| $t$      | 10                     | 1,000     | 1.0               | $\frac{10 + .2}{1,010} = .0101$                       |
| $t+1$    | 150                    | 1,000     | 15.0              | $\frac{10 + 3}{1,150} = .0113$                        |
| $t+2$    | 250                    | 1,000     | 25.0              | $\frac{10 + 5}{1,250} = .0120$                        |
| $t+3$    | 300                    | 1,000     | 30.0              | $\frac{10 + 6}{1,300} = .0123$                        |

This example is an extreme case, but it clearly shows that, when we are dealing with material which is not absolutely homogeneous, the fact that the select table does or does not run into an aggregate table is not a complete criterion as to the duration of selection. For, turning to Table I, we see that it would be quite possible to mix the exposed for all durations, excluding the first few years, so as to make the spurious select tables run into an aggregate table at any point. For instance, if the aggregate table, excluding  $t+1$  years, were formed in the imaginary experience in

the ratio of, say, 200:1,000, this would mean that it could be concluded that selection had entirely worn off at duration  $t+2$ , though the examination of the rates of mortality in the final column would make the duration of selection longer, and both would be wrong.

Again, if the parts of the table where selection really holds are mixtures of equal proportions, and any part where selection has worn off is a mixture of unequal proportions, then the aggregate table will either be reached too early or too late, according as there is more or less of the lighter mortality in the mixture.

An example of the application of the theory is the one which caused me to look into the subject, namely, the British Offices' O<sup>[M]</sup> Experience. This table is the amalgamation of "Old" and "New" assurances, and the former have a far heavier mortality than the latter. We wish to decide whether this amalgamation affected the duration of selection. Does selection appear to last for a greater number of years in the amalgamated table than it does in the two parts?

Table II, following out the theory indicated above, explains itself. E' relates to the Old, and E to the New, experience:

TABLE II.

| Attained age<br>$x$ | Duration<br>$t$ | $100 \frac{E'_{[x-t]+t}}{E_{[x-t]+t}}$ | $K_{xt}$ | $(q' - q)K^*$ | $Q_{[x-1]+t+1} - Q_{[x]+t}$                        | $Q_{[x-1]+t+1} - Q_{[x]+t}$ |
|---------------------|-----------------|----------------------------------------|----------|---------------|----------------------------------------------------|-----------------------------|
|                     |                 |                                        |          |               | G. F. Hardy's<br>Graduation of<br>O <sup>[M]</sup> | Ungraduated †               |
| (1)                 | (2)             | (3)                                    | (4)      | (5)           | (6)                                                | (7)                         |
| 40                  | 5               | 17                                     | ·027     | ·00006        | ·00034                                             | + ·00064                    |
|                     | 6               | 21                                     | ·022     | 5             | 32                                                 | - ·00033                    |
|                     | 7               | 24                                     | ·027     | 6             | 30                                                 | + ·00058                    |
|                     | 8               | 28                                     | ·028     | 6             | 28                                                 | - ·00022                    |
|                     | 9               | 33                                     | ·030     | 7             | 13                                                 | ...                         |
|                     | 10              | 39                                     | ...      | ...           | ...                                                | (average ·00017)            |
| 50                  | 5               | 19                                     | ·032     | ·00007        | ·00056                                             | - ·00089                    |
|                     | 6               | 23                                     | ·037     | 8             | 51                                                 | + ·00046                    |
|                     | 7               | 29                                     | ·027     | 6             | 45                                                 | - ·00009                    |
|                     | 8               | 34                                     | ·031     | 7             | 42                                                 | + ·00115                    |
|                     | 9               | 40                                     | ·039     | 9             | 19                                                 | (average ·00016)            |
|                     | 10              | 48                                     | ...      | ...           | ...                                                | ...                         |

\* A further amount of spurious selection is introduced by the "existing," see pp. 228 and 229.

† Obtained from the table on p. 478 of Whole-Life Unadjusted Experience.

The ratios of the exposed were worked out for other ages, but as they did not differ much from the above it is unnecessary to give them.

The value of  $q' - q$  can only be found approximately, and it varies with the age and term: if we take it as .0022 we shall probably not err very greatly, and this value was used in column (5) above. It is perhaps too large for age 40, and too small for age 50. The figures in col. (5) are actually small, but they represent an error of from 13 per-cent to 50 per-cent in the differences on which selection really depends, and are certainly worth noticing. We must also remember that the graduated figures—on which these percentages are based—do not afford a correct comparison, because, in order to make the select table join the aggregate, Mr. G. F. Hardy overstated the mortality in the durations 8 and 9, and to some extent exaggerated the selection as judged by  $q_{[x-1]+t+1} - q_{[x]+t}$ . It was for this reason that the approximate ungraduated figures were added. Probably the cause of the graduation difficulty was the amalgamation of the Old and New experiences, which made the table, excluding the first ten years of assurance, show a proportionally heavier rate of mortality than the select, because more of the Old business was included. This is shown by the following table, giving the ratios of the exposed in the full aggregate table:—

TABLE III.

| Age | $O^M$<br>$100 \frac{E'_x}{E_x}$ | $O^{NM}$<br>$100 \frac{E'_x}{E_x}$ |
|-----|---------------------------------|------------------------------------|
| 25  | 6                               | 8                                  |
| 35  | 19                              | 19                                 |
| 45  | 49                              | 44                                 |
| 50  | 76                              | 63                                 |
| 55  | 118                             | 87                                 |
| 60  | 178                             | 114                                |
| 65  | 266                             | 152                                |

The above table seems to indicate that the mortality is proportionally much heavier at the end of the table than at the beginning, and perhaps this may be the real cause of the impossibility of finding a Makeham curve to graduate the  $O^M$  Table. This explanation seems more probable than the one originally given, namely, that, as the end of the curve is common to both the  $O^M$  and  $O^{M(5)}$  Tables, it is unreasonable to expect a Makeham curve



to fit both ; but surely the only thing we can decide is that an identical equation cannot be used for both. I cannot see the *a priori* impossibility of finding two Makeham curves, having different constants, and giving values which differ widely at the start, and then converging sufficiently to give identical values for the expected deaths at the end of the tables.\* Is it theoretically necessary for an aggregate table like the  $O^M$  and a table like the  $O^{M(5)}$  to be exactly equal after a certain age ?

We may now turn to a rather interesting point in connection with the Mortality Investigation, which may help to bring out my difficulty in following the method. The observations beyond age 80 obtained from the "New" policies were very few, and the mortality at the end of the table could not therefore be found without using the "Old" policies. This was presumably why they were used, and the practical effect is that a method closely analogous to that used in census statistics has crept into the recent investigation. The "Old" policies in 1863 (*i.e.*, "Old" entrants) and the "New" business in 1863, taken together, closely represent the "population" in assurance offices at their policy anniversaries (1863), and the existing at the end of the experience (1893) represent the "population" thirty years later ; the intermediate populations are not known, but the exposed to risk may be considered from our present point of view as a very accurate (practically exact) method of obtaining the average population corresponding to the deaths in the thirty years. This means that, if the "existing" at the end of the observations and the "entrants" at the beginning were subject to the same rates of mortality, the eccentricities of the select tables would not be noticeable (the mortality rates being constant). I do not wish to imply that a modified census method is necessarily unsound, at any rate in aggregate tables ; but if it is used, the term of the investigation should, I think, be short. A method sufficiently accurate for an aggregate table may, however, be unsuitable for a select table.

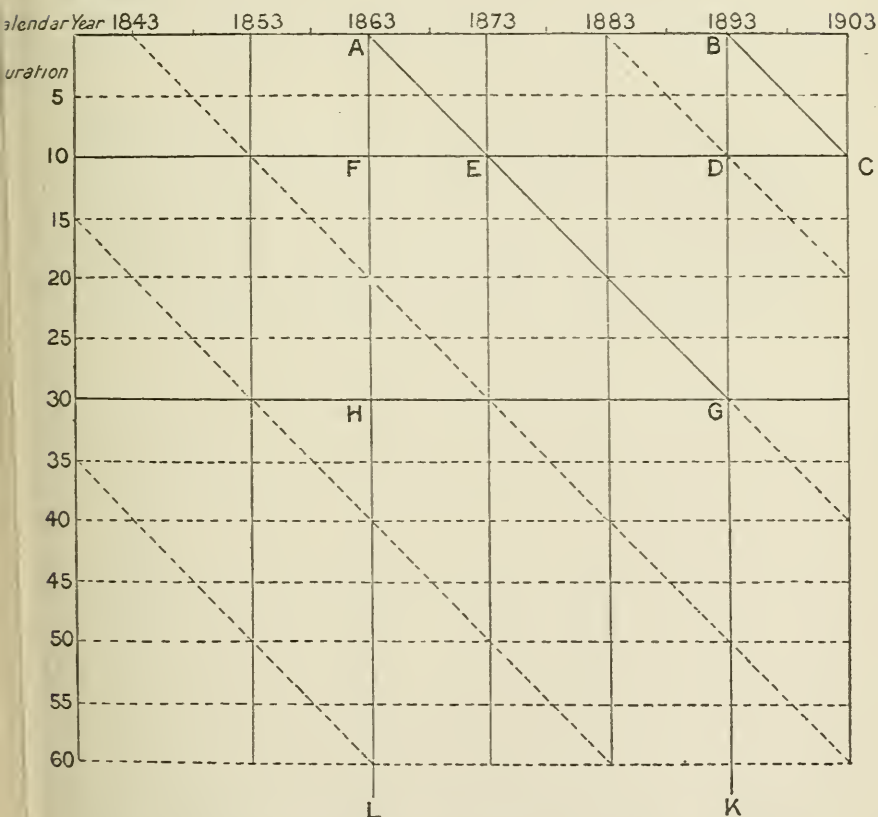
We have not, however, yet exhausted the amount of spurious selection that may arise when mortality tables are amalgamated;† but, before proceeding to deal with some of the remaining points, the exact nature of the problem must be examined.

\* See p. 310.

† It is interesting to note that the use of nearest age at entry has a slight effect on select mortality and after attained age 35 tends to exaggerate the selection. The error is, however, extremely small ; it only affects the fifth place in  $qx$  when five years of age are grouped together as in the table on p. 478 of the whole-life experience.

An investigation into select mortality is an attempt to estimate the rates of mortality shown by policies, according to the number of years they have been in force. If, therefore, the observations end in 1893 (say) and select tables are worked out for 10 years after entry, only those cases which entered before 1883 should be included. In other words, in the recent investigation it would be necessary to take "New" assurances plus an experience formed from the "existing" at the close of the observations, in order to estimate selection, for we do not know what effect on the select mortality has resulted from ending the experience at a certain date.

The following diagram, representing cases included in investigations for any attained age, may help to explain the position:



The recent investigation took all cases represented by A B K L into account; A G K L represents the "Old" assurances and

A B G the "New" assurances. The select tables for the first 10 years of assurance were based on the experience represented by A B D F, while the experience with which we now wish to deal is A B C E.

The effect of the "existing" on selection can be measured by our formulæ, but, in order to avail ourselves of them, we have to arrange our material in two experiences, and then see if an amalgamation could affect the result, and in what direction. That is, an imaginary table of exposed must be formed, by continuing the "existing" as if the experience had not ended at a fixed date, and the resulting figures must be compared with the exposed shown in the published tables. In using our formulæ, it must be remembered that the experience in this case having no selection is the combination, and that showing spurious selection either part of it. The amalgamated table would be an experience in which there were no "existing."

In order to give some idea of the result, I calculated roughly an exposed to risk by continuing the "existing" among "New" assurances, as if they had been a separate experience; deductions for withdrawals and deaths were made from the "existing" at ages at entry 30 to 40, and the percentages of the resulting exposed to that of the O<sup>M</sup> "New" policies were calculated for attained age 40. I made a similar calculation for the withdrawals, and the following table shows both results :

TABLE IV.

| Duration | "Withdrawals" Exposed<br>÷<br>Exposed (New Assurances) | "Existing" Exposed<br>÷<br>Exposed (New Assurances) | "Existing" Exposed<br>÷<br>Exposed (Old + New)* |
|----------|--------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------|
| 0        | 1%                                                     | Nil                                                 | Nil                                             |
| 1        | 8                                                      | 3%                                                  | 3%                                              |
| 2        | 14                                                     | 7                                                   | 6                                               |
| 3        | 18                                                     | 10                                                  | 9                                               |
| 4        | 22                                                     | 14                                                  | 12                                              |
| 5        | 25                                                     | 18                                                  | 16                                              |
| 6        | 29                                                     | 22                                                  | 18                                              |
| 7        | 31                                                     | 28                                                  | 22                                              |
| 8        | 35                                                     | 33                                                  | 25                                              |
| 9        | 39                                                     | 39                                                  | 30                                              |
| 10       | 42                                                     | 47                                                  | 34                                              |

\* This column shows the effect of the "existing" on the select tables, on which the monetary tables are based; the two previous columns show the effect on the "New" part only.

These ratios are increasing, so our formulæ tell us that in each case the selection will appear to last longer or shorter than it really does, according as the "existing" (or withdrawals) are subject to a lighter or heavier mortality than that shown by the "New" assurances. Now there is little doubt that the "existing" have a very light mortality, so the duration of selection probably appears longer than it really is, owing to the mortality being lighter at the end of the period included in the recent experience than it was during that period.

Assuming that the difference between rates of mortality shown by the "New" assurances and the "existing" is less than the difference between the rates shown by the "Old" and "New", the following table has been formed to give a rough idea of the total amount of spurious selection in the O<sup>M</sup> Table:

TABLE V.

| Attained Age | Duration | SPURIOUS SELECTION        |                                                   |                                                   |
|--------------|----------|---------------------------|---------------------------------------------------|---------------------------------------------------|
|              |          | Owing to "Old Assurances" | Owing to "existing" and "Old"                     |                                                   |
|              |          |                           | Effect of "existing" taken as $\frac{1}{2}$ "Old" | Effect of "existing" taken as $\frac{1}{2}$ "Old" |
| (1)          | (2)      | (3)                       | (4)                                               | (5)                                               |
| 40           | 5        | ·00006                    | ·00008                                            | ·00009                                            |
|              | 6        | 5                         | 7                                                 | 8                                                 |
|              | 7        | 6                         | 8                                                 | 9                                                 |
|              | 8        | 6                         | 8                                                 | 9                                                 |
|              | 9        | 7                         | 9                                                 | 10                                                |
| 50           | 5        | ·00007                    | ·00009                                            | ·00010                                            |
|              | 6        | 8                         | 11                                                | 12                                                |
|              | 7        | 6                         | 8                                                 | 9                                                 |
|              | 8        | 7                         | 9                                                 | 10                                                |
|              | 9        | 9                         | 12                                                | 13                                                |

If we bear in mind that the average ungraduated differences with which these figures can be compared are ·00017 and ·00016 at ages 40 and 50 respectively, we can see that the amount of selection has probably been exaggerated. The withdrawals will also have some effect, unless their mortality is exactly the same as that of the main part of the experience; but, as it is impossible to tell even whether the rates of mortality are heavier or lighter among withdrawals than among the rest of the business, I did not think it worth while to calculate the percentage when "Old" and "New" policies are combined. It may be noticed, however, that if, as is generally supposed, the withdrawals have a lighter

mortality than that prevailing in the rest of the business, selection is again overstated. The lighter mortality of the "existing" very probably accounts to some extent for the long continuance of selection in the Endowment Assurance experience. On the other hand, with regard to the short duration of selection in the without-profit experience, I find that up to age 40 at entry the "Old" and "New" policies do not show a very different mortality for durations 5-10, and after age 40 the proportions of "Old" and "New" business in the aggregate tables are smaller than in the with-profit experience (*see* Table III), which suggests that the number exposed might possibly be mixed in the aggregate table in such a way as to cause the select table to pass too soon into the aggregate (*cf.* our first example). It must also be remembered that the "Old" and "New" rates, so far as can be judged by the aggregate tables, do not differ quite so much as the corresponding with-profit rates.

Up to the present it has been assumed that the experience represented by A B C E would give a true measure of selection in an experience, but a distortion may occur, unless the mortality rates in the community from which the experience is drawn have not fluctuated. To put the matter in another way, we have assumed that heterogeneity can only be introduced through the time of entry of assurance, but it can also be brought in through the time of existence of assurance; heterogeneity is, in fact, a function of both the calendar year of entry and the calendar years passed through. In order to make a further step in the examination of the problem, we must therefore take into account the effect of variations in the rate of mortality, exhibited by the general population during the period covered by the investigation. A very simple case may show how the distortion in select rates might arise:

TABLE VI.  
*Table of Exposed to Risk at Attained Age  $x$ .*

| Duration. | CALENDAR YEAR         |                           |                           |                           |                           |                           |
|-----------|-----------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
|           | $1800 + n$<br>(·0084) | $1800 + n + 1$<br>(·0083) | $1800 + n + 2$<br>(·0082) | $1800 + n + 3$<br>(·0081) | $1800 + n + 4$<br>(·0080) | $1800 + n + 5$<br>(·0079) |
| $t$       | 1000                  | 1100                      | 1200                      | 1300                      | ...                       | ...                       |
| $t + 1$   | ...                   | 900                       | 1000                      | 1100                      | 1200                      | ...                       |
| $t + 2$   | ...                   | ...                       | 800                       | 900                       | 1000                      | 1100                      |

NOTE.—The rate of mortality for the year is given below the year in brackets.



In this case, if there is no selection, the rate of mortality at duration  $t$  is obviously—

$$\frac{\cdot0084 \times 1000 + \cdot0083 \times 1100 + \cdot0082 \times 1200 + \cdot0081 \times 1300}{1000 + 1100 + 1200 + 1300} = \cdot00825$$

Similarly, for duration  $t+1$  the rate is  $\cdot00815$ .

„ „ „  $t+2$  „  $\cdot00805$ .

This indicates that an approximate estimate of the increase or decrease in selection, owing to an increase or decrease in the population rates of mortality during the period covered by the investigations, would be the annual alteration in those rates.

The following table of annual death rates per 1,000 of the population is taken from p. xciv of the Sixty-fifth Annual Report of the Registrar-General on Births, Deaths, and Marriages.

TABLE VII.

| Period      | Age 35 | Age 45 | Age 55 |
|-------------|--------|--------|--------|
| 1861-65     | 13·2   | 18·9   | 32·8   |
| 66-70       | 13·8   | 19·6   | 33·5   |
| 71-75       | 14·3   | 20·3   | 34·8   |
| 76-80       | 13·4   | 19·8   | 34·9   |
| 81-85       | 12·8   | 19·3   | 34·2   |
| 86-90       | 12·0   | 19·4   | 35·2   |
| 91-95       | 12·0   | 19·6   | 35·9   |
| Decrease .  | + 1·2  | -·7    | -3·1   |
| Per annum . | +·04   | -·02   | -·10   |

Mr. King's remarks in the discussion on Mr. M. D. Grant's paper on "Canadian Vital Statistics," should, however, be borne in mind in considering the table (*see* p. 148 of the present volume of the *Journal*).

This table seems to indicate that "attained calendar year" was not a very important item; but even if it were in some particular case worth consideration, I am doubtful whether it should be eliminated, except, perhaps, for purposes of comparison with other tables. We merely want, for practical work, to secure homogeneity as regards time at entry, for we assume that the sequence of events that held in the past will continue in the future, and part of that sequence is change in population rates.

I fear that the above examples do something to discount the value of the recent investigation as evidence of selection: if

spurious selection *may* be present in a table, one begins to doubt the evidence it affords as regards selection. Incidentally, I may remark that there does not seem very strong evidence in the ungraduated rates that the "New" experience is subject to selection for more than eight years; but in ungraduated form judgment is impossible without a very lengthy investigation, which I have not been able to make.

If select mortality can be affected to so great an extent by heterogeneous material, is it wise to lay much stress on valuation by select tables? The subject of valuation has been brought forward, and ingenious methods suggested by Messrs. King, Ackland and Diver, but, in view of the above criticisms, I have, personally, some doubt if the time has yet come for departing from the aggregate table in valuation work, even if other arguments against select table valuations are neglected.

It can, of course, be urged that the "Old" mortality gradually changed into the "New", and that the heterogeneity is not so serious as it seems; but it is not difficult to see that if the number exposed is gradually increasing, and the rate of mortality decreasing, the same distortions in the duration of selection occur as those we have already noticed. As a final example, we shall deal with this point. The following table gives what is supposed to be the same series of exposed as that given in Table VI; but while in that table we were dealing with an experience like A B C E, we are now dealing with one like A B D F. (*See* Diagram, p. 227.) In Table VI, we assumed that the mortality rates varied with the calendar year actually being passed through. In Table VIII, we assume that the rates vary with the calendar year of entry.

TABLE VIII.  
*Table of Exposed to Risk at Attained Age x.*

| Duration | CALENDAR YEAR OF ENTRY.   |                           |                       |                           |                           |                           | Rate of Mortality which would be shown by a Select Table |
|----------|---------------------------|---------------------------|-----------------------|---------------------------|---------------------------|---------------------------|----------------------------------------------------------|
|          | $1800 + n - 2$<br>(·0086) | $1800 + n - 1$<br>(·0084) | $1800 + n$<br>(·0083) | $1800 + n + 1$<br>(·0082) | $1800 + n + 2$<br>(·0081) | $1800 + n + 3$<br>(·0080) |                                                          |
| $t$      | ...                       | ...                       | 1,000                 | 1,100                     | 1,200                     | 1,300                     | ·00815                                                   |
| $t - 1$  | ...                       | 800                       | 900                   | 1,000                     | 1,100                     | ...                       | ·00825                                                   |
| $t - 2$  | 650                       | 700                       | 800                   | 900                       | ...                       | ...                       | ·00836                                                   |

NOTE.—The rate of mortality for the year of entry is given below the year in brackets.

A difficulty that seems likely to occur in examining these tables is, that it is generally assumed that the whole of the selection exercised at entry wears off. This need not be the case. It is possible that assurance offices have gradually learnt how to choose those lives which are permanently better than the average, owing to healthier occupations, or better personal or family history. The selection that wears off may be that which is purely medical; recently examined lives are known to be physically healthy, but it does not necessarily follow that they have constitutions which will be likely to show longevity. Mathematically, selection may be of the form  $A + \phi(xt)$  where  $A$  is a constant and  $\phi(xt)$  depends on the attained age and duration elapsed since the date of the assurance.

We may conclude by summarizing the results at which we have arrived :

- (1) Spurious selection may arise from the amalgamation of two tables.
- (2) The fact that select tables do, or do not, run into an aggregate table after a certain number of years, is not a sufficient criterion that selection has, or has not, ceased.
- (3) The "existing" and "withdrawals" may also be causes of spurious selection.
- (4) With regard to the  $O^{[M]}$  Table, spurious selection is probably present to a quite appreciable extent, and if the amounts of it due to the "Old" assurances and the "existing" are both considered, it is likely that selection does not last more than seven years.
- (5) Variations in population rates may have to be considered, but they have probably not affected the  $O^M$  Tables to any great extent.

These views, which were impressed on me while working on a different line (graduation), are not put forward in a controversial spirit, for I fully appreciate that there may be other influences at work besides those I have investigated, though

I confess that my search for them has, so far, been fruitless. I merely wish to suggest that heterogeneity in life assurance data is worth more consideration than it has yet received, and that the possible existence of spurious selection, even in carefully constructed tables, shows how much care is needed in dealing with such a delicate subject as select mortality, and that a study of the correlation between rates of mortality, and calendar years of entry, and current calendar years, respectively, might lead to very important and interesting results.

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#### ABSTRACT OF THE DISCUSSION.

MR. GEORGE GREEN, M.A., said the subject of selection was one which had created great interest in the past, and one on which many different opinions had been expressed. It made its appearance so long ago as 1849 in the first volume of the *Journal* (p. 179), and it was evident from the paper under discussion that the last word had not yet been spoken on the subject; in fact, the more the question was looked into the greater seemed the difficulties in the way of reaching finality. The author of the paper had first put forward, in a very elegant form, a theorem with which he thought all the members could agree, namely, that if two distinct mortality experiences with different rates of mortality were amalgamated, by that process it was possible to introduce the appearance of selection where none really existed, or destroy the evidence where it did exist. But when he sought to apply the theorem, there seemed to be considerable difficulty in determining what aspect of the subject of selection the author wished to put forward. Sometimes it seemed as though it was to be an analysis of what he might call the strictly scientific methods of tracing selection; sometimes it had the appearance of a criticism of the practical methods of dealing with selection for the purposes of constructing tables of mortality for general use in life assurance business. Owing to the great importance of the new  $O^M$  Tables, he thought the latter aspect was of greater interest, though he recognized the necessity of the former, if a complete grasp of the subject was to be obtained. One of the first statements found in the paper was that in the  $O^M$  Table there was an example of an amalgamation of Old assurances and New assurances, and that the former showed far heavier rates of mortality than the latter. They were not told what proofs there were of that statement, and he had tried in the time at his disposal to test it in some way. To do that, he added together the values of  $q_{[x]+t}$ , unadjusted, for all values of  $x$  from 25 to 44, for each value of  $t$  from 0 to 14 inclusive for the Old assurances, New assurances and combined Old and New assurances, and also the corresponding values for the  $O^{[M]}$  Table as graduated by Mr. Hardy. The results seemed fully to bear out the author's statement, the figures being as follows:—

*Rates of Mortality O<sup>M</sup>.*

$$10^5 \sum_{x=25}^{x=44} q_{(x+t)}$$

| <i>t</i> | Old<br>Assurances. | Graduated<br>Tables. | New<br>Assurances. | Combined<br>Old and New<br>Assurances. |
|----------|--------------------|----------------------|--------------------|----------------------------------------|
| 0        | —                  | 7,469                | 7,601              | 7,601                                  |
| 1        | 9,642              | 11,576               | 11,115             | 11,068                                 |
| 2        | 15,321             | 13,472               | 13,281             | 13,402                                 |
| 3        | 16,751             | 14,879               | 14,504             | 14,684                                 |
| 4        | 18,901             | 16,212               | 15,092             | 15,545                                 |
| 5        | 20,014             | 17,572               | 17,057             | 17,501                                 |
| 6        | 21,238             | 18,998               | 18,940             | 19,349                                 |
| 7        | 23,139             | 20,490               | 20,316             | 20,865                                 |
| 8        | 23,777             | 22,063               | 21,429             | 21,984                                 |
| 9        | 24,926             | 23,723               | 21,606             | 22,475                                 |
| 10       | 25,517             | 25,162               | 23,264             | 23,934                                 |
| 11       | 25,783             | 26,395               | 25,347             | 25,547                                 |
| 12       | 29,008             | 27,742               | 26,823             | 27,524                                 |
| 13       | 32,546             | 29,214               | 27,964             | 29,746                                 |
| 14       | 33,120             | 30,823               | 28,550             | 30,475                                 |
| Total .  | 319,683            | 298,321*             | 292,889            | 301,700                                |
| Ratios . | 107.2              | 305,790†<br>100.0    | 95.8               | 98.7                                   |

\* Excluding  $t=0$ .† Including  $t=0$ .

No doubt a better method would have been to calculate the expected deaths by means of the graduated values, but time had not been available for such an investigation. A comparison of the aggregate tables, such as was given at the end of the volume of unadjusted data, was quite useless, because in the New, only durations 0—29 were included, whereas in the Old, duration 0 was missing, and the data for short durations were present only in very small proportions. If the full tables as set out were taken, they would thus be comparing one table in which selection was present in its full force with a table in which selection had, to a large extent, been excluded. On what grounds were they to distinguish between the Old and New? The New experience itself was surely an amalgamation, in the sense in which the author used the phrase, of very many experiences, namely, the lives selected, say, in 1863, in 1864, and so on. He did not quite follow on what grounds they should draw the line so sharply between 1862 and 1863, and say that the one must be excluded, and the other included. He thought the fair statement of what had been done was that all the whole-life policies, with profits, that were in force between 1863 and 1893 were regarded as forming part of the experience they wished to tabulate. No doubt the ideal method of tracing selection would be to have a complete experience of lives, all examined on the same day, and observed from that day to the end of life; but even then they would have to go so far back that the results when obtained would be out of date. It



was an obvious necessity that the Old and New assurances should be combined for practical purposes, in order to obtain sufficient data for the early years of assurance, and also for the later years of life. Had the  $O^M$  and  $O^{NM}$  experiences been amalgamated, he thought there would have been much more justification for criticism, because those experiences showed different rates of mortality, and there would have been nothing gained in combining them.

Turning now to the Select Tables, the author stated that probably it was necessary to increase the rates of mortality at durations 8 and 9 in their graduation because of the combination of the Old and New assurances. He thought that statement overlooked a somewhat important point. In his account of the graduation of those tables, Mr. Hardy had pointed out that the duration of selection varied very greatly according to the age at entry. Had he been seeking to graduate the tables merely for the purpose of studying selection, it was impossible to suppose that Mr. Hardy would have chosen ten years as the fixed period during which selection was assumed to be operative. That period was chosen for practical reasons, and it seemed to be only what one might expect that the junction of the select and aggregate rates of mortality could only be made smooth by raising rates here and depressing them there, though it was not found necessary to do so to any large extent. The next point was the suggestion that the real cause of the impossibility of finding a Makeham curve for the  $O^M$  Table was the combination of the Old and New assurances. In the first place, was it proved that all tables of mortality could be graduated by means of Makeham's formula? Did not that tacitly assume that there was a definite law of mortality, a theory which was surely now discredited? In the case of the Annuitants' Experience, it was found possible to graduate the Male Table but not the Female Table by that formula, though the tables were constructed on identical principles. Further, the  $O^{(M)}$  Table had been graduated in that way, and yet that was a combination of Old and New assurances. Then, again, Mr. Hardy had informed them in his account of the graduation that he found the  $O^M$  Table did not lend itself to graduation by Makeham's formula, apart from the question of being identical with the  $O^{M(5)}$  Table at the higher ages. The next point the author raised was as to a comparison of the combined experience with tables based on population statistics, but was not this exactly what they wanted? The tables were required for use in life assurance work, and they were exactly adapted to their ends. If they wished to study selection only, aggregate tables would, he imagined, have no place at all in the investigation, and the select tables would be graduated separately to their full extent. It was with some surprise that he found the question of aggregate tables brought up so prominently in the paper, as they were at their best only a compromise, though a necessary and satisfactory one for practical uses. The author then further discussed the probable effect on the appearance of selection of the withdrawals and "existing", and pointed out that the "existing" would probably be subject to a lighter rate of mortality than the office population during the period of observation. He was sure they all hoped it

would be so, otherwise their mortality profit would be likely to disappear. He could not, however, quite agree that the "existing" accounted for the long period during which selection was effective in the endowment assurance tables. It was known, from the Annuityants' Experience, that self-selection was a strong influence, and bearing that in mind they should rather account for the three different periods of selection in the  $E^M$ ,  $O^M$ , and  $O^{NM}$  Tables by the fact that in the first the office and the assured exercised selection in the same direction, in the second the assured did not exercise so strong an influence, and in the third the action of the office and the assured was in opposite directions. He would only draw attention to one other point which was put forward in the paper and in the summary of conclusions, namely, that it was likely that selection did not last more than seven years. That statement was, he submitted, not justified, either by what had gone before, or by what they knew of the  $O^{[M]}$  Select Tables. They should surely rather come to the conclusion that the difficulties in tracing selection were too great to be entirely overcome, even if that form of selection which was due to the assured could be eliminated; but that such indications as there were pointed to its being powerful for some years, then less powerful, that its duration was a function of the age at entry and the calendar year of entry, and that, for practical purposes, its influence might be ignored after ten years.

MR. JAMES BACON said that the author discussed a question of theory, with the principles of which perhaps all were in agreement, but introduced also a criticism of the  $O^{[M]}$  Tables, which, if the author's numerical conclusions were accepted, might create a great deal of mental disturbance among the members of the Institute. It was therefore of importance to ascertain whether the author's conclusions as to those tables were wholly justified. Mr. Elderton had given a proof, applicable only to durations where the influence of selection was presumed to have disappeared, that, where there were two tables, one showing a heavier rate of mortality than the other, an amalgamation of the two might produce what he termed spurious selection. The measure of error found for that special case was then applied by the author to the quite different case where it was not agreed that selection had disappeared, that was to say, the example given in the paper dealt with durations 5 to 10; and it might be shown that, unless the influence of selection actually had disappeared at those durations, or that it was identical under both the Old and New tables, the deductions drawn would be to a certain extent vitiated; in fact, that another form of spurious selection might also have occurred which might operate, either in the same direction as that shown by the author, or in the other direction. It appeared to him that, except in the one case where the influence of selection was identical under both Old and New tables, the author's measure of "spurious selection" was not applicable, but there seemed very little evidence that the influence of selection was consistently different under the two tables. He had had access to certain data, where groups of five ages were taken, and the actual unadjusted value of  $(q' - q)$  was obtained, and, the same process being followed as that adopted by the author in constructing Table II, the results from

those unadjusted facts were found to be very irregular. On the whole, however, the spurious selection brought out was not nearly so great as that shown in Table II. Referring to a point on p. 225 as to the difficulty found in graduating the tables, Mr. Hardy stated that there was very little difficulty at the end of ten years in forming a fair junction between the select tables and the combined select experience, excluding the first ten years. He (Mr. Bacon) thought they could hardly expect to effect an easy junction between the select tables and the full aggregate O<sup>M</sup> Table. With reference to Table III, the author said the table seemed to indicate that the mortality was proportionally much heavier at the end of the table than at the beginning. He would like to ask whether that might not be due, not only to an increased percentage of the Old assurances, but also to a decreased percentage of select data in both New and Old assurances, having durations of less than ten years. He could not understand the statement on p. 229, as to the "existing" having a much lighter rate of mortality than those cases that passed out before the close of observations. He assumed that was due to their being subject to the rates of mortality at the end of 1893, which might perhaps be supposed to be lower than in 1863 or the earlier years of the experience. If that was not so, he should be glad to hear any other reason. He was afraid that he had not been able to understand Tables VI and VIII, especially as the rates of mortality for the same calendar year were different in the two tables. The author set forth his conclusions at the end, and he thought all could agree with those numbered (1) (2) and (3), and also to a certain extent with conclusion (4), although, from the data he had been able to see, he thought the author had over-estimated the importance which was to be attached to the question of "spurious selection" in the O<sup>M</sup> Tables. With regard to conclusion (5), "Variations in population rates may have to be considered, but they have probably not affected the O<sup>M</sup> Tables to any great extent," it seemed to him that the difference in the mortality between the Old and New Tables would point to the fact that the changes in population rates, at least as regards the "population" included in the life offices, would have had some influence, because, apart from the question of the effect of the "existing", he could see no reason why those changes should take place, unless there had been either changes in the population rates over that period, or changes in the influence of medical selection. There did not seem to be much reason for believing that the latter had had very much effect, and therefore he presumed that the greatest importance was to be attached to the difference in the rates of mortality from 1863 to 1893 of the "population" included in the life offices. The author appeared to bear out that conclusion to a certain extent when he stated at the end of the paper "that a study of the correlation between rates of mortality and calendar years of entry and current calendar years respectively might lead to very important and interesting results."

With regard to the criticism of the experience necessarily involved in the paper, he thought the members ought to ask themselves exactly what the Mortality Committee set out to do. It seemed to



him the primary question was to obtain the rates of mortality experienced from 1863 to 1893 by assured lives, and he took it that no question would be raised as to the accuracy of the results so brought out. But there was another question, which he thought was of secondary importance, namely, that of selection, which was mainly of interest in connection with questions of option and contingent survivorship premiums, and, perhaps, select valuations; and he thought that the author had proved that there might be a certain amount of error in the results shown, looked at from that point of view. Another point raised, which seemed to be of importance, was that the author had shown that the mortality in the Old table, and presumably among the earlier entrants of the New table, was greater than among the New assurances generally. It would therefore appear that the mortality shown by the "Combined" table was higher at practically all ages than would be likely to be experienced in the future, although correctly representing the experience of the last thirty years. That was also brought out from an examination of the full aggregate tables. Unfortunately they had not before them the separate aggregate tables for the Old and New assurances, excluding the first ten years of duration; but, looking at the full aggregate table, it would be found that the ratio of the New to the Old rate of mortality, taking the combined ages 18-22, 23-27, 28-32, &c., commenced at about 72 per cent at central age 20, dropped to 59 per-cent at central age 25, and then rose fairly consistently to 100 per-cent at age 85, whilst the ratio of the expectations of life rose gradually from 94 per-cent at age 20 to 100 per-cent at age 75. Apart from the question of the introduction, and possible unequal distribution, of select data in these two tables, it would thus appear that the mortality at the higher ages tended to be equal under both the Old and the New tables. The question raised in his mind was whether the improvement in mortality, which they were told had taken place at the young ages only (an improvement due probably to the results of sanitary and other science in recent years), had had time to make itself shown in the New tables at the higher ages, and whether in the future it might not be found that, at those ages, the rate of mortality would be appreciably lighter than that shown by the aggregate tables. He hoped some member of the Mortality Committee would be able to throw more light on that question.

MR. T. G. ACKLAND thought the Institute was indebted to the author for introducing a valuable paper to the notice of the members, in which he directed attention to some important principles connected with the compilation of statistical data. But the arguments of the paper did not appear to him to be always fully or clearly stated. No doubt those arguments were well based, and the author's conclusions in most cases fully justified, but they were sometimes not able to judge whether this were so or not, because Mr. Elderton did not give them the benefit of the lines of argument which led to his conclusions. He was glad that the word "spurious" in the title of the paper was not taken in the dictionary sense of meaning "counterfeit, illegitimate, or untrue", but that it was defined in the text of the paper, and in a foot-note, in a special technical sense, which was not perhaps familiar to all present. The general arguments of the paper, with which he

thought they must be in substantial agreement, were that, where there was a mixture of non-homogeneous data, in which the rate of mortality was markedly and consistently higher in one set of data than in the other set of data, the effect of the mixture in select assurance tables, might be materially to disturb the measure of selection, in passing through years of duration; and this more particularly, where the data having the heavier rates of mortality were in increasing proportions, as successive years of duration were passed through. That was a proposition which although it might, on consideration, be deemed self-evident, they were glad to have formally demonstrated by mathematical analysis. He agreed with a previous speaker that, whilst the author had rigidly and satisfactorily, as he thought, proved mathematically that the effect in question took place where selection had expired, he had in no way analyzed the case where selection was still in operation; and it seemed to be questionable to apply his analysis to the case of the years of assurance 5 to 10, where, as he understood, the author himself admitted that selection was in operation, at any rate, for some portion of that period. The case still required to be investigated, as to the precise effect of mixing the data, where selection was still in operation. No doubt there might, even then, be a certain element of spurious selection brought in; but at least there was no proof that it would be quite on the lines of the author's formulas. Mr. Elderton's general proposition was, he thought, a counsel of perfection. He compared the work of the Mortality Committee to the taking-out of a mortality investigation in connection with census returns at two different dates. He supposed the more satisfactory plan, in the case of a population, would be to trace the whole of the population, if it were possible, from birth throughout life until death, and that appeared to be, in effect, what the author desired to attain in the case of an assurance experience. If he understood the author correctly, he would exclude from the experience any data which passed out in the course of the period by any other cause than death. That was to say, if there were a case under observation in the fourth year of the experience, and at the end of that year it ceased to proceed further, and was included amongst the "existing", or if it were withdrawn by lapse at that point, it should be excluded from the experience, because it would not be right to compare it with the reduced data in the fifth year of assurance, which would not include the cases that had thus passed out other than by death. That might possibly be sustainable as a matter of theory, but he was afraid that in practice it would be quite unattainable, and he thought, moreover, that such selective processes were somewhat dangerous. In the compilation of the data in connection with the O<sup>M</sup> Table, great care had to be taken, in considering as to the exclusion of cases of a particular class, to make sure that cases were not excluded from the exposures without excluding the corresponding deaths, and *vice versa*; and he was afraid that, if arbitrary selective principles were brought into practice, they might lead to similar results which would be theoretically indefensible.

The data for the Old and New assurances were combined by the Joint Committee, after very careful consideration of many facts and figures that were laid before them, as to the relative rates of mortality



arising in those two classes of experience; and he also ventured to think that, having invited from the contributing companies the data in reference to cases existing as in 1863, as well as to cases entering in the subsequent thirty years, there was practically no alternative before the Committee to that of combining the data, and thus obtaining the largest available body of facts. If the volume of Unadjusted Data were studied, it would be found that, when the cases came to be analyzed at ages of entry and durations of assurance, with the further separation into participating and non-participating whole-life assurances, the data, even when combined, were very scanty at many points, and that it was rather an exceptional thing to get three figures in the deaths at an individual entry age and duration. He thought that pointed to the conclusion that the only plan to adopt was to combine the data in such a way as to get a good body of facts. It would not, of course, be right to combine them so as to bring in any error, but he ventured to hope that that was not done quite to the extent that the author led them to anticipate. The Committee, rightly as he thought, set out the data fully and separately for the Old and New assurances, but the graduated results were based upon the combined data. He had investigated an important section of the data, including the 35 attained ages from 28 to 62, both inclusive, the 35 ages being chosen so that there were seven quinary groups, with central attained ages, 30, 35, and so on, up to 60. He investigated the experience at these ages, over the first ten years of assurance (omitting the first year, in which New data were alone included) in order to ascertain, first, what proportion of Old and New assurances there were in that considerable body of data, and, secondly, what the relative rates of mortality might be. Summarizing the results, he found that, out of sixty-three cases of duration that were investigated, the mortality of the Old assurances was in excess of that of the New in forty-six cases; in the remaining seventeen, the New mortality was in excess of the Old. So that, to begin with, they were not quite justified in saying that the Old rates were consistently heavier than the New. Then, getting nearer to the author's point, the *differences* between the rates of mortality (representing the effect of selection), preserving the same attained age, but passing down by duration, were investigated in sixty-three cases; and it was found that in twenty-eight of these, the Old selection was greater than the New, and that in thirty-five the New selection was greater than the Old; so that there, again, no very marked consistent characteristic was noticed about the Old rates, as compared with the New. Then, applying the very severe test (referred to by Mr. Green) of taking out the Old exposures, and applying to them at every age and duration, the New unadjusted rates of mortality, he obtained over the first ten years of assurance 4,291 deaths, as compared with 4,799 actual Old deaths, so that these would have been less, by a little more than 10 per-cent, if the New experience had been in operation over the Old cases. At first sight, this difference of 10 per-cent seemed rather considerable; but allowance had also to be made for the proportions of Old data which entered into the Combined experience. Looking at these proportions, he found, first of all, that in the first year of assurance (as already stated) there were no Old assurances at all, and

in the second year of assurance (over the 35 attained ages dealt with) the Old assurances actually experienced a lighter mortality than the New; so that, in regard to the first two years, the author's point did not arise. In the following years of assurance, the proportion of the Old data included in the Combined varied from 5 per-cent in the third year to 25 per-cent in the tenth year of assurance; so that, in considering the excess mortality of about 10 per-cent, they had also to allow for these varying proportions of the Old data. After giving due weight to these proportions, he found that, in respect of the first ten years of assurance, at all attained ages from 28 to 62 inclusive, instead of 1,000 actual deaths in the Combined data, there would have been 983, if the New rates of mortality had been in operation throughout. The results are shown in detail in the following Table:—

O<sup>[M]</sup> TABLE. UNADJUSTED SELECT DATA.

TABLE showing, in respect of each of the first ten years of assurance, and at the undermentioned ages attained, the number of Deaths (relative to 1,000 Actual Deaths included in the Combined Experience) which would have been included, upon the assumption that the Old Assurances had experienced, throughout, the rates of mortality obtaining, at identical ages and durations, amongst the New Assurances.

| Curtate Duration | EXPECTED DEATHS, DEDUCED FROM GROUPED DATA<br>AT ATTAINED AGES |       |       |       |       |       |       |       | Curtate Duration |
|------------------|----------------------------------------------------------------|-------|-------|-------|-------|-------|-------|-------|------------------|
|                  | 28-32                                                          | 33-37 | 38-42 | 43-47 | 48-52 | 53-57 | 58-62 | 28-62 |                  |
| 0                | 1,000                                                          | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 1,000 | 0                |
| 1                | 998                                                            | 1,007 | 1,001 | 1,011 | 987   | 1,010 | 977   | 1,001 | 1                |
| 2                | 993                                                            | 999   | 985   | 995   | 991   | 985   | 1,006 | 993   | 2                |
| 3                | 963                                                            | 988   | 985   | 1,006 | 1,006 | 1,013 | 1,003 | 992   | 3                |
| 4                | 980                                                            | 961   | 998   | 965   | 991   | 993   | 938   | 977   | 4                |
| 5                | 986                                                            | 976   | 960   | 990   | 961   | 1,016 | 1,001 | 981   | 5                |
| 6                | 958                                                            | 984   | 1,018 | 984   | 958   | 1,025 | 1,008 | 990   | 6                |
| 7                | 901                                                            | 943   | 999   | 969   | 970   | 1,014 | 989   | 970   | 7                |
| 8                | 957                                                            | 972   | 983   | 984   | 960   | 978   | 955   | 972   | 8                |
| 9                | 992                                                            | 946   | 951   | 1,002 | 942   | 983   | 1,009 | 972   | 9                |
| 0-9              | 978                                                            | 976   | 986   | 989   | 972   | 1,001 | 989   | 983   | 0-9              |

He thought all would agree that those results pointed to the fact that the effect of mixing the Old and New data was, on the whole, insignificant, as regards the resulting rates of mortality, and also as to any spurious or unreal selection that might be brought in. Referring to Table II, he had not been able to find out why the author took an average value for the difference in the Old and New rates of mortality, which he set out at .0022. He (Mr. Ackland) preferred to take the actual value of the difference in the rates of mortality, appropriate to each particular attained age and duration,

and he found that the change very materially affected the author's results, so much so that he had no hesitation in saying that at age 40 the value of the error was greatly exaggerated, and probably stated at double its true value; although at age 50 the author's figures were in closer agreement with his own. Instead of the average value of  $\cdot 0022$ , he (Mr. Ackland) obtained at age 40 a value of  $\cdot 001$ , and at age 50 a value of  $\cdot 0027$ . If it were desired to deduce a constant or average value for the difference in the rates at a given attained age in respect of all durations, he submitted that it should be obtained by comparing the rates of mortality at ages 40 and 50, as deduced from the Old and New ultimate Tables, excluding the first ten years' experience. If that process were followed the difference came out at  $\cdot 00093$  at age 40, and  $\cdot 00213$  at age 50. The result of his own investigation, taking the true values of the differences of the rates of mortality, was that no definite conclusion could be deduced, as to any selection improperly introduced. The results varied according to sign, the minus sign indicating that selection was under-stated, while the plus quantities, which were very minute indeed, never exceeding  $\cdot 00014$ , indicated in other cases excess selection. If the ultimate table were taken, in comparing the Old and New experience over the same range of ages, 28 to 62, it would be found that, in durations exceeding 10 years, the Old assurances, tested by the New rate of mortality, would bring out 927 deaths instead of 1,000 actual deaths. So that there was a rather material difference in the ultimate table; and he ventured to think that, if there was a point of criticism which could fairly be launched against the O<sup>[M]</sup> Table, it was in respect of the ultimate table, and the fact that the Old assurances were included in an increasing proportion as age advanced, and made a somewhat material difference in the rate of mortality. He might finally refer to a footnote on page 226, relative to nearest age at entry, in which conclusions were stated which appeared to be of some importance, but the arguments and justifications for which were not stated by the author.

After some remarks from Mr. H. P. CALDERON,

Mr. S. G. WARNER referred to the extreme difficulty and complexity of the subject of selection. It was exceedingly difficult to form a satisfactory theory on the subject, which was, of course, an important one, because obviously selection existed, and had to be reckoned with. But it was more difficult apparently in this case than in almost any other connected with actuarial science, to approximate, by the only methods which could be used, to the ideal result; which could only be obtained by having under observation a number of lives entering at the same age, and tracing them throughout their entire subsequent experience to see precisely what effect the medical selection exercised had upon their mortality. The same kind of difficulty arose elsewhere; with regard, for instance, to population statistics, where a life table could not be constructed by observing, in Dr. Farr's famous phrase, "a generation of men passing through time"; but it seemed possible to proceed more satisfactorily in that case. He would like to understand a little more clearly what the author meant by the "amalgamation" to which he referred. Some interesting remarks were made by Mr. Bacon on the subject of



improvement in general longevity, and its incidence. He thought statistical investigation showed that, as Mr. Bacon supposed, the improvement was mainly in the younger years of life, due to sanitary regulations prolonging many childish lives which would otherwise have died earlier—prolonging them, perhaps, not for very many years, but sufficiently to have an appreciable effect on the general body of statistics. It remained to be seen how far those improving tendencies would prolong themselves into the later years of life. At present he thought it was pretty clear they had not done so. Select tables were probably most used for quoting competitive rates; it was desirable, therefore, that any defects which they might possibly possess should be brought forward and carefully considered. It seemed to him, on the whole, that even if the worst the paper suggested were granted, viz., that selection might only exist for seven years instead of ten, on the facts embodied in the tables, probably that was not of such very great importance in view of the general, steady, slow but sure improvement in the entire mortality of the community. When it was borne in mind how far back many of the facts embodied in the tables went, and the improvement just mentioned was considered, he thought they might continue to use the tables, as the continuous improvement in general mortality would tend to neutralize such small over-statement of the effect of selection as might exist. In that connection it was interesting to note Mr. Elderton's suggestion that the selection function might take the form of a constant plus a variable. They might find salvation in the constant. Mr. Elderton had probably made a criticism of permanent value in pointing out that the method necessary to the construction of assurance experience tables took no note of the lapse of time, but proceeded on the implicit assumption that all entries at the same age were synchronous. When it was considered that the experience forming the basis of the New tables might include as extremes a man aged 80 in 1863, and a child aged 10 in 1893, it was impossible to deny that the criticism might have much force.

The PRESIDENT then proposed a hearty vote of thanks to Mr. Elderton for his interesting and valuable essay, which was put, and carried with acclamation.

MR. ELDERTON, in reply, said that nothing was further from his thoughts in writing the paper than the desire to criticize. He wrote the paper with the intention of bringing out a point closely analogous to an interesting statistical point that had been dealt with before. He found the subject interesting, and hoped it might interest others. That was the sole object he had in view. It was, therefore, extremely gratifying to him to find that every speaker agreed with his main conclusions, which were numbered (1), (2) and (3) in the summary at the end of the paper. He merely mentioned the question of the graduation, because it was when considering graduation that he first came across spurious selection. Mr. Green mentioned, in connection with one of the examples, that the New assurances in the amalgamation were themselves an amalgamation, and seemed to think that his (Mr. Elderton's) argument was in some way affected. He had, however, dealt with the question of gradual change in the rates of mortality according to year of entry, and he

thought that his treatment of this point really answered practically the whole of Mr. Green's remarks and criticism. With regard to Mr. Bacon's remarks on the changes of population-rates of mortality in the community, he thought the point to be discussed was whether the rates were more closely correlated with the year of entry or with the attained year. In reference to the query by the same speaker, as to whether, when selection was known to be still in operation, there was any spurious selection introduced in a mixture of tables, he believed that his investigations pointed to the conclusion that there was. Mr. Ackland had pointed out that his arguments were not always clearly stated. He was quite prepared to plead guilty to having omitted some steps in his argument, and was afraid that when one knew exactly what one wished to express, one was a little apt to overlook that other people had not been looking at the things exactly on the same line. With regard to Mr. Ackland's remarks in connection with the word "spurious", it was naturally used in the statistical and not in the dictionary sense. He was very glad that Mr. Ackland agreed with his main propositions, and, as Mr. Ackland remarked, the suggestions he urged were a counsel of perfection; a fact he perfectly well recognized. With regard to the point as to whether withdrawals should be excluded, in trying to work out the ideal result, he did not think their exclusion possible, even though he knew perfectly well that they might introduce spurious selection. That fact simply emphasized Mr. Warner's point, that the subject was an extremely complex one, and that the more one thought of it the more difficult it seemed to become. Mr. Ackland remarked that his (the author's) value of  $q' - q$  for attained age 40 was double what it ought to be. He was very sorry if that was so, but he did not take attained age 40 alone in fixing that figure. He took the neighbouring ages so as to get an idea of the general trend of the results, because he wanted to give an example of the theory at which he was working, not a criticism of a particular table. The reason he put the footnote on page 226 was that he thought it was one of those causes operating the other way, to which he referred at the end of the paper. There seemed to be a difficulty, to which both Mr. Green and Mr. Warner referred, in regard to his remarks as to following the "existing." They seemed to think that if one followed the "existing" it was necessary to start with a large number of calendar years of entry, and follow this large experience through its whole duration. He did not see the necessity of doing that. In census work they took a ten-year interval, and if they desired to measure the effect of selection by the ideal process they would not take a much longer period. They might want a long period for other purposes, but to measure the exact effect of medical selection such a long period ought not to be taken. The recent experience was got out for use in valuations, and for other purposes, such as deducing premium rates; and a method that might be useful for some might be unsatisfactory for other purposes. The exact circumstances in which the table was going to be used afterwards made a considerable difference to the method employed. For instance, in census work, if a large number of children were taken at birth, and followed throughout life, a table would be obtained which might be



useful for the purpose for which Professor Pearson used the  $d_x$  column in his investigation of the English Life Table (Males) No. 4 (see Mr. G. F. Hardy's review, *J.I.A.*, vol. xxxiii, p. 535). For most purposes it was, however, much more reasonable to use the method adopted in practice, and it seemed to him that the same argument applied to life office mortality tables. In reply to Mr. Warner's enquiry as to what he meant precisely by the amalgamation of the Old and New data, he might explain that he found two tables which had different rates of mortality, and employed them as an example in his paper; afterwards showing that if there was a gradual change, it would have, practically speaking, the same effect as the amalgamation of the two main bodies of data. Mr. Warner's remarks as to the great lapse of time was the very point to which he had previously referred.

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For further remarks by Mr. Elderton on the discussion, see pp. 309-312.

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*Some Aspects of Registration of Title to Land.* By JAMES ROBERT HART, F.I.A., of the Pelican and British Empire Life Office.

[Read before the Institute, 26 March 1906.]

**Introduction.** OUR English system of registration of title to land has been actively discussed and severely criticized among certain professions, but certainly cannot be said to have attracted general interest. The subject is too often regarded as a purely legal one. Yet, looking to the fact that London properties of enormous value in the aggregate have already been affected by the drastic changes introduced by the Land Transfer Act, 1897, and that the whole country may sooner or later be brought within their influence, it is difficult, I think, to overrate the public importance of the question. Just at present, particularly, there is call for a more widespread acquaintance with the principles and working of the system. In the first place, the general public is left responsible for its extension, inasmuch as the popular veto on its application to any particular area is to be exercised through the County Councils. Secondly, the system in respect of many of its details is on trial. From time to time modifications of the original scheme are introduced, as practical experience of its working shows them to be really necessary; and where the system causes inconvenience, adjustments are being made to remove it. The opportunity therefore should not be lost of influencing these changes in the direction of meeting the

requirements of any special section of business. Then, again, strenuous efforts are being made by opponents of the system for the reversal of what they consider a huge mistake. Even if this is not effected, the application of the system may be kept within such limits as seriously to diminish its value; and it is therefore important to understand the reasons for this opposition, and to be able to weigh the arguments put forward. Further, there seems to me great objection to leaving to the legal profession most of the activity in regard to registration of title. Such has been the situation up to the present. From the nature of English land tenure, the design of the scheme and the variations it has undergone have chiefly, if not entirely, been due to lawyers; and that there is a danger that registration may fail to effect many of the objects desired, so long as their point of view is so predominant, is, I think, clearly shown by the history of the subject both here and elsewhere.

In submitting the following notes to the Institute of Actuaries, it would appear unnecessary to emphasize the attractions of the ideal conditions aimed at by registration of title. This is to reduce dealings with land to the simplicity of those with other classes of property, and obviate the delays and expense under the old system of conveyancing. This practical end we can appreciate as business men, while the reasonable methods by which it is sought to be attained will doubtless appeal to our scientific turn of mind. But there are other reasons why we, as actuaries, should give attention to the subject. We have, or are supposed to have, an elementary knowledge of legal principles. From some points of view, this may be a danger; but it should at any rate bring us into closer touch with the present situation than the vast majority of the public. Further, when we remember that British life offices have many millions of their funds invested in mortgages and real property, it may, I think, be claimed that we are very practically concerned with registration of title.

A brief reference to the principal features of two systems in force in other countries and the causes that have facilitated dealings with land under them, not only bears on some of the points of controversy to which the English Act has given rise, but shows the direction in which we should move, if we are to improve on the present conditions. The Scottish Deed Registry and the Australian Torrens system of registration of title have both stood the test of many years' experience and are remarkable for the

success and public appreciation with which they have been worked. It is well, perhaps, to emphasize the chief difference between a register of deeds and a register of title, as sometimes there is confusion on the point. The first is a public record of dealings, and the second of ownership. Our Middlesex and Yorkshire registries are examples of the former, but as they appear to fall far below the Scottish Deed Registry in efficiency, I propose to confine my remarks to the last mentioned.

The Scottish Deed Registry. The main features of the Scottish Deed Registry are (1) the necessity for registration of every conveyance or charge before ownership or incumbrance can be completed, and (2) a search sheet to assist investigation of title. While the entry on the register does not secure a Government guarantee of title, it practically gives what has been described as a negative warranty, inasmuch as it declares that any deed outside the register is not of equal validity. No memorial of a deed is required, as in our Middlesex Registry, but a full copy is taken, thus making the register a complete record of title.

Though under some circumstances such a registration system would go very little way to effect the main objects of registration of title, yet under the conditions under which the Scottish Deed Registry is worked, the benefits of the former are to a great extent secured. Indeed, the Scottish system has been referred to as a half-way house to registration of title. The search sheets take the place of an Abstract; and an inspection of them and the register often gives in five minutes all the information required as to title. When the Keeper of the Register records a conveyance, he practically gives the owner what under our new English system is called a Possessory Certificate. It is true that title has to be re-investigated on each dealing, but this is a simpler process than with us. At the same time, it must not be forgotten that the facilities and protection afforded by the Scottish Registry are greatly assisted by the conditions of the land laws in Scotland, where the drawbacks of the feudal system do not prevail to the same extent as in England. One effect of the registry is that fraud in dealings with land is almost unknown.

Whether with conditions which go far towards registration of title, the complete change to that system should be made in Scotland has been considered by the legal profession. There seems no doubt that under the circumstances such a change would not involve so much disturbance as in our case. In many instances Absolute titles could be registered at once without

difficulty. And we have the statement of the Keeper of the Register himself that when it is clearly shown that such a change is desired, the existing Register House will not offer any obstacles. Up to the present, however, the legal profession, at any rate, seem to have come to the conclusion that the resulting advantages would not be sufficient compensation for the cost and trouble; and the Scottish Deed Registry will therefore probably remain for many years an admirable example of what an accurate and complete system of deed registration can accomplish.\*

The Australian  
Torrens  
System.

Registration of title in England is never criticized without some reference to the Australian Torrens system. In fact, no satisfactory study of the former can be made without such reference; for, in the controversies aroused by the introduction and extension of the system here, its striking success in the Colonies has been one of the chief arguments used to promote its favourable reception.

Briefly stated, the main principles of the Australian Torrens system are (1) a Government Registry with Government guarantee of title; (2) compulsory registration of all land granted by the Crown after the Act, and voluntary registration for earlier grants; (3) the registration of indefeasible or Absolute titles only; and (4) the restriction of dealings to those on the register.

The simplicity of Crown grants doubtless assisted the introduction of registration as well as its subsequent extension. A grant could be at once put on the register without any investigation of title, and at a minimum cost. Moreover, many of the titles voluntarily put on the register would also be of a fairly simple nature, as the time elapsed since the grant would often be short. And in facilitating the issue of Certificates of Absolute title only, the provision for an assurance fund, out of which claims arising from mistakes or fraud could be paid, was an important feature of the scheme.

While there is very scanty evidence as to the drawbacks of the Torrens system, which has been worked for fifty years, it would be easy to quote many opinions of weight as to its success and public appreciation. I will mention one only, embodied in a letter to the *Solicitors' Journal* in December last. The writer, who was a barrister and solicitor for 16 years in New Zealand and had 28 years' experience of the working of the system there,

\* Since the above was written, a Royal Commission has been appointed to enquire and report upon the expediency of instituting in Scotland a system of Registration of Title.



clearly sets out the conditions of its practical working, and states that it is so simple that he believes it got into working order the very first week after the Act came into force in 1870, and that the public is absolutely confident that if the land is under the Act the title must be right. He further goes on to mention the practice of advertising, as an advantageous feature of a property for sale, that it is held under a registered title. This practice is pretty generally known, and speaks strongly in favour of the system. It is even said on good authority that many dealers would not attend a sale unless the title were so announced in the advertisements. In regard to mortgages, the evidence is equally favourable. The simplicity of the procedure enables many lenders to carry out their loans themselves; and although, so far as I can ascertain, Australian Life Offices in lending on mortgage of land, do not take advantage of the system to this extent, there is not wanting evidence that its benefits are experienced by them in the facility with which their loans can be carried out. Though a solicitor is employed, his work is simple and very quickly done, and the costs are moderate. Not only do landowners and men of finance speak strongly of the benefits of registration, but even lawyers who at first opposed its introduction as strongly as some have done in this country, are equally emphatic on the same side; and as further evidence of value, it may be mentioned that following the lead of South Australia, the system was adopted by the Australian Colonies one after another, and subsequently introduced, in various forms, in some of the Provinces of Canada.

Possibly a more intimate connection with the practical working than one can get at this distance from the Commonwealth would disclose individual cases of difficulty. It may be mentioned that in Australia, estates for life and in remainder can be registered, and not the fee simple only. And while trusts are theoretically excluded, a practice seems to have grown up of allowing these to appear on the register, with the result that it is not so free of complication as it might be. It has even been said that the Torrens system is a fruitful source of litigation. Difficulties have been created in regard to what is known here as an equitable interest; as, for example, the rights of the purchaser of a property to call for a conveyance from the vendor's representatives in case of the latter's death before the purchaser has obtained registration. Evidence unfavourable to the Act is, however, very scanty.



**The English  
Land  
Registry.**

Passing now to our English system, it may be mentioned that registration of title became a question of real importance here as the result of the compulsory provisions of the Land Transfer Act 1897. For many years the subject had exercised the minds of lawyers, and two statutes, the 1862 and 1875 Acts, had been passed with the object of bringing the system into use. Under these Acts the main lines of the Torrens system were followed; that is, a Government register and guarantee were instituted. But registration was voluntary, and was taken advantage of to a very small extent. The Act of 1862 had special features which rendered its system impracticable, and so little was done under it, that it need not be considered. But the 1875 Act contains almost the whole of the scheme now in force, this being applied compulsorily by the 1897 Act.

The chief features are (1) the establishment of a Government registry, where titles to land are to be registered, and (2) a Government guarantee of title to facilitate dealings. The intention of the Acts was (1) to obviate the necessity of investigating past transactions, the effect of which is exhausted; (2) to relieve a purchaser from responsibility regarding settlements and trusts affecting the vendor's title; (3) to simplify the method of effecting transfers and charges, and (4) at the same time to serve the purpose of a deed registry in preventing fraud. On and after 1st January 1899, registration was made compulsory in certain districts on all sales of freeholds and leaseholds having forty or more years to run or two lives to fall in. It was first applied to a small portion of London, and subsequently extended to the whole of the metropolis, including the City.

**The Register.** It may be useful to give a brief description of the Register. This is kept in three divisions, the Property Register, the Proprietorship Register, and the Charges Register. The Property Register contains the description of the land, giving the parish, the name of the house or estate, with a reference to a filed plan, and notes relating to mines and minerals or rights and liabilities. The Proprietorship Register states the nature of the title, and contains the name, address, and description of the proprietor of the land, and cautions, inhibitions, and restrictions affecting his right of disposing of it. Then we have the Charges Register, containing incumbrances prior and subsequent to registration, and covenants and rights adversely affecting the land, and all such dealings with

registered charges and incumbrances as are capable of registration.

**Procedure in  
Registering.**

The procedure to obtain registration is quite simple in most cases. The applicant, or his solicitor, attends with the purchase deed on which the application is founded, and identifies the property on an index map showing the position and extent of every registered property, which is open to public inspection. He then arranges for the preparation of the filed plan and the draft entries for the register. The fees paid, there is nothing further to do, unless some question arises in the course of the registration. In a large number of cases of first registration, the procedure takes from eight to ten days, and in the case of dealings, only three or four days. If a special survey has to be made, as sometimes happens, where there is no plan sufficiently accurate for registration, a longer time is required. Through the courtesy of one of the officials, I have had the opportunity of going through the office and seeing the process. Everything seems arranged to allow of registration being carried through as quickly and accurately as possible; and apparently one of the chief aims of the department is to meet the public desire for expedition. When the necessary processes have been completed inside the office, a Certificate of Title, called the Land Certificate, is prepared and despatched. This is held by the proprietor of the land. It contains duplicates of the pages of the register, showing by inspection the state of the title; and as it has to be produced and brought up to date on every dealing involving a change of ownership or on the entry of a charge, it can for most purposes be relied upon as evidence of title.

It was felt that to enforce a system in England under which landowners must register with an Absolute title would be too drastic a change, and practically impossible to carry out. The scheme was therefore designed of allowing one of three classes of title to be registered, namely, either an Absolute, Qualified, or Possessory title.

**Absolute Title.**

To obtain an Absolute title, the applicant has to produce an abstract in the usual form, with all deeds and documents relating to it, just as under the non-registration system. The title is then examined by the Registrar, and advertisements inserted in the newspapers for objectors, if any, to come forward within two months. When this period has expired, and the title has been approved, the applicant and his solicitor make a statutory declaration to the effect that all deeds, writings

and instruments of title, and all leases, charges and incumbrances affecting it, and material facts, have been disclosed, that the plan comprises the land in question, that the rents are actually received as set out in the title, and that the value of the land does not exceed £——. The registration is then completed, and the Land Certificate handed to the proprietor.

**Its effect.** The effect of first registration of freehold land with an Absolute title seems to be to vest in the proprietor an estate in fee simple, subject (1) to the incumbrances on the register, and (2) to rights and liabilities (such as succession and estate duty, taxes, tithes, quit-rents, rights of common), but free from all other estates and interests whatsoever. The proprietor has a Government guarantee, and can then dispose of his land, it is said, without delay, risk or trouble. In future there should be no deeds to hold nor title to investigate; and a purchaser cannot require any further evidence of ownership than that afforded by an inspection of the register and a statutory declaration as to those matters that are not dealt with as incumbrances. Subject to certain adjustments referred to later on, dealings can be carried out on short printed forms.

To get full advantage of registration, an owner should register with Absolute title. But there is also the Qualified title, which implies an investigation as for the Absolute title, but under which the Certificate sets out specifically the reservations to which it is subject. Objection has been taken by some of the legal profession to such a form of Certificate, on the ground that the title is thus labelled as a defective one, and is in a worse position than under the old system. I am not prepared to say whether this view is sound, or whether it is chiefly based on the hyper-critical attitude taken up by some solicitors; but holders have not up to the present availed themselves of the opportunity to register with the Qualified title. It may eventually prove to be a valuable feature of our system; but as it has no practical importance just now, I do not propose to refer to it again.

**Possessory Title.** It is not compulsory under the Act for a purchaser to register with either of the above forms of registration. He may register under what is called a Possessory title. This does not involve any investigation by the department; but a certificate is granted on the following evidence that the applicant is the proprietor of the land, namely, either

- (a) a conveyance, assignment, or lease to the applicant, or other document entitling him to apply for registration ; or
- (b) a statutory declaration by the applicant, or his solicitor, to the effect that the former is entitled to the property, accompanied by the latest documents of title in his possession.

The first of these alternatives is the simplest and that most usually adopted. The application must also contain sufficient particulars by plan, or otherwise, to enable the land to be fully identified on the ordnance map kept at the Registry. No advertisements for objections are inserted. When the registration is complete, a certificate in the same form as in the case of the Absolute title is handed to the proprietor, except that the following warning is added :

“The Possessory Title hereby certified does not affect or prejudice the enforcement of any estate, right, or interest, adverse to or in derogation of the title hereby certified, which was subsisting or capable of arising on the       day of       , being the date of first registration.”

The effect of first registration with such a title  
*Its effect.* is the same as that with the Absolute Certificate, except as to anything adverse to the title existing at the time of registration. At each subsequent dealing with the land the same forms of transfer and charge can be used as in the case of an Absolute Certificate ; and the only evidence of dealings since registration should be the entries in the register. While no guarantee is given as to the title prior to the point of time chosen, a line is drawn at a certain date providing for future dealings being carried out through the register. But besides obviating the necessity of investigation of future dealings, the fact of registration goes a good way in most cases towards rendering the title in the future clear as to the past, so that an Absolute Certificate could, without difficulty, be granted. In addition to the advantages mentioned, the immediate benefit of Possessory registration is to prevent fraud as regards future dealings and to provide for a clearer and more accurate definition and description of the property than is often the case under the old form of conveyancing.

*Good Leasehold Title.* To meet the case of leaseholds, new rules were introduced in 1903. Provision was made for registration with what is called a Good Leasehold Title. To



obtain this, it is necessary, where there have been dealings with the lease prior to registration, for an investigation of title to be made by the Registry. In many cases, however, the Good Leasehold certificate can be procured with very little trouble. Where the original lessee applies for registration, the certificate is granted to him on his satisfying the Registrar that he has not encumbered or dealt with the land. The freeholder's title is not affected by the registration. His interest may or may not be registered with Absolute title; but as the purchaser of a lease is precluded from enquiring into the freeholder's title, and in mortgages it is unusual for a mortgagee to do so, the Good Leasehold is practically an Absolute title. On future dealings there need be no investigation beyond an inspection of the register and an enquiry into certain matters kept off the register.

When property has been registered with Possessory title immediately on the grant of a lease, the effect of such certificate will obviously differ to a very small extent from that of the Good Leasehold. There is, however, an advantage in the latter. Solicitors' costs, in carrying out future dealings where a property is registered with Good Leasehold title, are regulated by a reduced scale prescribed for by the Act, instead of being chargeable by item as in the case of a Possessory title. There is practically no expense in making the change from the Possessory to the more complete Good Leasehold certificate. All that is required is a declaration that the property has not been dealt with by the lessee. From my own experience, however, it would appear that solicitors do very little to get such Possessory title converted, and unless the client knows sufficient of the system—which can rarely be the case—to take the necessary steps himself, he fails to get the benefit I have mentioned.

This provision for registration with a Good Leasehold title is of great importance in view of the character of land tenure in London. Leaseholds are dealt with very much more frequently than freeholds. At least three times as many of the former were registered in the first four years' working of the Land Registry; and it is evident that in many cases a long time may elapse before the freehold interest will be brought on to the register by sale. For example, land held by the Corporation or County Council, by such bodies as the Ecclesiastical Commissioners or Colleges, is sometimes retained by the freeholders for very long periods; and such holders of ground rents as insurance companies and trustees have generally purchased them with the idea of holding their



investment as long as possible. The result is that in the case of a large proportion of the land in London, the leasehold interest only is brought on to the register; and unless the freeholder's title has been investigated and an Absolute certificate granted to him, the leaseholder cannot obtain such a title. Hence, there must always be a small number of Absolute as compared with Possessory or Good Leasehold titles. When the system has been extended to those parts of the country, where the freehold is the interest dealt in, there will be a better opportunity of increasing the number of Absolute Certificates; and it might prove a satisfactory experiment if the area to which compulsory registration is next applied, is one where leasehold tenures are not common.

**Mortgages.** In respect of the changes introduced with regard to mortgages, insurance companies have not been so directly affected as banks or building societies. Our transactions are not so frequent, nor do they call for such rapid completion. We leave more work to our solicitors. Hence, the actual working of registration has not been brought before us in the same way as before these other investors in mortgages; indeed, the new procedure in connection with our own loans has probably been carried out without being brought to our notice at all, the Charge Certificate having been included in a bundle of deeds and thus escaped attention. But though our office arrangements have not yet been altered, it is, I think, useful for us to understand some of the legal aspects of our securities under the registration system, and to know to what extent its benefits have been, or are likely to be, attained. Perhaps this is even more necessary for us than for other classes of lenders. The long periods for which our loans are granted allow for many more unforeseen risks arising, and call for greater safeguards than usual. Having regard to the size of our loans, it cannot be expected that, even when the system has reached the stage of simplicity of the Australian Torrens system, we shall carry out our mortgages without the aid of a solicitor, but we may secure some of the advantages the Australian life offices are said to experience. Moreover, the saving of expense that should accompany the registration system, may, in some cases, allow of our obtaining a higher rate of interest. The costs of a permanent mortgage not infrequently weigh with a borrower when he is considering whether he should remove a temporary loan from his bankers to an insurance company.

Registered land can theoretically be charged by the execution of the following simple form—

“ In consideration of £                      I, A.B., of                      hereby  
 “ charge the land comprised in the title above referred to with  
 “ the payment to C.D., of                      , on the                      day of                      19                      ,  
 “ of the principal sum of £                      , with interest at                      per-cent  
 “ per annum payable half-yearly (quarterly) on the                      day of  
 “                      in every year.”

Practically it is generally considered necessary to insert additions to this form. For instance, as it stands, the charge does not imply a covenant to repair or to insure against fire. Both these require to be added, and the latter is sometimes expanded to provide for the stipulation as to the amount of the fire insurance, the way in which the policy monies are to be applied, and the name of the office. Nor does the charge include a conveyance of the legal estate, which in some cases is considered necessary to a mortgagee; nor is provision made for a penal rate of interest. Both can, however, be provided for by the addition of a few words. In the case of the more complicated mortgages to which we are accustomed, the form sometimes requires some further adjustment.

The necessity for these additions has led to the practice, very general among solicitors, of having a mortgage in the old form containing the special covenants and provisions and the conveyance of the legal estate, together with the statutory charge to conform to the Registry's requirements. This is open to the objection that there is an additional deed to be kept by the mortgagee and possibly disclosed on subsequent dealings. The statutory charge itself is sent in and retained by the Registry. On registration, a note is made in the Charges Register, and a corresponding note in the borrowers' Land Certificate; and a Certificate of Charge is then prepared and handed to the lender.

An interesting and important feature of the statutory form of charging is the introduction of a change in the relative positions of borrower and lender. Hitherto, under the English mortgage to which we are accustomed, the land is conveyed by the mortgagor to the mortgagee, and becomes the property of the latter, subject to the former's right to call for a reconveyance on repayment of the loan. But this has not been the position in most other countries. In Scotland, for example, the lender has only a charge over, not a vested interest in, the land. From a mortgagee's point of view, it is well of course to get as much as one can in the way of security; but looking at the matter broadly,

the relative positions under the new charge seem to be fairer. A borrower should not be obliged to divest himself of his ownership. As the result of the requirements in this respect under our English mortgage, the assistance of the Courts has frequently been requisitioned to protect him from the full effects of a complete surrender of his interest. But under the new form the borrower retains his registered estate in the land, and should in most circumstances keep his Land Certificate.

It is said that without the ownership of the property, or what is called the "legal estate", the mortgagee is without certain powers. Apparently he cannot distrain for rent, and it is doubtful whether he can bring an action for ejectment of tenants, which in some circumstances may be necessary; nor, in the case of leaseholds, can he rectify a breach of covenant. In what proportion of cases he would require to make use of such rights it is difficult to say, but probably, in the great majority of small mortgages at any rate, there is no necessity to provide for them. The circumstances of the case must of course be allowed for. Where, for example, a mortgage is taken over a block of residential flats, it may prove very important to a mortgagee in possession to be able to eject a tenant whose tenancy may damage the whole security; and it seems to be a question whether in all cases where trustees are lending, they would be entitled to take any risk incurred by the omission of a conveyance of the legal estate.

There is, of course, the alternative method of arranging a mortgage by the lender taking an absolute transfer of the land into his own name. An ordinary mortgage deed would be executed to set out the rights of the parties and a caution lodged to protect the mortgagor's equity of redemption. It has been urged by some advocates for registration that a system is to be preferred which allows only of mortgaging in this way and excludes the Charges Register altogether. But I think the general opinion is that the power of charging on the register is an important and valuable feature of the Torrens system.

Among other points affecting mortgages, the following may be of interest. The statutory charge confers on the chargee powers of entry, sale and foreclosure; and to exercise these, he does not require to hold the borrower's Land Certificate. On a sale, for example, a purchaser from him gets full rights as proprietor, and a new Land Certificate. Although the Act provides for the contrary, it is the practice, it seems, for

solicitors to stipulate for the borrower's Certificate to be handed over on completion of a mortgage. One effect of this is that when a landowner wishes to raise a loan on second mortgage from another person, he is put to the expense of getting the Land Certificate produced by the first mortgagee. As the second lender cannot get his charge registered until the Land Certificate has been noted with the second incumbrance, the facilities for charging, aimed at by the Act, are to some extent interfered with.

The process of foreclosure has remained unaltered by registration, except that on getting his order, the mortgagee takes it to the Registry and gets registered as proprietor. It is to be regretted, however, that foreclosure has not been simplified by being removed from the Courts to the Registry. Under present conditions few lenders care to face the expense, delay, and trouble of a foreclosure action so long as any other course can be taken. It has been suggested that when a chargee's power of sale has arisen, he might be registered as the proprietor of the land, and allowed to deal with it as his own, but a definite time would even in that case have to be fixed within which the borrower could come to redeem his property; otherwise he would be able to exercise to his own benefit an unfair option with regard to future fluctuations in its value.

On repayment of a loan, a statutory form of discharge is signed by the chargee and registered; and where there is no reconveyance of the legal estate this need only bear a *1d.* stamp. According to the legal Press, the Inland Revenue authorities seem to have become alarmed at the loss of stamp duty that would result from the use of this form, and consulted the Land Registry, with the view of devising some scheme by which the discharge could still be held to require an *ad valorem* stamp. The point was, however, eventually dropped.

Before the discharge is registered, there is sent to the holder of the charge a notice to the effect that such an instrument has been left at the office for registration, and that, in the absence of any objection being sent in, the registration will be completed at the end of three days. It must be observed that this notice is sent to the chargee himself and not to his solicitor; and it seems to me that the time allowed—three days—would in some cases be too short for the protection of the lender's interest. His absence abroad might preclude him from taking any necessary action; or, in entire ignorance of the Land Registry system, he



might neglect to send off the notice immediately to his solicitor for attention. In both cases there is the risk of his charge being fraudulently discharged.

In respect of a mortgage of leaseholds, the chargee under the new form is in a position neither of under-lessee nor assignee; but he avoids the liability for repairs and other covenants which he would incur as the latter, and, on his power of sale arising, he can transfer the whole term of the lease. Under the old arrangements, a mortgage of a leasehold is usually carried out by demise—that is, an under-lease for the whole term less one day, and a trust for the last day—in order that the mortgagee may be free from liability for covenants.

Comparison of  
English with  
Australian  
System.

In comparing these arrangements in England with those in Australia, it will be remembered that in both cases the Torrens system, in which the main idea is absolute ownership of land, as in the case of stocks and shares, has been super-imposed on a system of land tenure and legal conditions based on entirely different principles. The result is that in both countries, what might be called its reasonable methods have been considerably hampered. In the Colony, registration of title has been designed on more drastic lines than ours, and has consequently effected a greater reform; but even there analogies have been continually drawn from the general law in the decision of the Courts.

But while, in respect of the difficulties just mentioned, both the Australian and English systems had to meet obstacles, it is often maintained that the Crown grants facilitated the bringing of land on the register to such an extent that no analogy can be drawn from the success of registration in the Colony. No doubt such grants were of great assistance. We are not, however, without a corresponding advantage over here. As our system provides that on the grant of every lease after the Act, it shall be placed on the register, we have a large number of properties which, except for the question of covenants, are almost, if not quite, in as good a position as those registered in Australia on grant from the Crown. And even in the case of freeholds, there must be a number of titles that can be as easily registered. Where, for example, there is a well known estate, held by one family for generations, and unencumbered, there should not be much difficulty in registering sales of portions of it. It is alleged that the provision for none but Absolute Titles is a feature in which the Australian system is greatly superior to



ours. It must, however, be remembered that our good Leasehold Titles, which form so large a proportion of the total, practically secure all the benefits of the Absolute Certificate; and that, although experience has shown that a more drastic system would probably have been more successful, the omission of the requirement for Absolute registration in all cases was specially designed to meet the greater number of complicated titles here.

Passing to other points of difference, we find, as already mentioned, that in England only a fee simple interest, in Australia, the interests of a life tenant and remainderman, can be registered. Our arrangement probably tends to greater simplicity. And it must be noted that we are free from the complication of the separate State legislation, which has produced a large number of different statutes, and the question of trusts. The last are rigorously excluded from our register. It must, I think, be admitted that as regards dealings with the "legal estate" off the register, we are in a position that leaves great room for improvement; and in respect of foreclosure, removed in Australia from the Courts to the Registry, we might have followed the example of the Colony.

Comparison  
with Scottish  
System.

In drawing a comparison between our English system and the Scottish, it may be remarked that even the most bitter opponents of registration of title are not unfavourable to a properly worked deed registry; while the majority of those who deal with land are strongly in favour of it. Even in our own experience, we have probably had occasion to appreciate the protection it affords. But while the Scottish deed registry must be credited with many advantages, it does not, of course, go so far as registration of title towards securing the benefits of cheapness and simplicity; and it is obvious that when providing for the introduction of a register, a good deal is gained if, in addition to the mere fact of record of deed, some step can be taken at the same time in the direction of guarantee of title. This has been attempted in our system by the device of Possessory title.

Practical  
Working of  
English  
System.

It would, of course, be valuable and interesting to have information from individual landowners and mortgagees as to the working of the English Act. But it would obviously be difficult to collect their experience; and, moreover, under present conditions, as already mentioned, so much is left to the solicitor that the client's experience would probably be meagre. Most of the evidence we require, therefore,

must be looked for from the legal profession. Two considerations should, I think, be borne in mind. First, the solicitor's point of view must in many instances be affected by his training and association with the complications of our land laws. I do not suggest that solicitors are opposed to reforms; but the aims and methods of registration are so much more closely allied to those of ordinary commercial dealings than are the technicalities of our legal system, that the lawyer can hardly be expected to appreciate even a perfect registration system quite so well as the business man. Secondly, similar opposition to that experienced here was offered by the legal profession, or a section of it, both in Australia and in other parts of the world on the introduction of the Torrens system. It is often said that solicitors would naturally have a prejudice against arrangements that aim at taking away their employment to some extent, and that their views must on this account be biassed. But an acquaintance with the history of the subject makes it quite clear that those who have opposed registration have done so from an honest conviction that it is against the public interest; and it must not be forgotten that the very nature of our arrangements on the old basis have tended to deprive anyone but a member of the legal profession of a full knowledge of most of the complications following such a radical change.

Among solicitors themselves, opinion on many points is not unanimous. There are some who take the view that the system is objectionable from every point of view. It has even been called by its most active opponent a "giant imposture." But there are those—I think an increasing number—who, while fully aware of the risks and difficulties to which the Act has given rise, and of the fact that their own profession must eventually suffer, are not unfavourable to the system. They advocate such changes as will really improve it, and look to the time when, free from certain objectionable features, it will really secure the benefits of simplicity and cheapness.

**Objections.** Two of the principal objections raised by the profession, and dealt with in the report of a Special Committee of the Law Society, are (1) the officialism introduced by the system, and (2) the compulsory provisions of the Act.

**Officialism.** The dislike of officialism is so general in this country that where it is simply a question of transferring work from the private individual to a Government department, there would be no question that the change would be open to

very great objection. There is a general impression that work is not, as a rule, so well or so quickly done; and further, there is the important disadvantage that it is much more difficult to fix responsibility and obtain redress in the case of loss. As to this, the advantage taken by one Government department of the unlimited State purse is so notorious as naturally to give cause for anxiety in other directions. But in the case of registration of title, the transfer of work from solicitors to the Land Registry is, or should be, accompanied by such benefits as far outweigh the disadvantages I have mentioned; and these benefits can only be obtained by the creation of a Government department. Moreover, it is quite clear that the officialism of the Scottish and the Yorkshire deed registries is not felt; nor can I find any evidence from the Colonies, where the people may be credited with similar feelings to our own on the subject, that officialism has in any way detracted from the merits of registration of title there.

**Compulsion.** As regards compulsion, the history of registration in other countries sufficiently refutes any argument that, apart from the question of cost, it is a hardship. To prevent fraud, anyone will willingly submit to compulsory registration on a deed registry; and, as compared with this, there is not much more delay or trouble to get the Possessory title. It is argued that the voluntary principle was tried for many years and failed to achieve any result. This was not, I think, because landowners were opposed to the system. The failure was due rather to the fact that a landowner would naturally object to come forward to benefit future owners of his property, unless he were satisfied that by the general application of the system, he himself would benefit in his dealings with other properties. It was clear from the previous attempts and from the experience of other countries, that to make the system universal and beneficial, the compulsory provisions were absolutely essential. Where the cost is light, very little is heard against compulsion. There seems to be no complaint on this ground either in the Colonies or in Germany. In our case there seems no doubt that the cost is felt too much; and it is, I think, important for the authorities to consider whether the fees for Possessory title could not be reduced. On Possessory registration, the fee, which is based on the value of the property, is £3 per £1,000 (or £7 for £3,000), with a graduated scale for other amounts; and, in addition, there are the charges to be paid to the solicitor for his work in connection with the registration. For Absolute or Good Leasehold titles, the fee is

three times as much, but these do not, as already mentioned, come within the compulsory provisions of the Act.

**Paucity of  
Absolute  
Titles.**

Another of the complaints against our present system is that, as it has secured the registration of so few Absolute titles, it must be considered a failure in respect of its chief feature. Figures would seem to justify this. Out of about 100,000 first registrations, only some 200 Absolute certificates have been granted. At the same time, it must be remembered that not only have the effects of registration with Absolute title been to a large extent secured by the Good Leasehold titles, or their equivalent, Possessory titles registered on grant of the lease, of which 25,000 have been registered, but that owing to the nature of land tenure in London, the number of Absolute titles will probably never be a very large proportion of the total. This consideration is not, I think, given sufficient prominence when the complaint is made that the Act has failed in the way just mentioned.

**Remedies.**

Notwithstanding this, however, it is a matter for regret that the paucity of Absolute titles has not allowed the full benefit of registration in so many cases to be attained. To remedy this, several important modifications of the system have been introduced since it came into force. The measures adopted to facilitate the grant of Absolute titles have taken two directions. In the first place, the cost to the first registered proprietor of obtaining the certificate has been very much reduced. Instead of the fee originally fixed at three times that required on Possessory registration, the new rules of 1903 provide for a payment down of only £2 per £1,000 (value) by the applicant; the balance of the fee being deferred and cleared off on subsequent dealings by an increase to the extent of half as much again of the fees paid on those dealings. This arrangement will be welcomed as an admirably fair one. It seems to some extent inequitable that the first proprietor should have to bear the whole cost of registering with an Absolute title for the benefit of all future owners. It is true that with an Absolute title his property should sell for a price increased by the amount of the legal charges that will be saved by the next purchaser; but it is doubtful whether a first registered proprietor will reap this advantage until the system gets more widely known. Another direction in which a change was introduced by the rules with the same object has been a relaxation in the stringency of the investigation. This provoked some outcry from a section of the



legal profession on account of the risks alleged to be run. Not from all sections, however, for some solicitors have expressed the opinion that they apprehend no evil from limiting the enquiry into title. It must be remembered that the elasticity of the new regulations is in correspondence with the practice adopted by solicitors under the old arrangements with respect to short titles. A sale of land by the London County Council, or other similar body, acquired by them by Act of Parliament, may be cited as an extreme example. The purchaser gets nothing beyond the conveyance from the Council, while there are many other instances where it is the custom to limit the investigation. In such cases there is, of course, a safeguard in the publicity of the transaction, and the fact that the same title is being dealt with in a number of sales. The general opinion, therefore, will be, I think, that to attempt to deal with registered titles on fixed rules as to stringency was bound to lead to failure, and that the change to more reasonable methods is a most valuable move in the right direction. The rule now is that (1) where land has been sold or purchased under an Order of the Court, or (2) where registration has been made with a Possessory or Qualified title for six years prior to the date of the application for Absolute title, the first proprietor having been a purchaser on sale; or (3) where the title has been fully investigated before the date of the application, the examination may be modified in such manner as the Registrar may think fit. In a fair number of the cases registered with Possessory title during the last seven years, an Absolute Certificate could be granted at once without risk or trouble. The main idea of the feature of Possessory title is that after a number of years such titles can, in very many cases, be treated as Absolute. There would, however, be obvious advantages if in such cases where there has been a lapse of some years from the grant of the Possessory Certificate, and the Registrar considered the circumstances to justify it, the title were definitely changed in the register from Possessory to Absolute. If this were done voluntarily, there would not be the same risk of an application being put forward by a person fraudulently trying to get himself registered as Absolute proprietor. And there would be another advantage in the Registry taking the initiative in this way. The benefits of the system would be brought before a number of landowners who at present know nothing more of it than that they have had to pay for Possessory registration.



Insurance  
Fund.

In this question of facilitating the grant of Absolute titles the insurance fund is an important element, and calls for some remark. Under the 1875 Act no provision was made for such a fund. The omission was, however, rectified by the 1897 Act, and the system is now to some extent on the same lines as the Australian Torrens; that is, a fund has been formed out of the fees paid by the landowners, from which claims can be paid to those injured by mistakes or entries in the register obtained fraudulently.

As yet there seems to have been no attempt made to fix a contribution commensurate with the risk, and the necessity for allowing sufficient cover may partly account for the high registration fees. The risk may, I think, be divided into (1) that attaching to Absolute titles in respect of title prior to registration, and (2) that attaching to both the Absolute and Possessory Certificate in respect of dealings subsequent to registration. In the case of the first, there have been so few Absolute titles that only a very small portion of the fees received to date could be required to cover the risk under them. The principal feature of such risk is the possibility of suppressed mortgages. Except in the case of Middlesex and Yorkshire, our old system of conveyancing provides for no protection against these; and experience has proved that loss may arise not only as the result of deliberate fraud, but may be caused innocently by persons of little experience. In estimating the premium to cover this risk, allowance must, of course, be made for the fact that in many cases the chances can be taken as absolutely nil.

In respect of dealings after registration, the risk is chiefly from a forged transfer. Having regard to the large number of dealings recorded during the seven years for which the system has been in force, it should not be difficult to arrive at a premium that could be relied upon to cover the risk. From the experience of Australia, it would seem to require a very small one. A few years ago, when the insurance funds of the seven Australian Colonies amounted to £405,000, only £14,600 had been paid in satisfaction of claims, showing that the contributions, which are generally about one-fifth per-cent, had been twenty-eight times in excess of what was required; or, comparing the amount of claims, £2,672, in the case of two Colonies, with the number of dealings, 443,000, we find that the risk would be met by a contribution of about  $1\frac{3}{4}d.$  per dealing. In Australia, it will be remembered, there is no risk in respect of title prior

to registration, while over here this is the chief one to be allowed for.

The only claim on our insurance fund has arisen in consequence of a forged transfer. The particulars of the case, *Attorney-General v. Odell* (21 T.L.R. 450), bear out, I think, the impression that the position of the fund is unsatisfactory. A Mrs. Connell was the registered proprietor of a charge for £300. Her solicitor forged her signature to a transfer of the charge in favour of Odell; the latter took the forged transfer to the Registry and got registered as proprietor of it. On the forgery being discovered, Mrs. Connell's name was reinstated and Odell's removed; the Registrar awarding the latter indemnity for the loss suffered by the rectification of the register. The forgery was not disputed, and any insurance company that had such a risk on their books would have at once paid the claim, especially in view of the practical consideration that the amount involved was only £300. But what did the Treasury do? As guarantors of the fund, they disputed the Registrar's decision and carried the question to the Courts. It was there held that under the Act the Registrar is only in the position of a judge of first instance, and that the trustees of the fund and the Treasury have a right of appeal to the Court as being "aggrieved persons" within the meaning of the Act. The Registrar's decision was, however, affirmed, notwithstanding the arguments put forward by Counsel for the Crown that the loss had been incurred by the act of Odell in taking the forged transfer to be registered, and that such act should disentitle him to indemnity. The case is to be taken by the Treasury to the Court of Appeal. This will do more serious harm in discrediting the system than many of the attacks of its opponents. What will be thought of the practical aims of registration if every claim is to be disputed by the Treasury on technical grounds? It must, however, be noticed that it was not the Land Registry that disputed the claim. The Registrar awarded compensation. But here the real danger of officialism came out. The authorities could well have afforded to treat the claim in a practical way, in view of the fact that very large sums have been collected from landowners in respect of dealings, and that a claim of £300 is a small one to set against the contributions. To remedy this state of things, steps should be taken to put the fund on the proper footing of average; a contribution should be fixed, kept apart, and accumulated

to meet the claims. The Treasury's guarantee may have to remain for a little while; but as soon as the fund is found to be ample, this could be dropped.\*

**Complications.** One of the most common complaints against our system of registration is that it has complicated rather than facilitated dealings with land. This calls for some consideration. It seems to me that there may be some ground for this charge; though it is important to bear in mind that the complicated cases of which solicitors have experience probably secure undue prominence as compared with the much larger number that work smoothly and call for no special attention, and that the reports of the Land Registry, although they might be considered biassed, state that the great majority of cases offer no difficulty or complication. The features of the system around which this charge principally centres are (1) the forms, (2) plans, (3) dealings off the register.

As regards the use of the simple printed forms for dealings, which it is the object of the Act to permit, solicitors sometimes complain of the difficulty of adapting these to the special circumstances in each case. In respect of mortgages, Building Societies freed themselves from such a difficulty by obtaining permission to use their own forms; but in the case of other lenders, the feature does, doubtless, sometimes arise. In the course of time this will probably tend to disappear; yet it would seem that, except for small mortgages, the forms can hardly be used in practice to the extent that, theoretically, they should be.

Any difficulty of boundaries and mapping is only incident to first registration, and is generally caused, I believe, by the loose and inaccurate descriptions under the old system. The system of maps and plans adopted by the Registry has been brought to my notice on more than one occasion as meeting with considerable appreciation.

**The "Legal Estate."** Then, our system has provoked a great deal of criticism in respect of its allowing dealings with the legal estate off the register. This, it seems to me, is its chief

\* Since the above was written, the Court of Appeal has decided in favour of the Treasury, apparently on the ground that, in accordance with the terms of the Acts, the insurance fund is available only for those whose titles have been submitted for verification by the Registrar, as in the case of first registration, and not for those who get on to the register by means of documents presented for registration in the ordinary routine (see *Solicitors' Journal*, vol. 50, p. 388). The result supports the view set out above that the system requires amendment in respect of the insurance fund.

failing, though practically it may prove to be of less importance than one might imagine. It does, however, create an obstacle to the general understanding and appreciation of the system; and moreover, so long as solicitors are left responsible for the carrying out of dealings, it tends to retain the technicalities we are anxious to get rid of. It would, of course, be presumptuous for me to attempt to go into these, but it seems necessary, for anyone who wants to understand the system at all, to be aware of the nature of the difficulty created.

This feature of our Act seems to have resulted from a desire to make our system less drastic than the Australian Torrens system which, as mentioned before, permits of dealings only on the register. It was brought into prominence by a case—*Capital and Counties Bank v. Rhodes* (Ch. Div. 1903, I, 631)—in which it was decided that although first registration was necessary to get the legal estate, that estate would afterwards pass by an unregistered deed, and that the register is not conclusive for ascertaining where the legal estate is. The effect of this seems to be that a proprietor registered even with Absolute title need not necessarily be the legal owner, and that really to ascertain the latter, it is necessary in all dealings to carry the title back either to the first registered proprietor, or a registered purchaser for value. If the result of this is that the deeds relating to dealings off the register will have to be kept and disclosed, we should be back again in the mesh of the old investigation of title. It has, however, been stated that the register is, as it ought to be, the only criterion of title. At any rate, there is one safeguard to restrict dealings to those on the register. In the case of a purchaser from a mortgagee under a deed off the register, the former might get the legal estate but could not obtain registration except through the last registered proprietor; with the result that the mortgagee selling might have to go to the mortgagor to get the registration conferred on the purchaser. To borrow a brief definition of the position from an able writer on the Torrens system, Mr. J. E. Hogg, it may be said that the effect of the English Act is to give a positive warranty only; that is, a deed on the register is valid, but one off is not necessarily invalid. The Australian system gives both a positive and negative warranty: a deed is valid if it is on the register, and one off the register is invalid. The Scottish Registry gives a negative warranty, in declaring every deed not on the register to be to a great extent inoperative. There can be no question that



both the Australian and Scottish systems are superior to our own, in leaving less doubt or uncertainty as to the real owner of the land.

With several of the minor objections raised by some solicitors, most of us will have little sympathy. Some complaint has been made that the Land Registry have sent what are called "touting" circulars to landowners, setting out the advantages of the system, and recommending registration with Absolute title. So far from there being any objection to this, it seems to me, as I have already indicated, a matter of great importance to spread an acquaintance with the Registry and the benefits it has to offer, far beyond legal circles. There is no question of professional etiquette. The system ought to be widely advertized, so that even where the public do not know how to deal with their land themselves, they may at least understand the forms, and so be saved the risk, for example, of lending on a Possessory title without investigation of the prior title.

**Banks and  
Building  
Societies.**

Practical evidence of great weight is available from representatives of banks and building societies. The former are, of course, intimately concerned with registration in their everyday practice of lending on deposit of title deeds; while the latter have been affected in their widely extended mortgage business. Both were very early in the field, and have been able to exert sufficient influence to have the system adapted to some extent to meet their particular requirements.

It is all the more important to consider how bankers' loans are affected, because it is claimed that one of the chief advantages of registration of title is the facility it offers for loans on deposit of the Land Certificate. With Absolute or Good Leasehold titles this may be so; but there seems to be no doubt that up to the present bankers' loan business has not been simplified by the Possessory title. They get, it is true, the benefit of protection from fraud and the better definition of the property; but any saving of trouble in respect of dealings subsequent to registration does not yet outweigh, it is said, the disadvantage of the cost and extra work involved. When there are more Absolute titles, or the Good Leasehold titles bear a greater ratio to those merely Possessory, the benefits may commence to be felt.

Nor have building societies' representatives, it must be admitted, very much to say in favour of the present position of registration in this country. They can speak with a good deal of authority, as they have taken more active interest and had



more practical experience than almost any other laymen. On the question of cost there has been considerable dissatisfaction. The representative of one of the largest societies stated, some time ago, that he had had experience of 1,300 registrations, which had cost their borrowers on an average two guineas each more than under the old system. And there has been complaint as to the delay the system causes them. Hence, while the general opinion of building societies is strongly in favour of a system of registration, the conclusion arrived at seems to be that the extension of our system to other areas should be opposed for the present. At the same time, it should be mentioned that as regards protection from fraud and the maps, there is an appreciation of the benefits of the Registry; and it seems probable that, as in the case of the banks, the lapse of time will bring into greater prominence the facilities of being able to ignore cancelled dealings.

*Conclusion.* Reviewing as a whole the present position of registration of title in England, it is evident, I think, that a good system would not only be preferred to the old methods of conveyancing, but would meet with general appreciation. While, however, our particular type of registration contains the main principles essential to secure the benefits of successful working, and is even free from some of the drawbacks of its model, it has not given complete satisfaction up to the present time. Some of the reasons for this partial failure have been brought out in the previous remarks, but summarizing them, we have—

First, the state of transition from entirely different methods necessarily leads to some complication and difficulty. This is chiefly due to the impossibility of getting rid at once of all the unsatisfactory features of the old system, and to the necessity of allowing the technical ways of dealing with land which have hitherto prevailed to continue alongside the new methods. That our complicated land laws are an important element in detracting from the benefits of registration is clear from the facility of dealing with land in Scotland, where there is a deed registry only. At the same time, with the spread of our system and a better understanding of its principles, things will tend to improve. It would be unreasonable to expect perfect conditions in the early stages of its working.

Secondly, there are certain drawbacks in the present system, already referred to, which call for amendment.

- (a) Permission to deal with the legal estate off the register leads to uncertainty as to the exact position of registered proprietors, and will tend to prevent the public availing themselves of the system to its full extent. The simplicity of the Australian Torrens system should be adhered to in this respect.
- (b) Although the disadvantage of the smallness of the number of Absolute titles is not so pronounced as is often made out, the full benefits of the system are not secured until titles are Absolute or Good Leasehold. The Registry should adopt a much bolder policy in granting these Certificates whenever practical considerations show that they can do so without risk or trouble. To assist such grants, the insurance fund should be put on a better basis, both as regards the contribution to be paid and the adjudication of claim by the Registrar.
- (c) The registration of Possessory titles, which has been the main work accomplished up to the present, has caused some dissatisfaction owing to the cost and delay being generally considered incommensurate with the advantages secured, with the result that the compulsory provisions of the Act have been felt as a hardship to some extent. As the result of a closer estimate of the insurance risk, the cost of Possessory title might be reduced; but, if necessary, the State should contribute to the expenses until it is found that the growth of the work of the department will allow of a much smaller fee being charged.

Opponents of the system have long clamoured for an official inquiry into its working. In the first few years this would have been a test of no value, and was very properly refused. But it seems probable that such an inquiry will be made before very long; and it will, I think, show that in spite of the disadvantages I have mentioned, registration of title has already produced substantial benefits. For its full advantages we must be content to wait; but in the meantime it is satisfactory to know that the Registry officials are glad to learn the views of various sections of the public—especially, I think, of such bodies as our Institute—and will welcome practical suggestions.

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## ABSTRACT OF THE DISCUSSION.

MR. W. C. SHARMAN said he wished to congratulate the author on the lucid and interesting manner in which he had treated a subject which was admitted to be both complicated and difficult. Registration of title to land affected vast interests, and it was a question worthy of more general consideration than it received. No doubt there was a tendency to look upon the subject as one purely connected with conveyancing, but the avowed aim of the Land Transfer Acts—to reduce dealings in land to a more scientific form, and to provide security and cheapness—should appeal to all classes and bodies interested in investments in real property. In any discussion on that subject, reference was generally made to the strong opposition evinced on the part of some of the legal profession to the system of the Acts, and their hostility was often attributed to self-interest. It should, however, be remembered that the amendments and simplification that the present system had undergone in the past, had been due almost entirely to lawyers. It was only necessary to refer to the reforms effected by the Conveyancing Act of 1881, to show that the legal profession had not been averse to any measures tending towards the simplification of title. Many of their objections were due rather to a knowledge of the pitfalls which might abound in tampering with the title to land. On the other hand, it must be admitted that the present system was capable of great improvement. Sir Howard Elphinstone had estimated that a fresh investigation of title took place on the average once in ten years, and if this were accepted as being even approximately correct, it would, he thought, be agreed that any method by which the labour involved in those frequent investigations could be reduced, would be of great benefit to the community. The author, on page 248, referred to the conflicting systems of Registration of Deeds, and Registration of Title, as exemplified by the Scottish and Australian systems, and clearly pointed out their difference. It was interesting to note that registration of deeds relating to freeholds, was introduced into England as early as 1536, by means of the Statute of Enrolments, but proved a failure, although it succeeded in further complicating the method of conveying land. A system analogous to registration of title had, however, been in existence since feudal times in the form of copyhold tenure. Registration of title had been aptly described as being identical in principle with copyhold tenure, relieved of its oppressive feudal incidents. The fines might have disappeared, but their place had not been unworthily filled by the fees of the modern system.

On referring to the working of the Land Transfer Acts, the main point apparent was the small number of absolute titles that had been granted, and Mr. Hart's contention that that was not necessarily evidence of the failure of the Acts was, he thought, fully justified. In addition to the point mentioned by Mr. Hart as to the large number of leasehold properties in which it was not possible to obtain an absolute title, it had to be remembered that, to a purchaser who was in no immediate expectation of having to deal with the property, a possessory title was all that was required. Although, under the new rules, the cost of obtaining an absolute certificate had been to a

great extent deferred, the initial cost was still greater than that incurred in obtaining a possessory title, while in addition there was always a fear that official requisitions might cause trouble or delay. Apart from the protection afforded by the insurance fund in respect of past transactions, there was no immediate benefit to be gained by an absolute registration, and, moreover, any advantage which might occur in the future would be shared with the next purchaser. As actuaries were aware, benefits of that contingent nature were liable to be somewhat heavily discounted; and, until it was possible for a purchaser to have the vendor's title investigated at the Registry, and an absolute certificate granted at less cost than under the present system of taking a conveyance and registering with possessory title, he did not think there would be any great increase in the number of absolute titles. In some cases, however, under the present arrangement, it might be advantageous to register immediately with an absolute title. For instance, a vendor and purchaser might agree to share in due proportion the cost of an absolute registration, the investigation to be carried through by the Registry, and the purchaser nominated as owner; or the owner of an estate to be developed might register with absolute title, and obtain land certificates, made out in the names of intending purchasers. Again, a purchaser might be in expectation of having in the near future to create a mortgage on the property. As in such a case he would have to bear both his own and the mortgagee's costs, the grant of an absolute title would result in a considerable saving in the cost of the mortgage. In the great majority of cases, however, the possessory title, which was a distinct step towards an absolute title, would seem to be sufficient, and such a title was all that was needed in the case of an insurance company making investments of a permanent nature. Mr. Hart had expressed a hope that the fees for possessory registration might be reduced. In view of the fact that that method of registration was compulsory, he (Mr. Sharman) thought the members would agree that the cost should be reduced to its lowest limits. He had seen it stated that the fees payable were practically double the amount of those that prevailed when possessory registration was voluntary. Of course the premium required for the insurance risk was included in those increased fees, but, in the face of the Australian experience, it seemed that the actual cost of the risk was a very small matter, and some reduction in the cost of this method of registration might be possible.

Referring to mortgages, it was interesting to learn from Mr. Hart that the saving of expense which should accompany the registration system might, in some few cases, allow of a higher rate of interest being obtained. While fully admitting that mortgages of land were a security somewhat less easily affected by the various influences that governed the rate of interest, one would have thought that the improved security resulting from the insurance fund, and the increased facilities for dealing with registered land, would have had a tendency rather to lower the rate. Perhaps that might be made clearer. If the market rate of interest fell, an existing mortgagor was precluded for some time from taking advantage of it, owing to the necessity of giving notice, and the heavy costs incurred in making



a transfer. If those costs were considerably reduced under a complete system of registration, mortgages of land would tend to become more fluid, and this would operate in the direction of benefiting the mortgagor rather than the mortgagee. A point might be noticed with regard to the power of chargees of registered land. Under the non-registration system, a mortgagor would lose his equity of redemption, if the mortgagee entered into possession and held the land for twelve years without acknowledging the mortgagor's right. It was, however, provided by the Act that no title adverse to, or in derogation of, the registered proprietor should be acquired by length of possession, and as the mortgagor still remained as registered proprietor, the right of the mortgagee to obtain the equity of redemption would be barred. The point was not of much practical importance, but was interesting in regard to the change in the relative positions of mortgagor and mortgagee effected by the Acts. A question, interesting perhaps to insurance companies, arose in connection with mortgages or purchases of immediate or reversionary interests. When the funds or part of them consisted of real property, held by trustees, the office would have no notice of any change in the investments. The 1875 Act included provisions whereby any person interested in unregistered land, such as entitled him to object to any disposition being made without his consent, might lodge a caution entitling him to fourteen days' notice of any proposed registration. It was stated in Mr. Brickdale's work on the Act that that clause was interpreted very liberally, but, on the other hand, if a caution were lodged without reasonable cause, the person lodging the same should be liable to make compensation. It seemed doubtful as to whether a caution lodged by an office in respect to mortgages of reversionary interests would be held to be reasonable. Certainly some provision enabling cautions to be lodged in respect of unregistered land in the compulsory area would be useful, inasmuch as the office would then have notice of any application which might be made to register the property on a sale. With reference to the case of *Attorney-General v. Odell*, he certainly thought there was some justification for the action of the Treasury. It was not so much the amount at stake as the principle involved. Odell had no title to the property upon the forgery being discovered, and his loss was not primarily due to an error in the Registry necessitating its rectification. This extended protection against forgery was somewhat comprehensive, and if the decision were affirmed, it would appear that the protection afforded to an innocent transferee under a forged transfer of land, was greater than that which obtained in the case of an innocent transferee under a forged transfer of stock. The question as to the further extension of the area within the scope of the Acts was of some interest, but owing to the great opposition such extension would not appear to be probable. At the present time the Corporation of the City of London were inviting information as to the working of the Acts from registered proprietors and conveyancing solicitors, and he found that the replies received had been nearly all unfavourable. As, however, the compulsory provisions of the Act had only been applied to the City for some three years, the benefits of the system would not have had time to become apparent. If he might express an opinion,



legislation permitting existing possessory titles to be converted, in suitable cases, into absolute or good leasehold titles, without further expense, would tend to make the system more popular, and would draw attention to its inherent advantages.

MR. J. S. RUBINSTEIN (a visitor) wished in the first instance to make a reference to the history of the subject, because he desired to satisfy the members—as he thought he would be able to do—that every practical expert was opposed to the system. As would be learned from the paper, the Land Registry system was not a new system in this country. It was first introduced in the year 1862 by the late Lord Westbury. At that time Lord St. Leonards opposed the measure as firmly as he could, and he need not remind the members that his lordship was undoubtedly the greatest real property lawyer of the century, if not of all centuries. Notwithstanding Lord St. Leonards' opposition, the system was brought into operation, and, as the author had stated in the paper, it proved a complete failure. The next expert who took the matter in hand was Lord Cairns, who came second to Lord St. Leonards as a practical expert in conveyancing. Lord Cairns believed at the time that it was possible for a registration system to be introduced into the country which would have beneficial results, and consequently he was responsible for the Land Transfer Act of 1875. Lord Cairns, in dealing with the subject, came to the conclusion that the reason why the 1862 Act had failed was because it only provided for the registration of absolute titles, as, having regard to the complicated titles that existed in this country, it was found practically impossible, owing to the expense and delay of investigating earlier titles, to place on the register absolute titles. Lord Cairns thought he could get over that difficulty by placing on the register three characters of title—Possessory, Qualified, and Absolute—and no doubt it was in his mind at the time that in the course of years the possessory title would, automatically as it were, mature into the absolute one. The Act of 1875 (of which Lord Cairns was the author) was brought into operation, but after four or five years' experience Lord Cairns discovered that it was absolutely unworkable. He found out that the possessory title that was registered would not mature into the absolute one, the result being that he set to work to reform conveyancing in another direction. As a result of those labours, the Conveyancing Act of 1881 was passed, the Act under which conveyancing was dealt with at the present time in this country. That Act of 1881, which was the Act the advocates of registration wished to repeal, was, in his opinion, as far as this country was concerned, an admirable one. It cut down the length of the titles, and further placed solicitors' remuneration on a sound, rational, and well-understood basis, the basis on which solicitors were now paid being a very moderate scale of 30s. per-cent up to £1,000, £1 per-cent from £1,000 to £5,000, and then less and less, until it became ultimately 5s. per-cent; so that if any of the members desired to sell property worth, say £100,000, they would have to pay the solicitor £295 for the work done in connection with the transaction. Lord Cairns, having placed the conveyancing practice of the country on that basis, did a marvellously good work for the whole of the public.

He mentioned these facts because two of the greatest experts in connection with the matter, Lord St. Leonards and Lord Cairns, deliberately set aside the registration system as being absolutely unworkable in this country, and proceeded on other lines. Although Lord Cairns gave up the Act of 1875 as unworkable, Lord Halsbury, the year after he took office, brought in a Bill to make the system compulsory. The result was the creation of the large office in Lincoln's Inn Fields. When the Compulsory Act was passed, it was stated that it was only to be tried in one county for three years as an experiment. The three years came to an end in December, 1901, and everybody naturally expected that an enquiry would be held, but such had not yet taken place. Another important consideration was, that opponents of the system were hampered in dealing with the question, owing to the absence of figures. For the first four years after the system came into operation a return was made annually, up to 1902, giving the number of titles registered as Possessory, Qualified, and Absolute, respectively; but, after 1902, notwithstanding the pressure brought to bear on the Government, they had absolutely refused to give any returns showing what the figures were. Dealing a little more practically with the system itself, in the first place the merit claimed for it was its simplicity. He held in his hand Mr. Brickdale's book of over 600 pages on the Act. Anyone who desired to make himself acquainted thoroughly with the nature of the system could do so if he read through the book. Referring to the statement on page 246, that changes had to be made from time to time when the need for them was apparent, he would remark that when the Act was first brought into operation in 1899, there were, in addition to the Act of 1875, which was made compulsory by the Act of 1897, 280 rules issued, with 67 forms; but in 1904 the whole of the earlier rules were repealed, and there were now in existence 371 rules, with 72 forms. That was the simple system which the authorities were urging should be extended all over the country; and yet the author had stated that the Land Registry proposed to make further alterations, as the need for them from time to time arose. One would have thought that, before the system was made compulsory, care would have been taken to draw up rules which would meet all cases, and not throw the expense that had been involved on people who had dealings at the present time with land. Lord Cairns' idea with regard to possessory certificates was that in course of time they would mature into absolute titles. As a matter of fact, it was now admitted by the Registry itself that that result could not be brought about. For instance, in the case of any deed which was prepared purporting to convey a property to a person, the Registry would not look behind that deed to see the state of the title; they expressly excluded any prior dealings with the property, but looked at that deed only. The Registry admitted that, at the time the deed was brought in, there might be a large mortgage on the property, but the Registry shut its eyes to the existence of that mortgage. He asked every reasonable person present how the certificate of the Registry, which did not disclose that there was a mortgage on the property, could ever become a good root of title, having regard to the fact that the Registry said they could not

guarantee what had gone in front of it. In his opinion, it was always necessary to have an independent investigation. The case of the *Capital and Counties Bank v. Rhodes*, referred to in the paper, was a very important one, and showed the dangers of the system. The matter was somewhat technical, but the Act of 1897 said that the legal estate could not be obtained unless the purchase deed was registered; but the Court of Appeal had said that, once the legal estate was on the register, the Registry could be ignored in subsequent transactions. The law required them to put the property on the register for the first time, and that having been done it could be dealt with outside of the Registry. He suggested that that decision cut the ground from under the whole system, and showed how absolutely unworkable it was. He was rather startled in reading the paper to notice the manner in which the author referred to mortgages. There was a form provided by the Act, but it was admitted by the author that the form did not meet a very large number of conditions and circumstances which the ordinary mortgagee, exercising ordinary precautions, was bound to adopt. A layman was not acquainted with the practical details which a practising solicitor was bound to understand; it required a law expert to approach a subject of the kind. The author remarked, on page 258, "It is said that without the ownership of the property, or what is called the 'legal estate', the mortgagee is without certain powers. Apparently he cannot distrain for rent, and it is doubtful whether he can bring an action for ejectment of tenants, which in some circumstances may be necessary; nor, in the case of leaseholds, can he rectify a breach of covenant." Those were things he could not do if he relied on the form issued by the Registry. "In what proportion of cases he would require to make use of such rights it is difficult to say, but probably, in the great majority of small mortgages at any rate, there is no necessity to provide for them." But how were they to know when making a mortgage whether anything of the kind was necessary? They did not know whether it would be necessary to serve notice on the tenants, nor whether it would be necessary to bring an action for ejectment. Personally, he would say that a solicitor was guilty of negligence who simply relied on the form, and did not take a proper mortgage. He thought he had said sufficient to show that even the 72 forms at present issued were not sufficient to enable a solicitor to carry out an ordinary mortgage transaction, which was a matter of every-day work in every solicitor's office. With regard to the question of absolute titles, the Registry said that although it was true that everybody was now registering possessory titles, that was not what they intended should happen; they must wait until an absolute title was obtained. He had, however, had some experience already of how registering absolute titles hampered business. A client of his, a land society, bought a property with an absolute title, but the difficulties connected with the matter had become so great that they had to ignore the fact that it was registered, and to grant their conveyances as before registration. In another case, which came under his notice not very long ago, a land society whose practice it was to issue their conveyances free of cost to the purchasers, negotiated for the purchase of an estate. After two or

three months' negotiation, terms were agreed upon, but when the contract for the purpose of carrying out the purchase was obtained, it was discovered that the land had been registered with an absolute title. The society declined to proceed further, because they came to the conclusion that their business would be greatly hampered if they bought an estate under such conditions. The Land Registry had not, however, been content with having only three kinds of titles. They found it necessary to invent a fourth, called a "good leasehold" title. Under Lord Cairns' Act of 1881, there was one title and one title only, the absolute title. In the present registration system there were now four sorts of title; and the fact that there was a choice of four titles instead of one did not make for simplicity. In every other country where a registration system existed there was one title only, the absolute title. The good leasehold title seemed to have been thought of and introduced, because the Registry Office had an idea, adopted by Mr. Hart, that when a lease was taken up the lessee was precluded by the laws from investigating the lessor's title. No person could be obliged to take up a lease without investigation unless he voluntarily contracted to do so. In practice few people took up the lease of a valuable property, without ascertaining something about the right of the lessor to grant the lease. The returns which had been made up to the present time showed that between £60,000 and £65,000 a year was expended in dealings with property in London, in addition to the cost under Lord Cairns' Act. He would like to have said a few words about the delay caused by the system, but time would not permit. Then it was said that the system of registration prevented fraud. In his opinion, however, it opened a wide door to fraud. Two cases had already come under the notice of the Courts which proved this statement up to the hilt—the cases of *Attorney-General v. Odell* and *Marshall v. Robertson*. Anyone could go to the Registry with a deed and receive it back marked "Registered," and also a certificate of registration. Thus two documents of title were created in place of one. This duplication of documents gave an easy opportunity of committing fraud. He would like Mr. Hart to listen to the facts of a case which had not been referred to in his paper. A person obtained a deed with reference to some land at Hampstead, purporting to convey the property to, say, Mr. Brown. That deed was taken into the Registry, and without any question, registered, and Mr. Brown brought out of the Registry his deed, which purported to show he was the owner of the property, together with a certificate showing the same thing. He promptly went and deposited the certificate with two ladies who lent a sum of money upon them. Unfortunately the true owner of the land was not aware of the transaction, but he became aware of the fact about nine months afterwards. He then brought an action against the two ladies to rectify the Register by removing their charge, and the Judge stated that although he was very sorry for them, as they had lent their money on documents which a lay person might believe to be of value, their charge was absolutely valueless. This was an instance of how, in his opinion, the present registration system facilitated fraud.

MR. J. B. GILLISON desired, in the first place, to express his



appreciation of Mr. Hart's services in giving to the Institute the results of his experience and researches on the subject. It was one of perhaps greater importance to lenders than to purchasers, for lenders did not, as a rule, trouble about what was being done with a property so long as the interest on the advance was regularly paid, and years might therefore elapse before any fraud was discovered. He was pleased to notice that Mr. Hart approved of a system of registration of title. What every purchaser and lender wanted was a good title. If there were no registration of title, the purchaser bought, and the lender advanced, on the opinion of his solicitor. A system of registration of deeds only enabled the solicitor to express that opinion more speedily and with less necessity for inquiry. However reliable and trustworthy a solicitor might be, there was always the risk—a small one no doubt, but still a risk—that the title might not be good, in which case the purchaser or lender suffered. With registration of title there was practically a Government guarantee, and consequently greater safety and expedition. He made those remarks notwithstanding what had fallen from Mr. Rubinstein, because he had had a long experience of the Australian Acts. An important consideration was the question of cost. There would always be a certain amount of opposition from solicitors, whose work and remuneration would no doubt be reduced in the long run by a system of registration of title. If one solicitor's opinion was as good as another's, why should a title be investigated up to a certain point more than once? There was clearly a saving to be made there. It had been stated that titles were investigated every ten years. Why should they be investigated more than once up to a certain point? Let that investigation be made by a properly qualified man, who would then give a certificate, and the matter was finished. When a Registrar of Titles had made his examination and expressed his opinion, a definite point had been reached, and future investigations need not go back further than that particular point. There was no reason why that examination should be more costly than that by a solicitor. He had had considerable experience of the working of the Torrens Acts in Australia, and could testify to the simplicity and value of the system. It was not at all unusual for a borrower to walk into an office and produce his title—not a big bundle of deeds, but a sheet of parchment with the land duly described and shown on a plan in the margin. Subsequent steps to obtain a loan were easy, and when it had been granted the company's solicitors did not take long to prepare a mortgage, register it, and pay over the money. The cost was much less than it would be under the general law, usually about one-half. Mr. Rubinstein stated that it cost £295 to transfer a property of the value of £100,000. He would like to know what some of the solicitors in England would say to the scale of mortgage fees in Australia, namely, £1 per £1,000 over £5,000. A mortgage could be prepared in Adelaide for 1½ guineas up to £500, and the scale of fees in connection with the transfer of land was exceedingly low. It was not unusual in Australia to insist at the time of lending that land under the general law should be brought under the Act, or to charge a slightly higher rate of interest if it were not. One reason for that was that there was much delay



and expense in the event of foreclosure if the land were under the general law. He had known of many cases in which the whole cost of foreclosure under the Act did not exceed £20, and that without vexatious delays. In Victoria, where a mortgage was taken under the general law, and the land was subsequently brought under the Act, it used to be necessary to obtain a fresh mortgage under the Act. If this had not been done, although the land was under the Act the mortgage was not, and foreclosure proceedings would therefore have been necessary under the general law. Now, however, under an Act passed in November, 1904, application might (not must) be made to the Commissioner of Titles instead of to the Court, thus simplifying the procedure and minimizing the expense. The examination of securities in the office was very simple. The certificate of title, with mortgage to the company duly registered, and a duplicate mortgage also marked registered, were sufficient. No big bundles of deeds with solicitor's list and certificate attached were required. That was a great saving of time to the auditors also. The great bulk of alienated real estate in Australia was held under the Torrens Acts. One of the main principles upon which Sir H. R. Torrens founded his plan was the registration of incumbrances, to be ascertained on inspection of the register. In Victoria the land described in a certificate of title was held subject, (1) to the reservations therein contained; (2) to any rights under adverse possession; (3) to any public rights of way; (4) to any easements acquired by enjoyment or user; (5) to any unpaid rates; (6) to any mining license. Members would particularly observe that the last five did not require to be registered on the title. When the Torrens system was introduced, even unpaid rates were not a charge on the land. Since then the Legislature had adopted the device of making wholesale statutory charges. In Melbourne there was a heavy liability for connection with the sewerage system and a special rate; under the Health Acts there was the cost of private roads and rights of way as well as sanitation; under the Vermin Destruction Acts there was the cost of rabbit destruction on the land, and half roads adjoining, as well as wire netting; under Water Acts there were irrigation charges; under Drainage Acts there was a share of charges; and under Thistles Acts there were charges for the destruction of noxious weeds, and there might be others. All these, however, could readily be provided for by inserting a clause in any new Act that all subsequent charges on the land must be notified as incumbrances. In Melbourne the building societies often took transfers (*i.e.*, conveyances) absolute in form, and a deed of defeasance was executed by the society. That was not illegal, though opposed to the policy and intention of the Legislature. Members would observe that a mortgagor had no apparent interest in the land on the register; while, at the same time, he had such an interest under the deed of defeasance which was not registered. Obviously, difficulties must arise, if judgment were signed against the mortgagor, and an attempt made to sell his interest in the land. He quite appreciated the necessity in this country for registration of leasehold titles. In Australia there had not been any great necessity. Indeed, he fancied the policy of the Torrens Acts had had something to do with the

disinclination of the people to have anything to do with long leaseholds. If the value of a leasehold were anywhere near the value of the freehold, the purchaser wanted the freehold. He was not disposed to allow the ground landlord to sit still and do nothing, while all the time a valuable "unearned increment" might be accruing. He recollected an estate in one of the suburbs, to be leased on building lots for 99 years. The owners could not dispose of it, as people went elsewhere, and he (Mr. Gillison) thought they were compelled to sell as freehold. In connection with the assurance fund, another instance occurred to him besides that mentioned in Mr. Hart's paper, that of *Messer v. Gibbs*. Mrs. Messer, the owner, lived in Scotland. She gave a Power of Attorney to her husband, who visited Australia and lodged it and the certificates of title with a solicitor. The solicitor forged a transfer (conveyance) from the attorney to a mythical person called Cameron, and then prepared a mortgage from Cameron for £3,000. The forgery and fraud were discovered, the solicitor absconded and could not be traced. Mrs. Messer was granted by the House of Lords a fresh certificate of title free of incumbrance, but the unfortunate lenders got nothing out of the insurance fund, and had to pay the costs. It seemed to him that the above was so close to a case that would be covered by the Insurance Fund clauses of the Act, that the lenders should have been indemnified. One of the preliminary steps to bring land under the Act was a survey, and it was not infrequent to find a considerable surplus over the area given in the old general law title—a surplus sufficient in value to cover the cost of bringing the land under the Act.

MR. H. ROSS GILES (a visitor) remarked that he was diffident of speaking, as he felt that, as a solicitor, it would be said that he was prejudiced, but he would endeavour to approach the subject on lines quite outside his profession. He did not know whether any of those present were land-owners, or probable purchasers of land, but he had no doubt that they were all tax-payers, and he would first take the question from that point of view. They had been told what would happen when all the properties were on the register, but he believed he was right in saying that at the present time, despite the fact that some £65,000 had been received in fees on dealings in land, the Registry was an annual cost to the Government, and, in addition, the building in Lincoln's Inn Fields had been put up at considerable expense. As tax-payers, he contended it was wrong for any Government to thrust upon them the burden of protecting land-owners or dealers in land. Approaching the subject from the point of view of a purchaser of a property, or the owner of property, what would happen under the present system when he desired to sell? The advocates of the Registry stated that the advantage of the system was that when land was sold purchasers would be able to go the Registry and transact the business without a solicitor. But was it any advantage to him, as a buyer, to know that a purchaser forty years hence of the property which he might then have to sell would be able to deal with that land at a lower cost? There were two grounds, therefore, on which he said that really the Registry could not make out a good case for its existence, or for imposing compulsory

registration upon the people of the country. Again, everybody knew that an official was apt to deal with any matter upon certain set lines. He was bound by rules, and was in difficulties at once if he departed from those rules. Everybody knew that at times owners of property were anxious to deal with that property at very short notice, but if Government officials had to be dealt with, owners of property would find that they had to take their turn; whereas, if a man were dealing with a solicitor, and said that time was a matter of moment, the solicitor would carry the work through, or if he would not, and made difficulties, the client would go to another one. Coming to another point, the author had said in the paper that the present system ought to be widely advertised, so that even where the public do not know how to deal with their land themselves, they may at least understand the forms, and so be saved the risk of lending money on a possessory title without investigation of the prior title. He had in his possession the official circular which had been issued by the Land Registry, headed, "Short statement of the objects and Practical Working of the System of Registration of Title, including particulars of the New Regulations, &c." There were eight or ten pages of the circular, but there was not a word in any of them of caution to property owners or the public. Mr. Gillison had asked where was the necessity for so much investigation of title; why not have a title investigated by an official, and then the matter would be settled for all time? If Mr. Gillison would study the case of *Marshall and Robertson*, he would find that even after the title had been investigated by an official it was possible for the purchaser or person lending money on the strength of the certificate to lose his money. Mr. Gillison had also said that in Adelaide the cost of a mortgage for £500 was £1. 11s. 6d., but it was impossible to come down to those figures in this country, because the official fee alone was 30s.

DR. A. W. FINDLAY said he felt rather diffident in taking up a line of argument against the paper. Land law reform during the past century had been carried on through two lines. The Fines and Recoveries Act, 1833, the Real Property Act, 1845, Lord Cranworth's Act, 1860, and the Conveyancing Acts, were all passed with the object of facilitating conveyancing by the shortening of deeds, incorporating by statute in conveyances and mortgages those common form covenants and clauses which previously had made these deeds of inordinate length. This line of reform met with immediate success. The Acts of 1862, 1875 and 1897, on the other hand, were passed with the avowed object of establishing a Registry which would ultimately become the *fons et origo* of titles to land, with the consequent effect that in the end no dealing with land could be carried out without an entry on the register. This system in its voluntary stage proved an absolute failure, and it was not until the Act of 1897 made registration compulsory that land-owners to any extent registered their land, and the onus of proving that compulsory registration was beneficial lay upon those who supported it. Land-owners had the advantage of utilizing all the facilities of the Land Registry under the Acts of 1862 and 1875. Under these Acts, it was practically impossible to get land-owners to register their titles, and those titles which were registered, and had been dealt with since,

had been a perfect plague to the conveyancers who had had to deal with them. He had had some considerable dealings with the Land Registry, being in the fortunate position of being both a member of the Institute and also of the profession which felt to a certain extent where the shoe pinched in connection with the registration of land. In the first place, in the ordinary course a purchaser had to get the conveyance, and then take it to the Registry. That was an additional step, and cost additional expense and time. A possessory certificate was obtained, which, as was admitted by the Registrar himself, could not in any case form the root of title, and consequently it was necessary to go back to the conveyance upon which that certificate was issued and prove the title to that upon any subsequent dealing with the land. Therefore, nothing whatever was gained by this registration. Compulsory registration only applied to purchases on sale in fee simple, on grants and assignments of leases having an unexpired term of forty years still to run, so that it only touched the fringe of conveyancing, and at present only applied to the County of London. There were thus at present in use two parallel systems of conveyancing. There was the ordinary system for transactions taking effect independently of the Register and the registration system, which was simply the ordinary system plus the additional machinery of the register; at any rate in the case of possessory registration, the only form which was compulsory. Even granting that the Land Registry and the owners would benefit in the course of time, he thought the onus was still upon them to show that the cost was worth the benefit the land-owners were going to get, and up to the present that had not been done in any shape or form. It consequently seemed to him that, before any further advance was made in connection with extending the registration system of this country, the long-deferred enquiry ought to take place. Further evidence was certainly wanted as to the present benefits acquired by land-owners from the registration of their titles, and what further benefits would accrue by making the system compulsory throughout the country.

Mr. J. E. FAULKS said that, in his introduction, Mr. Hart gave as one of the reasons why the subject of the paper should prove of special interest to actuaries, the fact that actuaries had, or, as he said, were supposed to have an elementary knowledge of legal principles. With all due deference to Mr. Rubinstein, he (Mr. Faulks) cordially welcomed the present paper, which he thought showed that the actuary's knowledge of legal principles was not always a matter of supposition, and in some cases at any rate could, from a layman's point of view, hardly be described as elementary. For the benefit of those who had not studied the subject, it might be well if he reminded the members that in the course of lectures on the Law of Mortgage delivered before the members of the Institute in 1900-1901, the lecturer, Mr. Hayter, drew attention to the subject of the Land Transfer Acts, and his sixth lecture might well be read as an introduction to the present paper. Those lectures were, of course, delivered before the rules of 1903 were made, and in Mr. Hart's paper the members had before them the effect, so far as was material to his purpose, of the alterations and extensions carried



out in the Land Transfer scheme by means of those rules, combined with a fuller and more detailed account of the subject generally, looking at it not only from the legal but also from the practical or business point of view. He had intended to refer to some points connected with the history of registration, but they had been dealt with by previous speakers, and he did not think he need go over them again. The consideration of the question from the actuary's point of view, or indeed from any but a strictly legal one, seemed to divide itself into two branches. First, was a system of registration of title desirable; was it possible; and what was the ideal system? Secondly, how far were the objections which were frequently raised to the present system valid, and how far could they be removed or mitigated? As to the desirability of some such system, if it were possible, the preponderance of business opinion seemed to be overwhelming, while as to its possibility the experience of other countries, Colonial and Continental, seemed at any rate to show that a full and fair trial was worth making, even if all due weight were given to the peculiarities of real property law, which had descended to them from the feudal system. Under an ideal system they would, he thought, have registration of absolute titles only, or at any rate of only absolute and good leasehold titles. Eventually the register would prove, he took it, analogous (though perhaps the analogy must not be pressed too far) to the stock or share registers of the public funds and trading companies. When once a plot of land had been identified with sufficient accuracy, it might be said to correspond to shares bearing distinctive numbers, and when the owner of that plot of land had once been registered with absolute title, the transfer of the land should be as simple as a transfer of shares. Real property would be in fact, to a large extent, made transferable in the same way as personal. In this connection, was there not something to be said for the action of the Treasury in *Attorney-General v. Odell*, which Mr. Hart referred to in his paper? If the facts had been, as in that case, except that the forged transfer was one of stock and not land, would it not now be held, on the authority of the House of Lords' decision in *Sheffield Corporation v. Barclay*, that the defendant, in tendering for registration a forged transfer, warranted its genuineness, and must therefore bear the loss? That, however, was a side issue, and he thought that the opinion of most actuaries would be that a land register on the lines of a stock register would greatly facilitate business transactions, and be an immense improvement on the old system.

One could not help wishing that it had been found possible to insist on registration only with absolute freehold or absolute leasehold title, and at the same time by arranging from the outset for a reduction of charges by means of deferred payments or otherwise, in something like the same way as (though possibly to a greater extent than) in the 1903 rules, make the burden easier to those registering, and, in effect, bribe landowners to register. With regard to the objections to the present system, no one who had read the memoranda and reports of the Law Society on the subject could fail to perceive that such objections were very widely and strongly held. Of course, to a mind that strove to be impartial, many of the arguments on both



sides were only worthy of being neglected. It seemed to him, if he might say so, absurd for the advocates of registration to suggest that the opposition of solicitors as a class was based solely or mainly upon a fear for their own pockets, and equally absurd for the opponents of registration to say that the faith of its supporters was largely founded on the desire to create an enjoy an extensive system of patronage. Then came a class of objections to the scheme that referred to facts or mistakes of temporary importance, that had really no bearing on the main issue. In the establishment of such a system, as in the placing of some 75,000 titles on the register, mistakes and delays must necessarily occur. An amusing instance might be quoted from the legal Press of 1903. There was a deposit of a land certificate by way of equitable mortgage. The equitable mortgagee died, and the probate was registered with the Land Registry. The mortgage having subsequently been paid off, all the executors signed and sent in a withdrawal of notice of deposit. The Land Registry issued a notice in the following form:—"To A. B. An application, dated 25th March, 1903, signed by your executors, and purporting to be a withdrawal of notice of deposit of land certificate has been left for registration. If you have any objection it should be stated in writing." Obviously, in setting up a system of registration mistakes of that kind must arise, and delays must also occur; but from experience in other directions, actuaries might feel assured that, as time went on, and details were perfected, such mistakes and delays would be eliminated, or at any rate their occurrence would be minimized. Sweeping aside objections of that character, they arrived, he thought, at three which might be referred to as (1) officialism, (2) compulsion, (3) deeds off the register. With regard to officialism, while he did not think that the members of the legal profession would agree with Mr. Hart's statement that the officialism of the Yorkshire Deed Registry, with which he presumed the author coupled the old Middlesex Registry, was not felt (as to which the statement of a very eminent lawyer, not a solicitor, Lord Davey, that the chief result was to confer valuable patronage on the Lord Chancellor, and to increase the cost of conveyancing, and occasionally furnish materials for litigation, was instructive), yet it did seem to him that the disadvantages of so-called officialism were vastly over-rated. At any rate the drawbacks of officialism did not seem weighty enough to counterbalance the advantages of registration. With regard to compulsion, Mr. Hart put the matter in a nutshell when he said that he believed the system to be good, but that it must have a fair field for its trial, which it had been found impossible to obtain under a voluntary system, and that therefore recourse must be had to compulsion. With regard to the third objection, deeds off the register, the result of the decision in the *Capital and Counties Bank v. Rhodes*, seemed to him at the moment to be the most vital objection to the system. Yet they should bear in mind that that case was largely decided upon a question of merger, and that, to quote the words of a very recent writer, "The value of the dicta in the judgments is lessened by the very evident fact that the members of the Court were not familiar with either the principles of the system or its practical working." Those words were written by Mr. Hogg,

to whom the author referred in his paper, in his last work on Registered Land. The general conclusion seemed to him to be that the time had now arrived when an impartial enquiry into the working of the system might with advantage be held, and, in spite of the strong opposition, he personally believed that the result of such an enquiry would be to show that, with certain alterations and improvements, the system seemed so likely to confer in the not far distant future substantial benefits on owners and chargees of land, that it should undoubtedly be persevered with.

THE PRESIDENT said the members extended a hearty welcome to Mr. Hart, who had, at considerable trouble, brought together a great deal of interesting and useful information on matters which touched a great many actuaries in their daily work. He asked the members to accord to the author a very hearty vote of thanks for his excellent paper.

The resolution having been carried with acclamation,

MR. HART, in reply, said that Mr. Faulks had summed up so well the views which he (the speaker) held, and had answered Mr. Rubinstein's and other speakers' objections to the system so thoroughly, that it was not necessary for him to deal fully with all the points that had been raised. Mr. Rubinstein dealt at great length with the paper, but fortunately he (Mr. Hart) had read all his arguments before and given them very careful consideration. There was hardly one of them that could not be controverted. One great objection raised was that the system was introduced by Lord Halsbury, who was not a conveyancing expert. That seemed to him to be a great advantage of the system. Sir Robert Torrens, who introduced the Australian system, was a Commissioner of Customs, and he was so struck with the facility with which ships could be transferred on a register that he set to work, when he was given an appointment in a Deed Registry in Australia, and devised his well-known and very much admired and appreciated system. Then Mr. Rubinstein, apparently, had a great admiration of the present system of conveyancing, and quoted the small sum which was charged for conveying a property worth £100,000, namely, £295. He did not pretend that, under present conditions, all the benefits of cheapness were obtained at the Land Registry, but when more absolute titles were registered he thought there was not the slightest doubt they would be better off than under present conditions. For instance, with a property worth £5,000, the vendor's costs for selling would be seven guineas, and with unregistered land they would be forty-five guineas; in the case of registered land worth £50,000 the cost of transfer would be twenty-two guineas, and for unregistered land 170 guineas, and the solicitor's costs were cut down from one and a-half guineas per-cent to half a guinea per-cent. Mr. Rubinstein took objection to his claim that in many cases they could do without the legal estate, and asked how they would be able to judge without a solicitor. Personally he should say that a solicitor was not the best man to say how one was to judge. A solicitor did not think of practical considerations which a commercial man took into account; he was too much attached to technicalities, and in some respects was obliged to give weight to those technicalities, because he was so

frightened of making a mistake for which he was responsible to his client. Then, Mr. Giles had recommended that the members should make a practical attempt to carry a transaction through the Registry. He himself carried a small one through, and was very much struck with the simplicity and ease with which he got a good leasehold title; it only cost a few shillings and the making of a declaration. He desired to thank the members for the kind reception given to his paper, and for the cordial vote of thanks passed to him.

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MR. J. ARTHUR DAWES (a visitor), who was prevented by the lateness of the hour from joining in the discussion, has sent us the following for publication:—

I should like to say a few words in reference to Mr. Hart's carefully-prepared and well-reasoned paper. I take a somewhat different view of the Land Transfer Act, 1897, from that taken by many of my professional brethren, inasmuch as I believe it to be an honest and, up to the present, a very fairly successful, attempt to deal with an extremely difficult problem. I imagine that our system of land tenure is nearly, if not quite, the most complicated one ever known, derived as it is from several sources and retaining in many respects its feudal characteristics, so that any attempt to deal with the subject heroically would have been foredoomed to failure. It is a matter of fairly universal agreement that improved facilities for dealing with land are highly desirable, and, while it is not fair to compare our land conditions with those obtaining in comparatively new countries, such as Australia and Canada, it seems to me not only undignified but rather cowardly to sit down and say that because nothing has been done in the past, nothing should be done in the future. This, however, is rather the attitude adopted by Mr. Rubinstein and the other out-and-out opponents of the Land Transfer Act 1897, as I have never heard of their putting forward any alternative proposal or suggestion for simplification of land transfer. Granted that the Acts of 1862 and 1875 have been failures; granted that the Middlesex and Yorkshire Registries have not been complete successes; granted that the Scotch system causes irritating if unavoidable delay; are these sufficient reasons for condemning the new system, before it has had time to get into working order? I will at once admit that the Act has caused the appearance of a number of new officials, primarily paid by the tax-payer, and I personally think the late Government would have been well advised to commence operations in some county other than the Administrative County of London. But the question is not whether we have, at the present moment, an ideal system, but will the Act, the Registry, and the officials justify their existence in the future. My own opinion is that they will. The system has not yet had a fair chance, as before first registrations, and even after first registrations, where a possessory title only exists, investigation of title is necessary. This, time will cure, but even now, in the case of a lease recently granted, and immediately registered, it will be found that, instead of a deed of assignment, which may or may not be a comparatively lengthy document, you can fill up a printed form,

register it, and your title consists of two documents, the original lease and the land certificate, and that however many subsequent dealings there may be, you will never have more than these, with possibly, the addition of a charge certificate. Further, in justice to the officials of the Registry, who have been so often attacked for causing delay, I should like to say that my experience has been that documents are returned promptly, and that every endeavour is made to facilitate business. I will not deal at any length with the allegation that under the new system fraud is possible, as the number of frauds perpetrated under the old system and the ease with which they were carried out, is a sufficient answer. It is obviously impossible within the compass of a debate, such as that on Mr. Hart's paper, to deal exhaustively with this important subject, but my strong view is that eventually the new system will greatly simplify the transfer of land; and although Mr. Rubinstein and those who think with him may not agree, anything that tends to that end must encourage dealings with property, and so incidentally (to look at the personal aspect for a moment) benefit the legal profession. Finally, I would say that it is far too early for the official enquiry and investigation so strenuously insisted on by Mr. Rubinstein. Give the system, say, ten years' fair trial, and then let there be the fullest and most searching enquiry as to whether its objects have or have not been attained.

## ACTUARIAL NOTES.

### I.

*Interpolation by Finite Differences (Two Independent Variables).*

By HERBERT H. EDWARDS.

TO find  $u_{x;y}$  in terms of  $u_{0;0}$ ,  $u_{0;5}$ ,  $u_{0;10}$ ;  $u_{5;0}$ ,  $u_{5;5}$ ,  $u_{5;10}$ ;  $u_{10;0}$ ,  $u_{10;5}$ , and  $u_{10;10}$ .

By Lagrange's interpolation formula,

$$u_{x;y} = \frac{y(y-5)(y-10)}{y \times 5 \times 10} u_{x;0} + \frac{y(y-5)(y-10)}{(y-5)(+5)(-5)} u_{x;5} + \frac{y(y-5)(y-10)}{(y-10)(-5)(-10)} u_{x;10}$$

$$= f(y)u_{x;0} + F(y)u_{x;5} + \phi(y)u_{x;10}$$

say,

$$\begin{aligned} &= f(y) \{ f(x)u_{0;0} + F(x)u_{5;0} + \phi(x)u_{10;0} \} \\ &+ F(y) \{ f(x)u_{0;5} + F(x)u_{5;5} + \phi(x)u_{10;5} \} \\ &+ \phi(y) \{ f(x)u_{0;10} + F(x)u_{5;10} + \phi(x)u_{10;10} \} \\ &= f(y)f(x)u_{0;0} + f(y)F(x)u_{5;0} + f(y)\phi(x)u_{10;0} \\ &+ F(y)f(x)u_{0;5} + F(y)F(x)u_{5;5} + F(y)\phi(x)u_{10;5} \\ &+ \phi(y)f(x)u_{0;10} + \phi(y)F(x)u_{5;10} + \phi(y)\phi(x)u_{10;10} \end{aligned}$$

where  $f(x)$ ,  $F(x)$  and  $\phi(x)$  are functions of  $x$  similar to  $f(y)$ ,  $F(y)$  and  $\phi(y)$ .



If we could readily determine the values of the products of the various functions of  $x$  and  $y$  the calculation would be greatly simplified.

To find these values we must first draw up a table similar to the following, showing the values of the functions  $f(x)$ ,  $F(x)$ , and  $\phi(x)$  when  $x$  equals 1, 2, 3, 4, 6 and 7.

TABLE I.

| $x$ | $f(x)$ | $F(x)$ | $\phi(x)$ |
|-----|--------|--------|-----------|
| 1   | ·72    | ·36    | −·08      |
| 2   | ·48    | ·64    | −·12      |
| 3   | ·28    | ·84    | −·12      |
| 4   | ·12    | ·96    | −·08      |
| 6   | −·08   | ·96    | ·12       |
| 7   | −·12   | ·84    | ·28       |

So that if we wish to determine  $u_{2;6}$ , the coefficients of

$u_{0;0}$ ,  $u_{5;0}$ ,  $u_{10;0}$ ;  $u_{0;5}$ ,  $u_{5;5}$ ,  $u_{10;5}$ ;  $u_{0;10}$ ,  $u_{5;10}$  and  $u_{10;10}$  would be  $f(6)f(2)$ ,  $f(6)F(2)$ ,  $f(6)\phi(2)$ ;  $F(6)f(2)$ ,  $F(6)F(2)$ ,  $F(6)\phi(2)$ ;  $\phi(6)f(2)$ ,  $\phi(6)F(2)$  and  $\phi(6)\phi(2)$ , or  $(-·08)(·48)$ ,  $(-·08)(·64)$ ,  $(-·08)(-·12)$ ;  $(·96)(·48)$ ,  $(·96)(·64)$ ,  $(·96)(-·12)$ ;  $(·12)(·48)$ ,  $(·12)(·64)$  and  $(·12)(-·12)$ , taking the values of  $f(6)$ ,  $f(2)$ ,  $F(6)$ ,  $F(2)$ ,  $\phi(6)$  and  $\phi(2)$  from Table I.

In a similar manner, Table II has been calculated, and in Table III the logarithms of the various coefficients in the second table have been entered.

TABLE of  $a_{xy}$  by OM 3 per-cent.

| Age of<br>younger<br>life $y$ | AGE OF ELDER LIFE $x$ |        |        |
|-------------------------------|-----------------------|--------|--------|
|                               | 20                    | 25     | 30     |
| 10                            | 20·543                | 19·706 | 18·731 |
| 15                            | 20·133                | 19·364 | 18·453 |
| 20                            | 19·588                | 18·901 | 18·070 |

If an arithmometer be used the second table enables us to effect the interpolation without writing down a figure. Another advantage of the use of these tables is that they enable us to deal with the functions themselves and not with their successive orders of differences. In practice this is a great consideration, and an appreciable amount of labour may often be saved even when



dealing with a function of one independent variable, but when using functions of two independent variables the difficulty of differencing is greatly increased and the superiority of the alternative method of working becomes more apparent.

To show the application of the tables we may first find  $a_{25:14}$  by means of Table II.

TABLE II.

*Coefficients for Interpolation in a Table of Two Independent Variables.*

| COEFFICIENTS OF |     |            |            |               |            |            |               |               |               |                  |
|-----------------|-----|------------|------------|---------------|------------|------------|---------------|---------------|---------------|------------------|
| $x$             | $y$ | $u_{0:0}$  | $u_{0:5}$  | $u_{0:10}$    | $u_{5:0}$  | $u_{5:5}$  | $u_{5:10}$    | $u_{10:0}$    | $u_{10:5}$    | $u_{10:10}$      |
|                 |     | $f(y)f(x)$ | $F(y)f(x)$ | $\phi(y)f(x)$ | $f(y)F(x)$ | $F(y)F(x)$ | $\phi(y)F(x)$ | $f(y)\phi(x)$ | $F(y)\phi(x)$ | $\phi(y)\phi(x)$ |
| 1               | 1   | ·5184      | ·2592      | −·0576        | ·2592      | ·1296      | −·0288        | −·0576        | −·0288        | ·0064            |
| 1               | 2   | ·3456      | ·4608      | −·0864        | ·1728      | ·2304      | −·0432        | −·0384        | −·0512        | ·0096            |
| 1               | 3   | ·2016      | ·6048      | −·0864        | ·1008      | ·3024      | −·0432        | −·0224        | −·0672        | ·0096            |
| 1               | 4   | ·0864      | ·6912      | −·0576        | ·0432      | ·3456      | −·0288        | −·0096        | −·0768        | ·0064            |
| 1               | 6   | −·0576     | ·6912      | ·0864         | −·0288     | ·3456      | ·0432         | ·0064         | −·0768        | −·0096           |
| 1               | 7   | −·0864     | ·6048      | ·2016         | −·0432     | ·3024      | ·1008         | ·0096         | −·0672        | −·0224           |
| 2               | 2   | ·2304      | ·3072      | −·0576        | ·3072      | ·4096      | −·0768        | −·0576        | −·0768        | ·0144            |
| 2               | 3   | ·1344      | ·4032      | −·0576        | ·1792      | ·5376      | −·0768        | −·0336        | −·1008        | ·0144            |
| 2               | 4   | ·0576      | ·4608      | −·0384        | ·0768      | ·6144      | −·0512        | −·0144        | −·1152        | ·0096            |
| 2               | 6   | −·0384     | ·4608      | ·0576         | −·0512     | ·6144      | ·0768         | ·0096         | −·1152        | −·0144           |
| 2               | 7   | −·0576     | ·4032      | ·1344         | −·0768     | ·5376      | ·1792         | ·0144         | −·1008        | −·0336           |
| 3               | 3   | ·0784      | ·2352      | −·0336        | ·2352      | ·7056      | −·1008        | −·0336        | −·1008        | ·0144            |
| 3               | 4   | ·0336      | ·2688      | −·0224        | ·1008      | ·8064      | −·0672        | −·0144        | −·1152        | ·0096            |
| 3               | 6   | −·0224     | ·2688      | ·0336         | −·0672     | ·8064      | ·1008         | ·0096         | −·1152        | −·0144           |
| 3               | 7   | −·0336     | ·2352      | ·0784         | −·1008     | ·7056      | ·2352         | ·0144         | −·1008        | −·0336           |
| 4               | 4   | ·0144      | ·1152      | −·0096        | ·1152      | ·9216      | −·0768        | −·0096        | −·0768        | ·0064            |
| 4               | 6   | −·0096     | ·1152      | ·0144         | −·0768     | ·9216      | ·1152         | ·0064         | −·0768        | −·0096           |
| 4               | 7   | −·0144     | ·1008      | ·0336         | −·1152     | ·8064      | ·2688         | ·0096         | −·0672        | −·0224           |

Turning to the tables giving the constants for interpolating in the case of two variables we see that the value of  $a_{25:14}$  is given by (see  $u_{4:6}$  in Table II)

$$- \cdot 0096 a_{20:10} + \cdot 1152 a_{25:10} + \cdot 0144 a_{30:10} - \cdot 0768 a_{20:15} + \cdot 9216 a_{25:15} \\ + \cdot 1152 a_{30:15} + \cdot 0064 a_{20:20} - \cdot 0768 a_{25:20} - \cdot 0096 a_{30:20}$$

which by the arithmometer gives 19·268. The true value is

19.267, and to give an idea of the labour involved it may be remarked that the interpolation took between two and three minutes.

As a second example we may take  $a_{28:16}$ , and to apply Table II we must take the coefficients opposite the line where  $x=2$  and  $y=4$ , because  $30-28=2$  and  $20-16=4$ , and this gives as the value of  $a_{28:16}$  the expression

$$\cdot 0576a_{30:20} + \cdot 4608a_{30:15} - \cdot 0384a_{30:10} + \cdot 0768a_{25:20} + \&c. = 18.760,$$

which agrees exactly with the figure in the published tables.

TABLE III.

*Logarithms of Coefficients for Interpolation in a Table of Two Independent Variables.*

|     |     | LOGARITHMS OF COEFFICIENTS OF |                   |                      |                   |                   |                      |                      |                      |                         |
|-----|-----|-------------------------------|-------------------|----------------------|-------------------|-------------------|----------------------|----------------------|----------------------|-------------------------|
| $x$ | $y$ | $u_{0:0}$                     | $u_{0:5}$         | $u_{0:10}$           | $u_{5:0}$         | $u_{5:5}$         | $u_{5:10}$           | $u_{10:0}$           | $u_{10:5}$           | $u_{10:10}$             |
|     |     | Log<br>$f(y)f(x)$             | Log<br>$F(y)f(x)$ | Log<br>$\phi(y)f(x)$ | Log<br>$f(y)F(x)$ | Log<br>$F(y)F(x)$ | Log<br>$\phi(y)F(x)$ | Log<br>$f(y)\phi(x)$ | Log<br>$F(y)\phi(x)$ | Log<br>$\phi(y)\phi(x)$ |
| 1   | 1   | 1.71467                       | 1.41364           | (2.76042)            | 1.41364           | 1.11261           | (2.45939)            | (2.76042)            | (2.45939)            | 3.80618                 |
| 1   | 2   | 1.53857                       | 1.66351           | (2.93651)            | 1.23754           | 1.36248           | (2.63548)            | (2.58433)            | (2.70927)            | 3.98227                 |
| 1   | 3   | 1.30419                       | 1.78161           | (2.93651)            | 1.00346           | 1.48058           | (2.63548)            | (2.35025)            | (2.82737)            | 3.98227                 |
| 1   | 4   | 2.93651                       | 1.83960           | (2.76042)            | 2.63548           | 1.53857           | (2.45939)            | (3.98227)            | (2.88536)            | 3.80618                 |
| 1   | 6   | (2.76042)                     | 1.83960           | 2.93651              | (2.45939)         | 1.53857           | 2.63548              | 3.80618              | (2.88536)            | (3.98227)               |
| 1   | 7   | (2.93651)                     | 1.78161           | 1.30449              | (2.63548)         | 1.48058           | 1.00346              | 3.98227              | (2.82737)            | (2.35025)               |
| 2   | 2   | 1.36248                       | 1.48742           | (2.76042)            | 1.48742           | 1.61236           | (2.88536)            | (2.76042)            | (2.88536)            | 2.15836                 |
| 2   | 3   | 1.12840                       | 1.60552           | (2.76042)            | 1.25334           | 1.73046           | (2.88536)            | (2.52634)            | (1.00346)            | 2.15836                 |
| 2   | 4   | 2.76042                       | 1.66351           | (2.58433)            | 2.88536           | 1.78845           | (2.70927)            | (2.15836)            | (1.06145)            | 3.98227                 |
| 2   | 6   | (2.58433)                     | 1.66351           | 2.76042              | (2.70927)         | 1.78845           | 2.88536              | 3.98227              | (1.06145)            | (2.15836)               |
| 2   | 7   | (2.76042)                     | 1.60552           | 1.12840              | (2.88536)         | 1.73046           | 1.25334              | 2.15836              | (1.00346)            | (2.52634)               |
| 3   | 3   | 2.89432                       | 1.37144           | (2.52634)            | 1.37144           | 1.84856           | (1.00346)            | (2.52634)            | (1.00346)            | 2.15836                 |
| 3   | 4   | 2.52634                       | 1.42943           | (2.35025)            | 1.00346           | 1.90655           | (2.82737)            | (2.15836)            | (1.06145)            | 3.98227                 |
| 3   | 6   | (2.35025)                     | 1.42943           | 2.52634              | (2.82737)         | 1.90655           | 1.00346              | 3.98227              | (1.06145)            | (2.15836)               |
| 3   | 7   | (2.52634)                     | 1.37144           | 2.89432              | (1.00346)         | 1.84856           | 1.37144              | 2.15836              | (1.00346)            | (2.52634)               |
| 4   | 4   | 2.15836                       | 1.06145           | (3.98227)            | 1.06145           | 1.96454           | (2.88536)            | (3.98227)            | (2.88536)            | 3.80618                 |
| 4   | 6   | (3.98227)                     | 1.06145           | 2.15836              | (2.88536)         | 1.96454           | 1.06145              | 3.80618              | (2.88536)            | (3.98227)               |
| 4   | 7   | (2.15836)                     | 1.00346           | 2.52634              | (1.06145)         | 1.90655           | 1.42943              | 3.98227              | (2.82737)            | (2.35025)               |

The table giving the logarithms of the coefficients is used similarly, the only point requiring care being the negative coefficients of some of the functions between which interpolation has to be effected. It should be noted that in Table III a bracketed logarithm indicates that the co-efficient is negative.

## II.

*Some Practical Hints on Two-Variable Interpolation.* By  
JOHN SPENCER, F.I.A.

THE problem of interpolating between values of a function of two variables is one that frequently arises in practice, and, considering its importance, it is surprising that so little reference to the subject is to be found in current actuarial literature.

Mr. W. M. Makeham gave a solution of the problem in his paper "On the calculation of the differential coefficients of a function from its differences . . ." (*J.I.A.*, vol. xvi, p. 98), and showed how tables for use in interpolation of this nature might conveniently be arranged. The only other contribution I have been able to find in the *Journal*, prior to publication of the present number, is Mr. T. G. Ackland's letter "On some practical applications of simple interpolation formulæ" (vol. xxxii, p. 286), though in an interesting contribution to *Biometrika* (vol. ii, part I) Mr. W. Palin Elderton has dealt with the problem somewhat more fully from the theoretical side. In the following remarks I hope to offer a few practical suggestions which may be helpful in certain cases which present themselves in interpolation work.

1. In the preceding note, which I have had an opportunity of seeing in proof, Mr. H. H. Edwards has ingeniously utilized Lagrange's formula in deducing a value for  $u_{xy}$ , expressed in terms of the original values. In many instances in practice, when the given values are equidistant, it is equally convenient for purposes of interpolation to employ the fundamental formula involving the successive orders of differences of those values. This formula may, as might be expected, be readily deduced by the powerful method of separation of symbols.

If the quantities available are  $u_{0,0}$ ,  $u_{1,0}$ ,  $u_{2,0}$  . . .  $u_{0,1}$ ,  $u_{0,2}$  . . .  $u_{1,1}$ , &c., and if by  $\Delta_x$  we denote the operation of

differencing with respect to  $x$ ,  $y$  remaining constant, a corresponding significance attaching to  $\Delta_y$ , so that

$$\Delta_x u_{0,0} = u_{1,0} - u_{0,0}$$

$$\Delta_y u_{0,0} = u_{0,1} - u_{0,0},$$

and so on, we have

$$\begin{aligned} u_{x,y} &= 1 + \Delta_x \cdot 1 + \Delta_y \cdot u_{0,0} \\ &= \left(1 + x\Delta_x + \frac{x \cdot x - 1}{2} \Delta_x^2 + \dots\right) \left(1 + y\Delta_y + \frac{y \cdot y - 1}{2} \Delta_y^2 + \dots\right) u_{0,0} \\ &= u_{0,0} + x\Delta_x u_{0,0} + \frac{x \cdot x - 1}{2} \Delta_x^2 u_{0,0} + \frac{x \cdot x - 1 \cdot x - 2}{6} \Delta_x^3 u_{0,0} + \dots \\ &\quad + y\Delta_y u_{0,0} + xy\Delta_x \Delta_y u_{0,0} + \frac{x \cdot x - 1 \cdot y}{2} \Delta_x^2 \Delta_y u_{0,0} + \dots \\ &\quad + \frac{y \cdot y - 1}{2} \Delta_y^2 u_{0,0} + \frac{x \cdot y \cdot y - 1}{2} \Delta_x \Delta_y^2 u_{0,0} + \dots \\ &\quad + \frac{y \cdot y - 1 \cdot y - 2}{6} \Delta_y^3 u_{0,0} + \dots \quad (1) \end{aligned}$$

The only point calling for remark is with reference to the quantities involving differencing with respect to  $x$  and  $y$  successively. These may be readily simplified as follows:

$$\Delta_x \Delta_y u_{0,0} = \Delta_x (u_{0,1} - u_{0,0}) = u_{1,1} - u_{0,1} - u_{1,0} + u_{0,0}$$

$$\Delta_x^2 \Delta_y u_{0,0} = \Delta_x^2 (u_{0,1} - u_{0,0})$$

$$= u_{2,1} - 2u_{1,1} + u_{0,1} - u_{2,0} + 2u_{1,0} - u_{0,0},$$

and so on.

The order of differencing is, of course, immaterial.

In actuarial work, when the necessity for two-variable interpolation confronts us, the data available generally consist of values of the function corresponding to combinations of two ages, each of which is a multiple of 5, and in most of the examples that follow it is assumed that such data alone are accessible. It will also be convenient, as it is in most cases sufficiently accurate, to neglect third and higher orders of differences.

When, as is sometimes the case, *e.g.*, in the British Offices' Two-Life Annuity Tables, the functions available are tabulated for each individual value of one variable, in combination with every fifth value of the other variable, the problem to be dealt with is not, strictly speaking, two-variable interpolation, but can be solved by the simpler method of interpolation applicable to functions of a single variable.

The following example will show how formula (1) may be applied.

From the "Short Collection of Actuarial Tables" we take (Carlisle 3%)

$$A_{30,60}^1 = \cdot 11669 \quad A_{35,60}^1 = \cdot 13190 \quad A_{40,60}^1 = \cdot 15494$$

$$A_{30,65}^1 = \cdot 09809 \quad A_{35,65}^1 = \cdot 11039$$

$$A_{30,70}^1 = \cdot 07812$$

To find  $A_{31,63}^1$ , we have  $x = \frac{1}{5}$ ,  $y = \frac{3}{5}$  and

$$\Delta_x = \cdot 01521 \quad \Delta_x^2 = \cdot 00783 \quad \Delta_x \Delta_y = -\cdot 00291$$

$$\Delta_y = -\cdot 01860 \quad \Delta_y^2 = -\cdot 00137$$

whence, performing the calculations, we obtain  $A_{31,63}^1 = \cdot 10776$ ; the correct value, taken from Orchard's Tables, being  $\cdot 10773$ .

2. A simple method of interpolation very commonly employed to find  $u_{x,y}$  is referred to by Mr. Ackland in his letter cited above. The work is accomplished in successive steps,  $u_{0,y}$  being first found, then  $u_{1,y}$ , and finally  $u_{x,y}$ , as follows:

$$u_{0,y} = u_{0,0} + y(u_{0,1} - u_{0,0})$$

$$u_{1,y} = u_{1,0} + y(u_{1,1} - u_{1,0})$$

$$u_{x,y} = u_{0,y} + x(u_{1,y} - u_{0,y})$$

This value of  $u_{x,y}$  will be found equivalent to

$$u_{0,0} + x\Delta_x u_{0,0} + y\Delta_y u_{0,0} + xy\Delta_x \Delta_y u_{0,0} \quad . \quad . \quad . \quad (2)$$

and, comparing this expression with formula (1), we see that the error involved as far as second differences is

$$\frac{x \cdot x - 1}{2} \Delta_x^2 u_{0,0} + \frac{y \cdot y - 1}{2} \Delta_y^2 u_{0,0}.$$



Employing formula (2) in the example given above, we obtain  $A_{31.63}^1 = .10822$ .

3. One has sometimes to interpolate between joint-life or last-survivor functions, when the ages of the two lives concerned do not differ by more than a few years. If this condition apply it is usually possible to select as origin a value corresponding to two lives of equal ages. In these circumstances formula (1) will usually admit of simplification, since if  $u_{0.1} = u_{1.0}$ , and  $u_{0.2} = u_{2.0}$  we have

$$u_{xy} = u_{0.0} + (x+y)\Delta_x u_{0.0} + \left(\frac{x \cdot x - 1}{2} + \frac{y \cdot y - 1}{2}\right)\Delta_x^2 u_{0.0} + xy\Delta_x \Delta_y u_{0.0},$$

$\Delta_x u_{0.0}$  being equal to  $\Delta_y u_{0.0}$  and  $\Delta_x^2 u_{0.0}$  to  $\Delta_y^2 u_{0.0}$ .

This expression may be written in the easily-applied form

$$u_{xy} = u_{0.0} + (x+y)\left[\Delta_x + \frac{x+y-1}{2}\Delta_x^2\right]u_{0.0} + xy(u_{1.1} - u_{2.0}) \dots (3)$$

For example, let it be required to find  $P_{39.33}$  from a set of values of joint life assurance premiums (O<sup>M</sup> 3 per-cent). Selecting as origin  $P_{35.35}$  the values we need are

$$P_{35.35} = 3.151 \quad P_{41.40} = 3.766$$

$$P_{35.40} = 3.471$$

$$P_{35.45} = 3.912$$

Here  $x = \frac{4}{5}$ ,  $y = -\frac{2}{5}$ ,  $\Delta_x = .320$ ,  $\Delta_x^2 = .121$ ,

and applying formula (3) we obtain  $P_{39.33} = 3.297$ , the correct value being 3.303.

It should be observed that formula (3) only holds good when the modification of one variable produces exactly the same effect as a similar modification of the other variable. Otherwise  $u_{0.1}$  would not be equal to  $u_{1.0}$ , nor  $u_{0.2}$  to  $u_{2.0}$ . The formula does not therefore apply to Contingent Survivorship Functions, nor to Joint Life Annuity-Values when the lives are subject to different rates of mortality.

4. Other special cases, in which formula (1) may be greatly simplified, not infrequently arise in actuarial work. Thus, when  $x+y=1$ , i.e., when the desired value relates to two ages, the sum

of which is a multiple of 5, we have, writing  $y=1-x$  in formula (1).

$$u_{x, \overline{1-x}} = u_{0,0} + \left[ x\Delta_x + (1-x)\Delta_y + \frac{x(x-1)}{2} (\Delta_x^2 - 2\Delta_x\Delta_y + \Delta_y^2) \right] u_{0,0}$$

and expressing the differences in terms of the original values, we obtain the formula

$$u_{x, \overline{1-x}} = u_{0,1} + x(u_{1,0} - u_{0,1}) + \frac{x(x-1)}{2} (u_{2,0} - 2u_{1,1} + u_{0,2}) \quad (4)$$

If, as in the case dealt with in paragraph (3),  $\Delta_x = \Delta_y$ , and  $\Delta_x^2 = \Delta_y^2$ , this is reduced to

$$u_{x, \overline{1-x}} = u_{0,1} + x(x-1)[u_{2,0} - u_{1,1}] \quad (5)$$

5. Again, if  $x+y=2$ , i.e., if the sum of the ages be, as before, a multiple of 5, a different origin being taken, the above formulae will be still further simplified, the result being

$$u_{x, \overline{2-x}} = \frac{x-1 \cdot x-2}{2} u_{0,2} - x \cdot x-2 u_{1,1} + \frac{x \cdot x-1}{2} u_{2,0} \quad (6)$$

It will be noticed that the three values employed in formula (6) are consecutive and equidistant values of the function  $u_{x, \overline{2-x}}$ , or, dealing with two-life functions, they are consecutive values in which the sum of the ages is constant; and a little consideration will show that instead of arriving at formula (6) by modification of formula (1) we might have deduced it directly, bearing the above fact in mind, by single-variable interpolation.

Thus, having given  $u_{0,0}$ ,  $u_{1,-1}$  and  $u_{2,-2}$ , we write at once

$$\begin{aligned} u_{x, -x} &= u_{0,0} + x(u_{1,-1} - u_{0,0}) + \frac{x \cdot x-1}{2} (u_{2,-2} - 2u_{1,-1} + u_{0,0}) \\ &= \frac{x-1 \cdot x-2}{2} u_{0,0} - x \cdot x-2 u_{1,-1} + \frac{x \cdot x-1}{2} u_{2,-2} \quad (7) \end{aligned}$$

which will be seen to agree with formula (6) when the necessary alterations are made in the suffixes.

In applying formula (7) it will, in certain cases, be possible to select an initial value such that  $u_{0,0}$  is equal to  $u_{1,-1}$  or to  $u_{2,-2}$ , and thus reduce the formula to one of two terms only.

6. It may be useful to give a few examples of the application of some of the foregoing formulæ.

(a) To find  $a_{26,14}$  by formula (4) from the data given on p. 290, we may take as origin  $a_{20,15}$ , giving

$$x = \frac{6}{5}, (1-x) = -\frac{1}{5},$$

whence

$$a_{26,14} = a_{20,20} + \frac{6}{5}(a_{25,15} - a_{20,20}) + \frac{6}{50}(a_{30,15} - a_{25,20}) = 19.265.$$

The correct value is 19.267, while Mr. Edwards' method (*see* p. 291), employing nine values, gives 19.268.

(b) By taking  $a_{20,10}$  as the initial value, so that  $x = \frac{6}{5}$  and  $2-x = \frac{4}{5}$ , we may utilize formula (6) and write

$$a_{26,14} = \frac{6}{50}a_{30,10} + \frac{24}{25}a_{25,15} - \frac{4}{50}a_{20,20} = 19.270$$

(c) It is also possible, on the data given, to employ a two-term formula, since taking  $a_{15,25}$  as origin and modifying formula (7) we obtain

$$a_{26,14} = \frac{36}{25}a_{25,15} - \frac{11}{25}a_{20,20} = 19.265.$$

(d) Taking an example from Mr. Ackland's letter (*J.I.A.*, xxxii, p. 287), we may find  $P_{33,42}$ , given

$$P_{30,40} = 4.433 \quad P_{35,40} = 4.688$$

$$P_{30,45} = 5.049 \quad P_{35,45} = 5.265$$

Using formula (7) with  $P_{30,35}$  as origin we need only two of these values since

$$\begin{aligned} P_{33,42} &= \frac{14}{50}P_{30,45} + \frac{36}{50}P_{35,40} \\ &= 4.789, \text{ the correct value being } 4.784. \end{aligned}$$

The rough but longer method of practice dealt with in paragraph 2 (*see* formula (2)) gives 4.823.

(e) The following is another illustration of the way in which formula (1) may sometimes be found of assistance.

Let it be required to find the value of  $a_{[63,56]}$  at  $2\frac{3}{4}$  per-cent interest according to the British Offices' Annuity Tables, the elder life being a male and the younger a female. The necessary data may be conveniently set out in the following form :

| AGES     |          | ANNUITY-VALUE    |       |                  |
|----------|----------|------------------|-------|------------------|
| <i>m</i> | <i>f</i> | $2\frac{1}{2}\%$ | 3 %   | $3\frac{1}{2}\%$ |
| 63       | 53       | 8.797            | 8.498 | 8.216            |
| 63       | 58       | 8.405            | 8.132 | ...              |
| 63       | 63       | 7.785            | ...   | ...              |

Employing formula (1) we have  $x = \frac{1}{2}$  and  $y = \frac{3}{5}$ ;

$$\begin{aligned} \text{while} \quad \Delta_x &= -.299 & \Delta_x^2 &= .017 & \Delta_x \Delta_y &= .026, \\ \Delta_y &= -.392 & \Delta_y^2 &= -.228 \end{aligned}$$

and we obtain—

$$a_{\left[\begin{smallmatrix} 63 \\ m \end{smallmatrix}, \begin{smallmatrix} 56 \\ f \end{smallmatrix}\right]} = 8.445.$$

The correct value is 8.444, while formula (2) gives 8.420.

7. A very useful device has been suggested to me by Mr. Elderton, to whom I showed my notes, by means of which the principle underlying formula (7), viz., that of selecting, from the given data, values which admit of the employment of the single-variable interpolation formula, may be extended to other cases.

Thus, if we choose from the given values  $u_{0,0}$ ,  $u_{1,2}$ , and  $u_{2,4}$ , we may, with their aid, write (compare formula (7))—

$$\begin{aligned} u_{x,2x} &= u_{0,0} + x(u_{1,2} - u_{0,0}) + \frac{x \cdot \overline{x-1}}{2} (u_{2,4} - 2u_{1,2} + u_{0,0}) \\ &= \frac{\overline{x-1} \cdot \overline{x-2}}{2} u_{0,0} - \overline{x \cdot x-2} u_{1,2} + \frac{x \cdot \overline{x-1}}{2} u_{2,4} \dots \quad (8) \end{aligned}$$

By carefully selecting the initial value, which, of course, need not in two-life cases necessarily relate to lives of equal ages, this formula will be found to apply to many cases not dealt with above on the one-variable basis.

For example, to find  $a_{21,22}$  we may take  $a_{20,20}$  as origin, and, writing  $x = \frac{1}{5}$ , we have—

$$\begin{aligned} a_{21,22} &= .72a_{20,20} + .36a_{25,20} - .08a_{20,40} \\ &= 19.211 \text{ (O}^M \text{ 3\%)}, \text{ the true value being } 19.212. \end{aligned}$$

Similarly, to find  $a_{33,34}$  we may choose  $a_{25,20}$  as starting-point, and write, ( $x = \frac{4}{5}$ )—

$$a_{33,34} = .12a_{25,20} + .96a_{35,35} - .08a_{45,40} = 15.932;$$

or, preferably in this case, we have (putting  $x = \frac{1}{5}$ )—

$$\begin{aligned} a_{33,34} &= .72a_{35,35} + .36a_{25,20} - .08a_{15,25} \\ &= 15.929, \text{ which agrees with the true value.} \end{aligned}$$

Again, if  $a_{41,55}$  were the desired value, we might use  $a_{35,35}$ ,  $a_{45,60}$ , and  $a_{55,65}$ , taking  $x = \frac{3}{5}$ .

8. I may mention that formula (8), when it applies, seems specially suitable in interpolating between two-life annuity-values. It is not always equally satisfactory in dealing with Joint-Life or Contingent Assurance premiums where the third differences are sometimes appreciable, but even here a very good result may usually be obtained by bringing in a fourth value. As an illustration let us take

$$\begin{aligned} A_{21,57}^1 &= .72A_{30,55}^1 + .36A_{25,65}^1 - .08A_{40,75}^1 \\ &= .13796 \text{ (Baker and Raisin, O}^{[NM]} \text{ and O}^{[a']} \text{ 3\%)}. \end{aligned}$$

By utilizing  $A_{45,85}^1$  also, and modifying formula (8), we get .13844, the true value being .13842. Formula (1) gives .13824 from 6 values.

It will be seen that there are various ways in which values may be selected in applying formula (8). We may, for instance, substitute for  $u_{1,2}$  and  $u_{2,4}$ ,  $u_{-1,-2}$  and  $u_{-2,-4}$ , or  $u_{1,-2}$  and  $u_{2,-4}$ , &c., obtaining solutions for  $u_{-x,-2x}$ ,  $u_{x,-2x}$  . . . respectively. A little experience will show which set of values is most likely to yield a satisfactory result in a given case.

I may point out that the method by which formulæ (7) and (8) were obtained is capable of considerable extension. Thus, one might interpolate between  $u_{0,0}$ ,  $u_{1,3}$ , and  $u_{2,6}$ , or between  $u_{0,0}$ ,  $u_{2,3}$ , and  $u_{4,6}$ , obtaining in each case a formula



analogous to (7) and (8). The student may, however, be left to investigate whether any useful results would accrue from the employment of these supplementary formulæ.

9. In conclusion, it may be observed that with the aid of the formulæ given in these notes, and of one or two others which are obvious, all cases commonly arising in practice of interpolation between two-life values can be dealt with by the simple one-variable formula, assuming a complete set of quinquennial values to be available. The various cases may be conveniently summarized as in the following table, where

$$m = \frac{x-1 \cdot x-2}{2}, \quad n = -x \cdot x-2, \quad \text{and} \quad p = \frac{x \cdot x-1}{2}.$$

| FORMULÆ FOR INTERPOLATION BETWEEN VALUES OF TWO-LIFE FUNCTIONS |                                               |
|----------------------------------------------------------------|-----------------------------------------------|
| Classification of Ages                                         | Formula                                       |
| One age a multiple of 5 . . . .                                | $u_{x,0} = mu_{0,0} + nu_{1,0} + pu_{2,0}$    |
| Equal ages . . . . .                                           | $u_{x,x} = mu_{0,0} + nu_{1,1} + pu_{2,2}$    |
| Sum of ages a multiple of 5 . . . .                            | $u_{x,-x} = mu_{0,0} + nu_{1,-1} + pu_{2,-2}$ |
| Other combinations of ages . . . .                             | $u_{x,2x} = mu_{0,0} + nu_{1,2} + pu_{2,4}$   |

The general second-difference formula, of which those tabulated above are particular cases, is clearly

$$u_{x,rx} = mu_{0,0} + nu_{1,r} + pu_{2,2r} \quad . \quad . \quad . \quad . \quad (9)$$

The values of the coefficients  $m$ ,  $n$ , and  $p$  are given by Mr. Edwards in his Table I (p. 290), but it may be useful to reproduce them here.

| $x$ | $m = \frac{x-1 \cdot x-2}{2}$ | $n = -x \cdot x-2$ | $p = \frac{x \cdot x-1}{2}$ |
|-----|-------------------------------|--------------------|-----------------------------|
| ·2  | ·72                           | ·36                | -·08                        |
| ·4  | ·48                           | ·64                | -·12                        |
| ·6  | ·28                           | ·84                | -·12                        |
| ·8  | ·12                           | ·96                | -·08                        |

## III.

*On the Law of Uniform Seniority.*

MR. THOMAS CARLYLE JONES, a Fellow in Mathematics at the University of Pennsylvania, U.S.A., has called our attention to further conditions under which the Law of Uniform Seniority holds good, involving an extension of the demonstration given in the *Text-Book* (Part II, Second Edition), on pp. 205-207, which follows generally the methods given by Mr. R. Henderson (*J.I.A.*, vol. xxxii, p. 293) and Mr. A. Levine (vol. xxxiii, p. 538). It appears that the extended application of the Law in question was announced by M. Albert Quiquet, a Corresponding Member of the Institute of Actuaries, in a note appearing in the *Transactions of the French Academy of Sciences*, for the year 1888,\* following on and further developing a demonstration, given by M. Bertrand in an earlier issue,† that Simpson's Law is based on the formulas of Gompertz and Makeham. M. Bertrand's demonstration appears also in his well-known *Calcul des Probabilités*.

Mr. T. C. Jones sends us an interesting demonstration of the extended application of the Law in question; but the following simple demonstration, for which we are indebted to Mr. Levine, will, we think, be more suitable for readers of the *Journal*.

Referring to vol. xxxiii, p. 539, of the *Journal*, and repeating the equations there set out—

$$k\mu_w = \mu_x + \mu_y + \mu_z + \dots \text{ (to } h \text{ terms).} \quad (a)$$

$$k \cdot \frac{d\mu_w}{dw} = \frac{d\mu_x}{dx} + \frac{d\mu_y}{dy} + \frac{d\mu_z}{dz} + \dots \text{ (to } h \text{ terms)} \quad (\beta)$$

$$\left. \begin{aligned} k \cdot \frac{d\mu_w}{dw} \cdot \frac{dw}{dx} &= \frac{d\mu_x}{dx} \\ k \cdot \frac{d^2\mu_w}{dw^2} \cdot \frac{dw}{dx} &= \frac{d^2\mu_x}{dx^2} \end{aligned} \right\} \dots \dots \dots (\gamma)$$

(with similar equations involving  $w$  with  $y, z$ , etc.), it will be noticed that, in addition to the more general solution given in the *Journal* and in the *Text-Book*, two special solutions of the equations ( $\gamma$ ) are given by—

$$(i) \quad \frac{d\mu_w}{dw} = 0 = \frac{d\mu_x}{dx}$$

$$\text{and} \quad (ii) \quad \frac{d^2\mu_w}{dw^2} = 0 = \frac{d^2\mu_x}{dx^2}$$

\* *Comptes Rendus*, vol. 106, p. 1465.

† *Comptes Rendus*, vol. 106, p. 1042.

Solution (i) gives the value of  $\mu$  as *constant at all ages*.

Solution (ii) gives the value of  $\mu_x = A + Bx$ ,

and 
$$w = \frac{x + y + z + \dots \text{(to } h \text{ terms)}}{h} + C.$$

Substituting in ( $\beta$ ), we find that, unless  $k=h$ , B is zero, which would bring us back to solution (i). Making  $k=h$ , and substituting in ( $\alpha$ ), we find  $C=0$ , so that the equivalent age  $w$  is the arithmetic mean of the  $h$  ages  $x, y, z \dots$

It follows that if, for  $h$  lives of different ages, we may substitute  $h$  lives of equal ages, then, as an alternative to Makeham's law, we may have the law  $\mu_x = A + Bx$ , provided the equal age of the substituted lives is the arithmetic mean of the original ages.

Mr. G. J. Lidstone, to whom we were glad to give the opportunity of perusing the above Note, has favoured us with the following remarks and alternative demonstration:—

It may be remarked that the special solution (i), referred to above, arises precisely when the general equation (on which the *Text-Book* demonstration proceeds) takes an indeterminate form, which could lead to no reliable solution. That general equation is:

$$\frac{\frac{d^2\mu_x}{dx^2}}{\frac{d\mu_x}{dx}} = \log c$$

which takes one of the forms  $\frac{0}{0}$  and  $\infty$ , if the above relation

(i) hold. But this general equation does not necessarily take an indeterminate form when (ii) holds, and it might, therefore, be expected that the complete solution of the general equation should include the form  $\mu_x = A + Bx$ , which is derived from (ii). This will be found to be the case, and the point at which this result has been missed in the *Text-Book* proof is at the passage from equation (vii) to equation (viii), *vide Text-Book*, Part II, page 207. Those equations are as follows:

$$(vii) \quad \frac{d\mu_x}{dx} = bc^x$$

$$(viii) \quad \therefore \mu_x = A + b \frac{c^x}{\log c}, \text{ where } A \text{ is the constant of integration.}$$

Now when  $c$  takes the limiting value unity (*i.e.*,  $\log_e c = 0$ ), the function  $\frac{c^x}{\log_e c}$  becomes infinite and discontinuous; *i.e.*, when  $\log_e c$  is a very *small* quantity,  $+$  or  $-$ , the function is a very *large* quantity of the same sign, and as  $\log_e c$  passes through the critical value zero, the function changes discontinuously from  $+\infty$  to  $-\infty$ . In such conditions the analysis leading from (vii) to (viii) fails, and we must retrace a step to (vii), which becomes, when  $\log_e c = 0$ , or  $c = 1$ ,

$$\frac{d\mu_x}{dx} = b$$

whence

$$\mu_x = A + bx,$$

which is the result obtained above, the arbitrary constant  $B$  being replaced by  $b$ .

The above may serve as a useful reminder to the student of the importance of watching carefully for zero values and discontinuous functions, the unobserved presence of which in mathematical analysis may easily lead to error or to loss of generality.

## CORRESPONDENCE.

### "SPURIOUS SELECTION."

*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—In the course of the discussion on Mr. Elderton's paper recently read before the Institute, I ventured to express the opinion that the measure of "Spurious Selection"  $(q' - q)K_{xt}$ , deduced by the author *on the assumption that at duration  $t$  medical selection had ceased to operate*, could not properly be applied, without modification, to the example chosen, namely, to durations 5–9 in the O<sup>[M]</sup> Table, where it is not generally admitted that the effect of such selection has vanished, and where, for at least durations 5–7, it is admitted by the author that medical selection is still effective. As the opinion was expressed in the subsequent discussion that the proof given applied to all cases, I shall be glad if you will allow me space for the following demonstration and remarks.

Let  $E'_t$  be the exposed to risk, and  $q'_t$  the rate of mortality, at duration  $t$  in one mortality experience, and  $E'_{t+1}$  and  $q'_{t+1}$  be the exposed to risk and rate of mortality in that experience at duration

$(t+1)$ , and let the unaccented symbols  $E_t$ ,  $q_t$ ,  $E_{t+1}$  and  $q_{t+1}$  refer to similar functions in a different experience, and also let  $Q_t$  and  $Q_{t+1}$  be the rates of mortality, at durations  $t$  and  $(t+1)$  respectively, in the experience formed by the amalgamation of the two separate experiences, the whole of the above functions applying to the same attained age.

Now,

$$Q_t = \frac{E'_t q'_t + E_t q_t}{E'_t + E_t}$$

and

$$Q_{t+1} = \frac{E'_{t+1} q'_{t+1} + E_{t+1} q_{t+1}}{E'_{t+1} + E_{t+1}}$$

$$Q_{t+1} - Q_t = \frac{E'_{t+1} E'_t q'_{t+1} + E_{t+1} E'_t q_{t+1} + E'_{t+1} E_t q'_t + E_{t+1} E_t q_t - E'_{t+1} E'_t q'_t - E_t E'_{t+1} q_t - E'_t E_{t+1} q_t - E_{t+1} E_t q_t}{(E'_{t+1} + E_{t+1})(E'_t + E_t)}$$

$$= \frac{(q'_{t+1} - q'_t) E'_{t+1} E'_t + (q_{t+1} - q_t) E_{t+1} E_t + (q'_t - q_t) E'_{t+1} E_t - (q'_t - q_{t+1}) E'_t E_{t+1}}{(E'_{t+1} + E_{t+1})(E'_t + E_t)}$$

If the effect of medical selection has not vanished, let

$$q'_{t+1} - q'_t = \Delta'_t \text{ and } q_{t+1} - q_t = \Delta_t.$$

Then,

$$Q_{t+1} - Q_t = \frac{E'_{t+1} E'_t \Delta'_t + E_{t+1} E_t \Delta_t + (q'_t - q_t + \Delta'_t) E'_{t+1} E_t - (q'_t - q_t - \Delta_t) E'_t E_{t+1}}{(E'_{t+1} + E_{t+1})(E'_t + E_t)}$$

$$= \frac{(q'_t - q_t)(E'_{t+1} E_t - E'_t E_{t+1}) + E'_{t+1} E'_t \Delta'_t + E_{t+1} E_t \Delta_t + E'_{t+1} E_t \Delta'_t + E'_t E_{t+1} \Delta_t}{(E'_{t+1} + E_{t+1})(E'_t + E_t)} \quad (A)$$

If for the given attained age and durations the effect of selection is the same in both tables, *i.e.*, if  $\Delta' = \Delta$ , expression (A) reduces to

$$Q_{t+1} - Q_t = \frac{(q'_t - q_t)(E'_{t+1} E_t - E'_t E_{t+1})}{(E'_{t+1} + E_{t+1})(E'_t + E_t)} + \Delta_t = (q'_t - q_t) K_{xt} + \Delta_t$$

and of this,  $\Delta_t$  may be assumed to be the true measure of selection under the "Combined" Table also; and in this case, as also in the case where selection has ceased to operate and  $\Delta_t = \Delta'_t = 0$ , we have  $(q'_t - q_t) K_{xt}$  as the measure of spurious selection arising from the amalgamation, thus agreeing with Mr. Elderton's result.



If, however, the effect of medical selection is not equal in the two experiences, and the true measure of medical selection in the Combined Table (including the "Spurious Selection" arising from causes other than the amalgamation of the separate experiences) is denoted by  $\Delta''_t$ , where  $\Delta''_t$  may be expected to lie somewhere between  $\Delta'_t$  and  $\Delta_t$ , expression (A) becomes

$$Q_{t+1} - Q_t = \Delta''_t + \frac{(q'_t - q_t)(E'_{t+1}E_t - E'_tE_{t+1}) + E'_{t+1}E'_t(\Delta'_t - \Delta''_t) + E_{t+1}E_t(\Delta_t - \Delta''_t) + E'_{t+1}E_t(\Delta'_t - \Delta''_t) + E'_tE_{t+1}(\Delta_t - \Delta''_t)}{(E'_{t+1} + E_{t+1})(E'_t + E_t)} \quad (I)$$

$$= \Delta''_t + a$$

and the measure of "Spurious Selection" arising from amalgamation is  $a$ .

I must confess that I do not at present see any exact method of measuring  $\Delta''_t$ , and consequently  $a$ .

If, however, we turn from the general question of the effect of amalgamating two experiences, with different rates of mortality, and showing different results of medical selection, and consider the particular case of the O<sup>(u)</sup> Tables it would seem unnecessary to determine either  $a$  or  $\Delta''_t$ .

The mortality table is of greater importance as a measure of what may be expected in the future than as a record of what has happened in the past, and the question suggested by the latter part of Mr. Elderton's paper is as to how far the inclusion of the Old experience detracts from the value of the "Combined" Table as such a measure.

In the New experience we have the best available standard of the effect of medical selection at the present time, since it comprises the very latest data, and is most free from those causes which would produce a spurious appearance of selection, and I think that a comparison of  $(Q_{t+1} - Q_t)$  by the "Combined" Table, with  $(q_{t+1} - q_t)$ , or  $\Delta_t$ , by the New Table, would give a fair idea of the error caused by the inclusion of the Old experience. If, therefore, in expression (B) we make  $\Delta_t = \Delta''_t$ , we obtain, as a measure of the divergence of the Combined experience, as to medical selection, from the true experience, on the above assumption,

$$(q'_t - q_t)K_{xt} + \frac{(\Delta'_t - \Delta_t)(E'_{t+1}E'_t + E'_{t+1}E_t)}{(E'_{t+1} + E_{t+1})(E'_t + E_t)}$$

$$= (q'_t - q_t)K_{xt} + \frac{(\Delta'_t - \Delta_t)E'_{t+1}}{(E'_{t+1} + E_{t+1})}$$

$$= (q'_t - q_t)K_{xt} + (\Delta'_t - \Delta_t)K'_{xt}.$$

In this connection the following Table may be of interest :

| uped<br>ained<br>ges | Duration<br>$t$ | $100 \frac{E_t}{E_t}$ | $K_{xt}$ | $100(q'_t - q_t)$ | $(q'_t - q_t)K_{xt}$ | $K'_{xt}$ | $100(\Delta'_t - \Delta_t)$ | $(\Delta'_t - \Delta_t)K'_{xt}$ | $[6] + [9]$ | $Q_{t+1} - Q_t$<br>Combined<br>O <sub>31</sub><br>Ungraduated |
|----------------------|-----------------|-----------------------|----------|-------------------|----------------------|-----------|-----------------------------|---------------------------------|-------------|---------------------------------------------------------------|
| (1)                  | (2)             | (3)                   | (4)      | (5)               | (6)                  | (7)       | (8)                         | (9)                             | (10)        | (11)                                                          |
| -32                  | 5               | 14                    | .0203    | .071              | .00001               | .1430     | .139                        | .00020                          | .00021      | .00114                                                        |
| ..                   | 6               | 17                    | .0197    | .210              | .00004               | .1628     | .230                        | .00037                          | .00041      | .00000                                                        |
| ..                   | 7               | 19                    | .0176    | .440              | .00008               | .1804     | -.263                       | -.00047                         | -.00039     | .00024                                                        |
| ..                   | 8               | 22                    | .0198    | .177              | .00004               | .2002     | -.142                       | -.00028                         | -.00024     | .00078                                                        |
| ..                   | 9               | 25                    | .0175    | .035              | .00001               | .2177     | -.040                       | -.00009                         | -.00008     | -.00115                                                       |
| ..                   | 10              | 28                    | ...      | ...               | ...                  | ...       | ...                         | ...                             | ...         | ...                                                           |
| ..                   | + Total         | ...                   | ...      | ...               | .00018               | ...       | .369                        | .00059                          | .00062      | .00216                                                        |
| ..                   | - Total         | ...                   | ...      | ...               | ...                  | ...       | .445                        | .00084                          | .00071      | .00115                                                        |
| ..                   | Average         | ...                   | ...      | ...               | .00004               | ...       | -.015                       | -.00005                         | -.00002     | .00020                                                        |
| -42                  | 5               | 17                    | .0247    | .231              | .00006               | .1697     | -.327                       | -.00055                         | -.00049     | .00086                                                        |
| ..                   | 6               | 20                    | .0271    | -.096             | -.00003              | .1968     | .099                        | .00019                          | .00016      | -.00055                                                       |
| ..                   | 7               | 24                    | .0256    | .003              | .00000               | .2223     | .074                        | .00016                          | .00016      | .00135                                                        |
| ..                   | 8               | 29                    | .0243    | .077              | .00002               | .2466     | .118                        | .00029                          | .00031      | -.00008                                                       |
| ..                   | 9               | 33                    | .0262    | .195              | .00005               | .2728     | -.105                       | -.00029                         | -.00024     | -.00104                                                       |
| ..                   | 10              | 38                    | ...      | ...               | ...                  | ...       | ...                         | ...                             | ...         | ...                                                           |
| ..                   | + Total         | ...                   | ...      | ...               | .00013               | ...       | .291                        | .00064                          | .00063      | .00221                                                        |
| ..                   | - Total         | ...                   | ...      | ...               | .00003               | ...       | .432                        | .00084                          | .00073      | .00167                                                        |
| ..                   | Average         | ...                   | ...      | ...               | .00002               | ...       | -.028                       | -.00004                         | -.00002     | .00011                                                        |
| -52                  | 5               | 19                    | .0282    | .325              | .00009               | .1893     | -.032                       | -.00006                         | .00003      | -.00048                                                       |
| ..                   | 6               | 23                    | .0342    | .293              | .00010               | .2235     | -.083                       | -.00019                         | -.00009     | .00254                                                        |
| ..                   | 7               | 29                    | .0324    | .210              | .00007               | .2559     | .023                        | .00006                          | .00013      | -.00058                                                       |
| ..                   | 8               | 34                    | .0267    | .233              | .00006               | .2826     | .062                        | .00018                          | .00024      | -.00060                                                       |
| ..                   | 9               | 39                    | .0292    | .295              | .00009               | .3118     | -.307                       | -.00096                         | -.00087     | -.00020                                                       |
| ..                   | 10              | 45                    | ...      | ...               | ...                  | ...       | ...                         | ...                             | ...         | ...                                                           |
| ..                   | + Total         | ...                   | ...      | ...               | .00041               | ...       | .085                        | .00024                          | .00040      | .00254                                                        |
| ..                   | - Total         | ...                   | ...      | ...               | ...                  | ...       | .422                        | .00121                          | .00096      | .00186                                                        |
| ..                   | Average         | ...                   | ...      | ...               | .00008               | ...       | -.067                       | -.00019                         | -.00011     | .00014                                                        |
| -62                  | 5               | 21                    | .0296    | -.011             | -.00000              | .2051     | -.092                       | -.00019                         | -.00019     | .00035                                                        |
| ..                   | 6               | 26                    | .0336    | -.103             | -.00003              | .2387     | .221                        | .00053                          | .00050      | -.00104                                                       |
| ..                   | 7               | 31                    | .0299    | .118              | .00004               | .2686     | .305                        | .00082                          | .00086      | .00060                                                        |
| ..                   | 8               | 37                    | .0324    | .423              | .00014               | .3010     | -.518                       | -.00156                         | -.00142     | .00520                                                        |
| ..                   | 9               | 43                    | .0341    | -.095             | -.00003              | .3351     | -.148                       | -.00050                         | -.00053     | -.00369                                                       |
| ..                   | 10              | 50                    | ...      | ...               | ...                  | ...       | ...                         | ...                             | ...         | ...                                                           |
| ..                   | + Total         | ...                   | ...      | ...               | .00018               | ...       | .526                        | .00135                          | .00136      | .00615                                                        |
| ..                   | - Total         | ...                   | ...      | ...               | .00006               | ...       | .758                        | .00225                          | .00214      | .00473                                                        |
| ..                   | Average         | ...                   | ...      | ...               | .00002               | ...       | -.046                       | -.00018                         | -.00016     | .00028                                                        |

An inspection of col. (6) and col. (11) of the above table will show that, when the facts are steadied by grouping the data at five consecutive attained ages, and the actual value of  $(q'_t - q_t)K_{xt}$  is compared with the actual ungraduated value of  $(Q_{t+1} - Q_t)$ , the ratio of "Spurious Selection" is much less on the whole than would be supposed from Mr. Elderton's Table II; whilst col. (10), which could of course have been deduced directly by taking the difference of the  $q$ 's under the New Table from that under the Combined Table, would appear to suggest that the influence of medical selection in the combined  $O^{(M)}$  Tables is *under-estimated*, as compared with that likely to prevail.

It appears to me, however, very unsafe to draw general deductions from the figures in either col. (6) or col. (10), for the following reasons:

- (1) The data, though those of five ages have been employed, are scanty, and the resulting rates of mortality and selection extremely irregular.
- (2) The rate of selection under the New assurances is not altogether reliable as a standard, as the data employed in the construction of that table is itself not homogeneous.
- (3) There may, as pointed out by Mr. Elderton, be a further error caused by the "existing" in the New assurances which are exposed to risk at one duration but not at the next, without having passed out by death; and a similar error may be caused by the inclusion in the Old experience of those cases existing in 1863 which came in at one duration without having been exposed to risk at the previous duration. The former error would be lessened to some extent by the inclusion of the "existing" in both the New and Combined experiences.

The above objections appear to me to apply with much greater force to the figures in Mr. Elderton's Table II, since in that table the value of  $K_{xt}$  is deduced from the data at one attained age only, and an average value of  $(q' - q)$  is taken, which, whether it be correct or not, certainly differs from the average of the values in either col. (6) or col. (7) of that table, and differs very materially from the ungraduated values of  $(q' - q)$  at individual durations. Although, by taking such an average value, a certain amount of smoothness is secured, it would appear to be at the expense of accuracy, and in this connection it is interesting to note that, on comparing cols. (5), (6) and (7) of the author's Table II, the "Spurious Selection" at age 40 varies between '00005 and '00007, whilst the difference between the average graduated and ungraduated values  $(q_{[x-1]+t+1} - q_{[x]+t})$  is '00010, also that at age 50 for a "Spurious Selection" of from '00006 to '00009 we have an average difference of no less than '00027 between the graduated and ungraduated figures. In this particular case graduation is certainly of more importance than the inherent error, which it is one of the functions of graduation to remove.

Whatever other result the above demonstration and illustrations may have, they will at least, I think, illustrate the very complex nature of the problem the author has attacked. They will

also, I trust, show still more clearly the debt of gratitude we are under to Mr. Elderton for his valuable and suggestive paper, and emphasize what I take to be its real message, that, failing some more scientific method of allowing for changes during the period of observations, such as is hinted at on page 226,

- (1) Only such data should be used for select tables, or tables of a similar nature, as can be traced from entry till the close of the select period, unless going out by the ordinary methods of office experience—death, withdrawal, or termination of the contract.
- (2) The period of observations should be as recent as possible; and
- (3) Subject to inclusion of sufficiently extensive data, the period of observation should be as short as possible.

I am, SIR,

Yours faithfully,

JAMES BACON.

5 & 6, Clement's Inn,

Strand, W.C.

5 March 1906.

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*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—I have carefully re-considered the discussion on my paper on "Spurious Selection", and it seems advisable before leaving the subject to bring out a few points with which I was only able to deal partially in my reply. The main point of my paper was to show the danger of amalgamating experiences or using heterogeneous material when dealing with select mortality, and the tone of the discussion gives me the satisfaction of feeling that my attempt to prove that such a danger exists was successful, although I confess that I was sorry some speakers considered it necessary to defend certain tables which I had no intention of attacking.

In the course of his speech Mr. Green asked if the "population method" at present in use is not exactly what we want. My answer is that its use leads to select rates that *may* tell us absolutely nothing about selection as defined in my paper, and the result would not give ideal premiums for office rates. The method would, I think, be more defensible for valuation purposes; and my remarks on this point, though they were, I fear, indistinctly expressed, were meant to suggest that the subject needed more consideration before additional work was incurred in office practice. The "population method" does not give an exact measure of selection, though it can give a measure which approximates closely to the truth when the term of the investigation is short; and the practical conclusion to be drawn from this is, I think, that in studying selection our statistics should only be used for a small number of years, as there is then a better chance of securing homogeneity.

Mr. Ackland asked me to give a proof of the second footnote on p. 226. The problem referred to is: "If we add together a number of groups having different rates of mortality, will there be any spurious selection"? Since the exposed to risk at nearest age  $x$  means the

summation of the exposed for all ages from  $x - \frac{1}{2}$  to  $x + \frac{1}{2}$ , the problem in the footnote is a particular case of the one just enunciated. The proof follows from that on p. 222; but the following hypothetical example is probably the simplest way of showing the result:—

| Attained Age | DURATION 5 |        |                   | DURATION 10 |        |                   |
|--------------|------------|--------|-------------------|-------------|--------|-------------------|
|              | Exposed    | Deaths | Rate of Mortality | Exposed     | Deaths | Rate of Mortality |
| 40           | 1,190,100  | 11,913 | ·01001            | 1,064,400   | 10,655 | ·01001            |
| 41           | 1,104,900  | 11,469 | ·01038            | 998,400     | 10,363 | ·01038            |
| 42           | 1,006,300  | 10,868 | ·01080            | 963,500     | 10,406 | ·01080            |
| 43           | 951,400    | 10,694 | ·01124            | 922,200     | 10,366 | ·01124            |
| 44           | 903,600    | 10,590 | ·01172            | 863,100     | 10,116 | ·01172            |
| 40-44        | 5,156,300  | 55,534 | ·010770           | 4,811,600   | 51,906 | ·010788           |

This shows that the results can be distorted, but, as I have already remarked, the difference is slight and may be neglected.

I may take this opportunity of giving an example illustrating my remark on p. 226, that I cannot see the *a priori* impossibility of finding two Makeham curves having different constants, so that the rates of mortality deduced from them differ widely at the start, and then converge sufficiently to give identical values for the expected deaths at the end of the tables.

| Age | RATES OF MORTALITY DEDUCED FROM                                |  | RATES OF MORTALITY DEDUCED FROM                                 |  |
|-----|----------------------------------------------------------------|--|-----------------------------------------------------------------|--|
|     | Colog $p_x = \cdot 0025575 + \cdot 0000472(1 \cdot 0939564)^x$ |  | Colog $p_x = \cdot 0005575 + \cdot 000053293(1 \cdot 092615)^x$ |  |
| 20  | ·00752                                                         |  | ·00202                                                          |  |
| 40  | ·00978                                                         |  | ·00551                                                          |  |
| 60  | ·02922                                                         |  | ·02591                                                          |  |
| 80  | ·1385                                                          |  | ·1377                                                           |  |
| 85  | ·2056                                                          |  | ·2056                                                           |  |
| 90  | ·3004                                                          |  | ·3008                                                           |  |
| 95  | ·4268                                                          |  | ·4268                                                           |  |

I certainly did not mean to imply that I think Makeham's formula universally true; but one reason given for its failure in the particular case seemed rather unsatisfactory, and was therefore mentioned. My remarks on the subject certainly do not "tacitly assume that there is a definite law of mortality", if that expression means that  $q$  or  $\mu$  can be expressed in a simple empirical form; but I may remind Mr. Green that it seems possible that "deaths" are more or less complex frequency-curves, and, after all is said and done, the use of a formula of approximate summation is making something suspiciously like the assumption of a mathematical law of mortality. This is, however, a side issue.

My conclusion (No. (4) of p. 233) gives my personal opinion, but it is impossible to dogmatize with the particulars available, and



my estimate may easily be too high or too low. Though I have no intention of going further into this matter, I should like to point out that, so far as practical work is concerned, the sooner select tables can be run into an aggregate table the better, and I hoped I was simplifying matters for the future by suggesting that on some future occasion we might be able to dispense with a few of the ten years of selection shown by the present tables.

I may now turn to Mr. Bacon's letter, which I have had an opportunity of perusing. He deduces a formula and then shows that, in the two cases in which he finds it possible to assign a definite value to the function he uses, he has no difficulty in reaching the expression I adopted, and it is not until he is unable to see "any exact method of measuring" his function that he questions my result. It seems to me that even in this last case the simple formula I used has a meaning, for it holds if we assume that a possible measure of the selection in an amalgamation is obtained by using the same proportions of exposed at duration  $t+1$  as those at duration  $t$ , and interpret the formula in which  $q'-q$  was constant as  $(q'_{t+1}-q_{t+1})K_{xt}$ . I have used the term "possible measure", because I do not see how any measure in such cases can be strictly called correct, and I think Mr. Bacon is probably in as complete agreement with this remark as I am with the three final conclusions of his letter. I think it a pity, therefore, that Mr. Bacon should proceed to apply his formula to the case in which he admits he cannot measure his function exactly, and the application is so unfortunate, that he concludes that it is "very unsafe" to draw general conclusions from his figures. It is perhaps partly because he has had to obtain the result by working on the difference between the differences of two rates, both of which are based on ungraduated heterogeneous material, that he is led to conclude that he has reached an unreliable result.

With regard to my own figures in Table II, I attempted to avoid the roughnesses of data to which Mr. Bacon refers by using a constant difference; anyone who prefers a smaller difference has merely to reduce my figures in proportion to see the effect. The roughnesses do not affect me, even in using one age only, because they occur in the deaths, which are few in number, rather than in the exposed, which are many.

So much for past events; may I turn for a moment to the future, and suggest some possible developments of the study? (1) Under what conditions will a table like the "old" assurances contain negative spurious selection? (2) Heterogeneity may be introduced by subtracting part of a homogeneous body of facts. If, therefore, all with-profit policies were formerly for the whole of life, the starting of a new class like endowment assurances *might* make the whole-life material over a long period heterogeneous. This is a very difficult point, and needs investigation. (3) If we could prove that selection takes the form  $A + \phi(xt)$ , as suggested on p. 233, a test for homogeneity might be formed by examining changes in  $\phi(xt)$ . I may here mention that of course  $A + \phi(xt)$  is negative and  $\phi(xt)$  may probably take either the form  $-\kappa t^{-m}(a-t)^n$  or  $-\kappa t^{-m}e^{pt}$  where  $m$  is a fraction and  $\kappa, m, a, n$  and  $p$  depend on the age at entry. (4) A further study in the effects of gradual change in rates of mortality on selection showing, for instance, the effects of a decrease followed

by an increase in rates would be useful. Such an investigation might show that variations in the exposed and rates of mortality can counterbalance when selection has worn off though this would not prove that the experience was homogeneous, or that the rates when selection was in operation gave a true measure of select mortality.

I feel strongly that at present we are far more likely to get information about the effects of heterogeneous material by the construction of hypothetical examples than by the examination of existing material, and are less likely to be misunderstood.

It is admitted by Mr. Bacon in his letter, and by the speakers in the discussion, that it is unsafe to rely on the selection shown by heterogeneous material. This was what I set out to prove, and the admission makes it unnecessary for me to go further into the subject at present. Perhaps the many other problems connected with the subject may be solved by some other student, who has more time to devote to their solution than I seem likely to be able to give at present.

I am, Sir,

Yours faithfully,

W. PALIN ELDERTON.

11, Lombard Street, E.C.

31 May 1906.

## THE INSTITUTE OF ACTUARIES.

### ALTERATION IN RULES OF EXAMINATION.

FOR the information of Members of the Institute, an early intimation is given of the following change recently made in the Rules governing the conditions of exemption from Part I of the Examinations:—

*Extract from Report of the Council for the year 1905-6:—*

“An alteration has been made in the Rules regulating the Examinations, which will permit the Council to grant exemption from the necessity of passing Part I of the Examinations, to Graduates in Mathematical Honours of any University in the British Empire. This privilege has hitherto been confined to Graduates of Universities in the United Kingdom, and its extension is justified by the important increase in the number of our Colonial Members.”

Clause 3 of the Rules prescribed by the Council regulating the Examination for the Class of Student, as revised in May 1906, will, in consequence of the above alteration, now read as follows:—

“3. Graduates in Mathematical Honours of any University in the British Empire may, at the discretion of the Council, be exempted from Examination in Part I of the Syllabus.”

In other respects, the Rules of Examination remain unaltered, as published in the *Journal* for October 1904 (vol. xxxviii, pp. 703-6).

# THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

## *Summary of the Life Assurance and Annuity Revenue Accounts.*

[Extracted from the Parliamentary Returns for 1905, published in 1906.]

| INCOME                                                                                                                                                                                        | Ordinary Companies | Industrial Companies | TOTAL       |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|----------------------|-------------|
| £                                                                                                                                                                                             | £                  | £                    | £           |
| Balance at the beginning of the Year. . . . .                                                                                                                                                 | 277,537,214        | 27,111,038           | 304,648,252 |
| Adjustments: for capital removed from Life Account in the case of three companies now transacting other business (—£170,988); and for two Industrial returns discontinued (—£41,055). . . . . | —170,988           | —41,055              | —212,043    |
| Premiums . . . . .                                                                                                                                                                            | 277,366,226        | 27,069,983           | 304,436,209 |
| Consideration for Annuities . . . . .                                                                                                                                                         | 24,547,873         | 11,093,268           | 35,641,141  |
| Interest and Dividends (less Tax) . . . . .                                                                                                                                                   | 1,793,736          | 3,846                | 1,797,582   |
| Increase in value of Investments . . . . .                                                                                                                                                    | 10,456,303         | 934,706              | 11,391,009  |
| Fines, Fees, &c. . . . .                                                                                                                                                                      | 169,897            | 1,529                | 171,426     |
| Capital Paid-up . . . . .                                                                                                                                                                     | 15,360             | 1,809                | 17,169      |
| Customs Timber Measuring, &c. . . . .                                                                                                                                                         | 25,000             | 102,986              | 127,986     |
| Transfers from other Accounts . . . . .                                                                                                                                                       | 3,744              | ...                  | 3,744       |
| Miscellaneous . . . . .                                                                                                                                                                       | 62,314             | 94,215               | 156,529     |
|                                                                                                                                                                                               | 33,514             | 22,898               | 56,412      |
|                                                                                                                                                                                               | 314,473,967        | 39,325,240           | 353,799,207 |
| OUTGO                                                                                                                                                                                         | Ordinary Companies | Industrial Companies | TOTAL       |
| £                                                                                                                                                                                             | £                  | £                    | £           |
| Claims . . . . .                                                                                                                                                                              | 17,866,469         | 4,255,966            | 22,122,435  |
| Cash Bonuses and Reduction of Premiums . . . . .                                                                                                                                              | 879,546            | 342                  | 879,888     |
| Surrenders . . . . .                                                                                                                                                                          | 1,636,373          | 79,775               | 1,716,148   |
| Annuities . . . . .                                                                                                                                                                           | 2,082,473          | 12,690               | 2,095,163   |
| Commission . . . . .                                                                                                                                                                          | 1,278,681          | 2,788,315            | 4,066,996   |
| Expenses of Management . . . . .                                                                                                                                                              | 2,034,624          | 2,039,335            | 4,073,959   |
| Bad Debts . . . . .                                                                                                                                                                           | 2,852              | 248                  | 3,100       |
| Decrease in value of Investments . . . . .                                                                                                                                                    | 258,645            | 5,019                | 263,664     |
| Interest on Capital and Dividends and Bonuses to Shareholders . . . . .                                                                                                                       | 467,445            | 557,282              | 1,024,727   |
| Transfers to other Accounts . . . . .                                                                                                                                                         | 258,329            | 188,582              | 446,911     |
| Miscellaneous . . . . .                                                                                                                                                                       | 9,166              | 8,107                | 17,273      |
| Balance* at the end of the Year . . . . .                                                                                                                                                     | 287,699,364        | 29,389,579           | 317,088,943 |
|                                                                                                                                                                                               | 314,473,967        | 39,325,240           | 353,799,207 |

\* This Balance includes the whole of the Life and Annuity Funds (£313,078,233), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

## Summary of the Balance Sheets.

| LIABILITIES                                                     | Ordinary<br>Companies | Industrial<br>Companies | TOTAL       |
|-----------------------------------------------------------------|-----------------------|-------------------------|-------------|
|                                                                 | £                     | £                       | £           |
| Paid-up Capital (including sundry Shareholders' Balances) . . . | 11,751,517            | 1,951,014               | 13,702,531  |
| Life and Annuity Funds . . .                                    | 284,533,609           | 28,544,624              | 313,078,233 |
| Fire Funds of Companies trans-acting Life Business . . .        | 14,422,783            | ...                     | 14,422,783  |
| Marine Funds of Companies trans-acting Life Business . . .      | 1,063,776             | ...                     | 1,063,776   |
| Reserve Funds . . .                                             | 4,636,578             | 1,540,793               | 6,177,371   |
| Other Funds . . .                                               | 3,227,673             | 263,840                 | 3,491,513   |
| Profit and Loss Balances . . .                                  | 4,808,119             | 1,216                   | 4,809,335   |
| Depreciation and Investment Balances . . .                      | 2,416,439             | 54,892                  | 2,501,331   |
| Globe Annuity (Liverpool and London) . . .                      | 1,654,200             | ...                     | 1,654,200   |
| Outstanding Claims . . .                                        | 4,416,628             | 9,738                   | 4,426,366   |
| Outstanding Accounts . . .                                      | 932,439               | 28,998                  | 961,437     |
| Temporary Loans . . .                                           | 399,049               | 29,213                  | 428,262     |
|                                                                 | 334,292,810           | 32,424,328              | 366,717,138 |

| ASSETS                                          | Ordinary<br>Companies | Industrial<br>Companies | TOTAL       |
|-------------------------------------------------|-----------------------|-------------------------|-------------|
|                                                 | £                     | £                       | £           |
| Mortgages . . .                                 | 92,225,684            | 3,868,827               | 96,094,511  |
| Loans on Policies . . .                         | 17,177,956            | 122,129                 | 17,300,085  |
| „ Rates . . .                                   | 31,855,782            | 10,780,528              | 42,636,310  |
| British Government Securities . . .             | 7,300,426             | 2,065,111               | 9,365,537   |
| Indian and Colonial Government Securities . . . | 19,251,114            | 605,234                 | 19,856,348  |
| Foreign Government Securities . . .             | 10,058,200            | 328,446                 | 10,386,646  |
| Debentures . . .                                | 62,458,636            | 2,867,757               | 65,326,393  |
| Shares and Stocks . . .                         | 39,409,659            | 1,055,029               | 40,464,688  |
| Companies' own Shares . . .                     | 631,287               | ...                     | 631,287     |
| Land and House Property and Ground Rents . . .  | 26,977,991            | 8,824,663               | 35,802,654  |
| Life Interests and Reversions . . .             | 9,390,331             | 6,592                   | 9,396,923   |
| Loans on Personal Security . . .                | 2,060,075             | 3,950                   | 2,064,025   |
| Agents' Balances and Outstanding Premiums . . . | 6,257,620             | 654,453                 | 6,912,073   |
| Outstanding Interest . . .                      | 3,165,185             | 281,355                 | 3,446,540   |
| Cash, Deposits, Stamps, &c. . .                 | 5,598,381             | 305,973                 | 5,904,354   |
| Customs Timber Measuring Balances, &c. . .      | 2,405                 | ...                     | 2,405       |
| Deficiencies, Establishment Expenses, &c. . .   | 472,078               | 624,281                 | 1,096,359   |
|                                                 | 334,292,810           | 32,424,328              | 366,717,138 |

INCREASE (+) or DECREASE (—) in the Chief Items of this Year's  
SUMMARY as compared with the corresponding Items for the  
previous Year.

|                                                                          | Ordinary<br>Companies | Industrial<br>Companies |
|--------------------------------------------------------------------------|-----------------------|-------------------------|
| <b>INCOME.</b>                                                           |                       |                         |
|                                                                          | £                     | £                       |
| Premiums . . . . .                                                       | + 644,085             | + 492,657               |
| Consideration for Annuities . . . . .                                    | — 88,236              | — *165,087              |
| Interest and Dividends (less Tax) . . . . .                              | + 460,721             | + 88,051                |
| Net Result of Realization and Re-valuation<br>of Investments . . . . .   | + 267,105             | + 5,590                 |
| <b>OUTGO.</b>                                                            |                       |                         |
| Claims . . . . .                                                         | + 1,139,165           | + 311,039               |
| Annuities . . . . .                                                      | + 52,442              | — *128,052              |
| Surrenders . . . . .                                                     | + 37,267              | + 18,421                |
| Commission . . . . .                                                     | + 32,736              | + 120,798               |
| Expenses of Management . . . . .                                         | + 3,148               | + 104,743               |
| <b>LIABILITIES.</b>                                                      |                       |                         |
| Paid-up Capital (including sundry Share-<br>holders' Balances) . . . . . | + 229,832             | + 5,394                 |
| Life and Annuity Funds . . . . .                                         | + 10,293,495          | + 2,269,289             |
| <b>ASSETS.</b>                                                           |                       |                         |
| Mortgages (including Loans on Rates) . . . . .                           | + 6,251,168           | + 1,172,122             |
| Life Interests and Reversions . . . . .                                  | + 408,158             | + 4,239                 |
| Loans on Policies . . . . .                                              | + 1,160,264           | + 30,421                |
| British Government Securities . . . . .                                  | — 243,072             | — 64,440                |
| Indian and Colonial Government Securities . . . . .                      | + 38,470              | + 105,090               |
| Foreign Government Securities . . . . .                                  | — 45,979              | — 74,431                |
| Debentures . . . . .                                                     | + 1,870,478           | + 309,090               |
| Shares and Stocks . . . . .                                              | + 229,354             | + 365,450               |
| Companies' own Shares . . . . .                                          | + 8,873               | ...                     |
| Land and House Property and Ground<br>Rents . . . . .                    | + 1,146,220           | + 702,568               |
| Loans on Personal Security . . . . .                                     | + 234,189             | — 217                   |

\* Due to "Nelson & Co." having gone into liquidation.

#### NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 96, of which 77 are classed as Ordinary, 10 as Industrial, and 9 appear in both Classes, the returns of these Companies showing the Ordinary and Industrial business separately. The accounts of the Consolidated and Hearts of Oak are included for the first time.

During the year two names have been removed from the Official List of Companies, viz.: British Endowment Tea; and Nelson & Co., which are in process of liquidation. And two names have been added, viz.: Gresham Continental Life Assurance Society, Limited; and the Ladies of the Maccabees of the World; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."



SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies*  
ORDINARY BUSINESS.

|                                     | WITH PROFITS |             | WITHOUT PROFITS |             | TOTAL     |             | Re-assur-<br>ances | Net       |
|-------------------------------------|--------------|-------------|-----------------|-------------|-----------|-------------|--------------------|-----------|
|                                     | No.          | Amount      | No.             | Amount      | No.       | Amount      | Amount             | Amount    |
| ASSURANCES.                         |              | £           |                 | £           |           | £           | £                  | £         |
| Whole Term of Life                  | 790,111      | 376,243,994 | 149,131         | 70,998,856  | 939,242   | 147,212,850 | 28,112,102         | 119,130,7 |
| Limited number of<br>Premiums . . . | 53,063       | 34,012,054  | 12,999          | 5,460,138   | 66,062    | 39,472,192  | 2,313,852          | 37,158,3  |
| Endowments . . .                    | 813,174      | 410,256,048 | 162,130         | 76,458,994  | 1,005,304 | 186,715,042 | 30,425,954         | 156,289,0 |
| Endowment Assur-<br>ances . . .     | 1,679        | 395,491     | 21,496          | 5,048,445   | 23,175    | 5,443,936   | 18,500             | 5,425,4   |
| Joint Lives . . .                   | 1,132,523    | 185,682,010 | 87,994          | 22,883,380  | 1,220,517 | 208,565,420 | 3,016,764          | 205,548,0 |
| Last Survivor . . .                 | 16,727       | 3,313,905   | 2,706           | 963,140     | 19,433    | 4,277,045   | 252,086            | 4,024,0   |
| Contingent . . .                    | 782          | 655,567     | 1,003           | 1,338,354   | 1,785     | 1,993,921   | 269,373            | 1,724,5   |
| Issue . . .                         | 32           | 67,676      | 4,256           | 6,727,501   | 4,288     | 6,795,177   | 1,731,729          | 5,063,4   |
| Miscellaneous . . .                 | 11           | 25,250      | 1,593           | 5,871,021   | 1,604     | 5,896,271   | 1,962,654          | 3,933,6   |
|                                     | 4,834        | 2,245,443   | 22,482          | 15,448,239  | 27,316    | 17,693,682  | 2,076,306          | 15,617,3  |
|                                     | 1,999,762    | 602,641,420 | 303,660         | 134,739,074 | 2,303,422 | 737,380,494 | 39,753,366         | 697,627,1 |
| ANNUITIES.                          |              |             |                 |             |           |             |                    |           |
| Immediate . . .                     | ...          | ...         | ...             | ...         | 38,770    | 2,044,697   | 53,356             | 1,991,3   |
| Deferred . . .                      | ...          | ...         | ...             | ...         | 15,142    | 414,182     | 22,250             | 391,9     |
|                                     | ...          | ...         | ...             | ...         | 53,912    | 2,458,879   | 75,606             | 2,383,3   |

## INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

|                                     | WITH PROFITS |        | WITHOUT PROFITS |             | TOTAL      |             | Re-assur-<br>ances | Net       |
|-------------------------------------|--------------|--------|-----------------|-------------|------------|-------------|--------------------|-----------|
|                                     | No.          | Amount | No.             | Amount      | No.        | Amount      | Amount             | Amount    |
| ASSURANCES.                         |              | £      |                 | £           |            | £           | £                  | £         |
| Whole Term of Life                  | 252          | 16,381 | 21,985,779      | 213,066,398 | 21,986,031 | 213,082,779 | 1,180              | 213,081,1 |
| Limited number of<br>Premiums . . . | ...          | ...    | 321             | 5,980       | 321        | 5,980       | ...                | 5,        |
| Endowments . . .                    | 252          | 16,381 | 21,986,100      | 213,072,378 | 21,986,352 | 213,088,759 | 1,180              | 213,087,3 |
| Endowment Assur-<br>ances . . .     | 140          | 42,475 | 1,734,905       | 16,803,104  | 1,735,045  | 16,845,579  | ...                | 16,845,0  |
| Joint Lives . . .                   | 45           | 2,599  | 532,594         | 5,319,276   | 532,639    | 5,321,875   | 88                 | 5,321,7   |
| Contingent . . .                    | ...          | ...    | 414,490         | 6,609,426   | 414,490    | 6,609,426   | ...                | 6,609,0   |
| Miscellaneous . . .                 | ...          | ...    | 4               | 1,910       | 4          | 1,910       | 400                | 1,0       |
|                                     | ...          | ...    | 2               | 2,100       | 2          | 2,100       | 1,000              | 1,0       |
|                                     | 437          | 61,455 | 24,668,095      | 241,808,194 | 24,668,532 | 241,869,649 | 2,668              | 241,866,1 |
| ANNUITIES.                          |              |        |                 |             |            |             |                    |           |
| Immediate . . .                     | ...          | ...    | ...             | ...         | 65         | 2,034       | ...                | 2,0       |
| Deferred . . .                      | ...          | ...    | ...             | ...         | 14         | 310         | ...                | 0         |
|                                     | ...          | ...    | ...             | ...         | 79         | 2,344       | ...                | 2,0       |

The above figures are based on Returns deposited, for the most part, during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of two Companies, viz.: the Customs Fund and the Northern, the amount of business at a more recent date has been included. The figures of the Colonial and Foreign Companies have been excluded as their Returns do not separately show the extent of business in the United Kingdom.

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| THE COMPILATION OF THE DATA, THE GRADUATION OF                      |   |    |    |
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| TABLES ... ..                                                       | 0 | 7  | 6  |
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## CONTENTS OF THIS NUMBER.

---

- I. On a form of Spurious Selection which may arise when Mortality Tables are amalgamated. By W. PALIN ELDERTON, F.I.A., of the Guardian Assurance Company.

Abstract of the Discussion on the preceding.

- II. Some Aspects of Registration of Title to Land. By JAMES ROBERT HART, F.I.A., of the Pelican and British Empire Life Office.

Abstract of the Discussion on the preceding.

- III. Actuarial Notes.

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- V. The Institute of Actuaries.

- VI. The Life Assurance Companies of the United Kingdom.

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## CONTENTS OF NO. 226.

|                                                                                                                                             | PAGE    |
|---------------------------------------------------------------------------------------------------------------------------------------------|---------|
| Reversionary Securities as Investments. By C. R. V. Contts, F.I.A.,<br>Assistant Actuary of the National Mutual Life Assurance Society..... | 317     |
| Abstract of the Discussion on the preceding .....                                                                                           | 332     |
| French Assurance Law, 1905 .....                                                                                                            | 343     |
| ORIGINAL TABLES :                                                                                                                           |         |
| Continuous Temporary Annuities, OM $2\frac{3}{4}$ % .....                                                                                   | 368     |
| THE INSTITUTE OF ACTUARIES :                                                                                                                |         |
| Examination Papers—Associate (Part I), April 1906 .....                                                                                     | 375     |
| „ „ „ (Part II), April 1906 .. .....                                                                                                        | 378     |
| „ „ Fellow (Part III), April 1906 .....                                                                                                     | 381     |
| „ „ „ (Part IV), April 1906 .....                                                                                                           | 385     |
| Proceedings of the Institute—Session 1905–1906 .....                                                                                        | 389     |
| Report, 1905–1906 .....                                                                                                                     | 390     |
| Revenue Account and Balance Sheet, for the Year ending<br>31 March 1906 .....                                                               | 392–393 |
| Results of the Examinations, 1906 .....                                                                                                     | 395     |
| Proceedings at the Annual General Meeting, 1906 .....                                                                                       | 397     |
| Additions to the Library, to October 1906 .....                                                                                             | 403     |
| Fifth International Congress of Actuaries .....                                                                                             | 416     |

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# JOURNAL

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*Reversionary Securities as Investments. By C. R. V. COURTTS, F.I.A., Assistant Actuary of the National Mutual Life Assurance Society.*

[Read before the Institute, 30 April 1906.]

THE previous papers on reversions read before the Institute have dealt mainly with the theory of the subject. It may be useful, therefore, to afford members who have to deal with reversionary securities as investments rather than mathematical problems, an opportunity of discussing some of the practical questions connected with them. In this I am following the example of Mr. Neil Campbell, whose paper, read before the Faculty of Actuaries in 1902, contained much valuable information of a practical character, and led to an interesting discussion.

Scope of Paper. I propose in the first place to refer to the advantages and disadvantages attaching to reversionary securities as compared with other investments, and then to deal in detail with some special points connected with their purchase and management.

That considerable difference of opinion exists as to the desirability of investing in reversions will, I think, be generally

admitted. The returns to the Board of Trade show that some offices invest 20 per-cent of their funds in this class of security, others less than 1 per-cent. This divergence is no doubt in some measure due to the exceptional facilities enjoyed by some offices for acquiring reversions on favourable terms. But, on the other hand, any office which desires to increase its holding in reversions can take advantage of the large and increasing open market for these securities.

**Mortality of  
Life Tenants.**

The absence of any reliable data as to the mortality among life tenants is, no doubt, one of the factors which deter many offices from purchasing reversions. The most common type of life tenant in the case of reversions offered for sale is a woman between sixty and seventy-five years of age, in receipt of an income large enough to keep her in comfort. It may be assumed that if she is known to be in bad health, the holder of the reversion will take advantage of the existing facilities for raising money on mortgage rather than sell outright. It might, therefore, be expected that the vitality of life tenants under reversions offered for sale would be found to surpass that shown by any existing table of mortality. Mr. Neil Campbell, in the paper referred to above, includes a mortality investigation based on 226 lives, among whom were 113 deaths. Although the numbers are too small to form a reliable table, it is interesting to notice that the mortality shown is lighter than the Government Annuitants' or Offices Annuitants' female experience, and that the extra vitality is more marked at the older ages. Would it be outside the scope of the Institute to make a joint investigation with the Faculty into the mortality among life tenants under reversions purchased by life offices and reversionary companies, and thus provide a table which would, no doubt, be adopted as a standard one in all reversionary transactions?

**Objections to  
Reversions.**

Perhaps the most important general objection to reversions as investments is the fact that, unless the fund is in Court, it is under the control of trustees, generally unknown to the office before purchase, who may fraudulently misappropriate the investments, or, through sheer incapacity or a desire to increase the income of the life tenant, re-invest in unsatisfactory securities and appreciably diminish the capital value. When the fund is invested in stocks registered or inscribed in England, a *distringas* may be placed on them which secures due notice of any proposed change, but does not enable

the holder of the reversion to interfere, provided the trustees are not exceeding their powers of investment.

In addition to these risks, there is always a possibility, until the reversion is actually realized, of some flaw in title being discovered. A prior charge may come to light of which due notice has been served on the trustees, but which has been overlooked by them. There is a greater danger of this if there have been changes in the trustees since the trust was constituted. A question may be raised as to the interpretation of the will or settlement under which the reversion is derived, and the opinion of the purchaser's counsel may be upset when the matter is brought before the Court for settlement. I will now pass on to consider what are the attractions which reversions offer, as compared with other investments, to set off the serious disadvantages mentioned above.

Rate of  
Interest  
Earned.

The most important advantage that reversionary securities possess is that they yield a higher rate of interest than that obtained on the bulk of life office investments. In this connection I should observe that I include in reversionary securities life policies, a comparatively modern form of investment for life offices, but one which has now been included in the syllabus for the Institute examinations.

Perhaps the most direct and convincing evidence of the high rate of interest earned by reversionary securities, notwithstanding the doubt as to the mortality and the various risks referred to above, is the general prosperity enjoyed by reversionary companies. These companies are burdened with the expense of an office and staff, and have only a limited capital, which prevents them from taking full advantage of every favourable opportunity of investment. But, in spite of these disadvantages, most of them are able to maintain a satisfactory dividend. In case it may be thought that, owing to the higher prices which have prevailed during the last ten or fifteen years, reversionary securities bought in the market at the present time are not likely to yield good results, I may perhaps be permitted to mention that a recent investigation of the financial results during the last ten years of a large batch of reversionary securities, nearly all purchased during that period in the open market, showed that the yield obtained was appreciably higher than the average yield from other investments, notwithstanding the very recent selection of the lives, which made the mortality lighter than may be expected after the effect of selection has worn off.

Classes of  
Reversions  
likely to be  
profitable.

The yield on reversionary securities depends to a great extent on the class of security purchased. In the absence of any statistics, it is only possible to give some general reasons why the following classes may be expected to prove more profitable.

- (1) Reversions depending on young lives.
- (2) Reversions depending on two or more lives.
- (3) Contingent reversions and reversionary life interests.
- (4) Life policies.

A higher yield may be expected under the first three of the above-mentioned classes, because there is not so wide a market for them as for absolute reversions depending on older lives. They can, therefore, be secured on more favourable terms; and in the case of reversions depending on two or more lives, the vitality is likely to be less pronounced, as there is not so much selection exercised against the purchaser as in the case of a reversion depending on one life alone.

Contingent reversions and reversionary life interests are specially attractive. The nominal yield, after providing policies at office premiums, is appreciably higher than that of absolute reversions; and the real yield on the basis of a net formula, assuming the policy set up by the purchasing office, is from  $\frac{1}{2}$  per-cent to 1 per-cent higher than the nominal yield. Again, it is more difficult for the holder to arrange a loan on a security of this character, and thus preserve the benefit accruing from impaired lives.

Life Policies  
as investments.

It may be convenient to discuss rather more fully the reasons which make life policies attractive investments. That offices should make large profits by purchasing policies in other offices rather suggests the proverbial case of the Sandwich Islanders, who made their living by taking in each other's washing; and if life offices were the only purchasers, an agreement might be arrived at to abstain from purchasing policies and allow them to be surrendered to the issuing office. But, unfortunately, reversionary companies and private buyers have discovered the high yield to be obtained on this class of investment; a self-denying ordinance on the lines suggested would, therefore, only deprive the purchasing office of an attractive investment without in any way benefiting the issuing office. It is, of course, open to any office to keep these policies off the market by offering surrender-values which compare favourably with the investment values. Whether the profit so

obtained would make up for the loss of profit from surrenders entailed by the adoption of a higher scale of surrender-values is an interesting problem which is outside the scope of this paper.

Mortality among lives assured. The chief advantage policies possess, as compared with other reversionary securities, arises from the rate of mortality which is found to prevail among the lives assured. That the mortality should be higher than among life tenants under reversions, might be expected in view of the difference in the lives. But to those unaccustomed to deal with policies as investments, it may seem strange to suggest that the mortality among the lives assured is higher than that of assured lives generally. It is usually assumed that, if a policy is surrendered, the life is in good health. This is no doubt generally the case if the policy is surrendered when it has only been a short time in force. But policies do not, as a rule, become marketable securities until they have been twenty years in force, and the lives are not less than fifty years of age. The vendor is most frequently the life assured or a mortgagee. It may not unreasonably be assumed that the life assured will not sell after maintaining the policy for twenty years or more unless :

- (1) He is in financial difficulties, and unable to bear the strain of maintaining the policy.
- (2) He has made other provision for those dependent on him, or has outlived those for whose benefit the policy was effected.

The first is the more general reason, and the mortality among this class of life probably accounts for a large part of the profit made from purchasing policies. But among those who sell their policies for the second reason given above, I have come across several cases where the life assured knew that he was in bad health, but preferred to take the cash-value of the policy rather than maintain it for others whom he had no desire to benefit. If the policy has been mortgaged and the mortgagor is in default, the mortgagee will, as a rule, prefer to realize his security at once, irrespective of the health of the life assured, rather than face the loss of interest and the payment of premiums under the policy.

The above considerations are offered not so much as reasons for assuming that the lives assured under marketable policies will be subject to a high mortality, but rather as an attempt to explain what is, I believe, the general experience of those who deal in this class of security.



Policies are not subject to the risks and disadvantages which attach to other reversionary securities owing to the funds being in the hands of trustees. But, as a set-off to this, there is a far greater risk of the life assured being lost sight of than in the case of a life tenant under a reversion. I propose to discuss various methods of keeping in touch with lives assured in a later section of the paper, but will now pass on to some of the special points which arise in connection with the purchase of reversionary securities. I have ventured to refer to many points of a somewhat obvious and elementary character, in the hope that they may be of use to junior members whose official duties have not brought them into touch with reversions.

**Index to enquiries.**

Reversionary securities are purchased either by private treaty or at sales by auction. If the purchase of reversions by private treaty is entertained, it will be found convenient to keep an index of all securities offered, as it is not unusual for the same reversion or policy to be offered from several sources in the course of a few weeks. Reversions may be conveniently indexed under the year of birth of the life tenant, policies under the name of the issuing office.

**Source of enquiry.**

As in the purchase of reversions so much depends on the good faith of the various parties concerned in the transaction, the source from which the reversion is offered should be carefully scrutinized.

**Form of application.**

It is usual before a quotation is made to require a form of application to be completed, giving full particulars of the reversion, the price required by the vendor, and, in addition, the names and addresses and occupations of the trustees. It should also contain a question as to whether the trustees are willing to answer enquiries. If this is answered in the negative, no quotation should be made, as, presumably, no purchaser would, in the ordinary way, agree to take the risk of prior dealings without any statement from the trustees.

**Should quotations be made?**

In many cases the price asked by the vendor will be more than the purchaser is prepared to give. The purchaser will then be asked to name a price. Judgment must be exercised as to the cases in which it is desirable to do so. There are obvious objections to quoting prices indiscriminately. The vendor frequently has no intention of selling, but merely wishes to obtain a valuation without paying a fee. In other cases the vendor will use the quotation obtained from one

purchaser as a lever to extort a higher price from another. In any case, therefore, where a price is quoted it should be made subject to withdrawal if not promptly accepted.

**Character of Fund.**

The character of the investment of the fund is, naturally, the point to which attention is first directed. It will sometimes be found that while the bulk of a fund is invested in first-class securities, a small portion is composed of inferior securities of a speculative character. As a rule these are investments made by the testator and left undisturbed by the trustees. But enquiry should be made on this point. If the trustees themselves have made the investments in question, the inference is that they are endeavouring to raise the income of the life tenant, and may proceed later on to re-invest the remainder of the fund in similar securities.

**Bearer Bonds.**

It is important to see that the securities are in the form of inscribed or registered stocks or bonds and not "bonds to bearer." But an arrangement can sometimes be made for bearer bonds to be deposited in a bank in the joint names of the trustees and the purchaser of the reversion.

**Redeemable Stocks.**

A special adjustment should, of course, be made in the valuation of any stocks standing at a premium but redeemable at par. If the date of redemption is within the period covered by the expectation of life of the life tenant at the date of purchase, these stocks should be written down below par to allow for the costs of re-investment on redemption.

**Mortgages.**

If the fund, or any part of it, is invested in mortgages, full details should be obtained as to the nature of the security, with valuation reports, if possible. The amount of margin kept will depend on the security. A small margin should be kept even in cases obviously well secured, to cover the cost of re-investment if the mortgage is paid off.

**Houses or land.**

Reversions to houses or land present several undesirable features. The value at the present time cannot be definitely ascertained, and is no safe criterion of the value when the reversion falls into possession. The trustees cannot be compelled to keep the property in first-class repair, and there is very often a tendency to sacrifice the interests of the reversioner to those of the life tenant, a tendency not likely to be less marked if the whole of the reversionary interest is vested in a life office or reversionary company.

**Ground Rents.** When the fund is invested in ground rents the position is very different. It is hardly possible for even the most careless trustee materially to damage the security; and apart from the possible liability to special taxation the value will automatically increase as the term of the lease runs off.

**Cash.** A small margin should be kept on any item of cash included in the fund, to cover the cost of investment. If the item is large it will be prudent to postpone the purchase until the amount has been invested in satisfactory securities.

**Charges on a Fund.** If the reversion is to a charge on a fund it is necessary to enquire if there are any other charges, and if so to ascertain the relative priority of the charges. It is also desirable to ask whether the charge bears interest from the date of death of the life tenant. If it does not do so, a more careful watch must be kept to prevent loss of interest when the reversion falls in.

**Residue.** Reversions to funds subject to prior charges are rather speculative, as any variation in the value of the fund is thrown on the residue which may form only a small proportion of it. A margin for possible depreciation should therefore be kept, not on the net amount of the fund, but on the whole fund, including prior charges.

**Funds subject to annuities greater than income.** In some cases the fund in reversion is subject to an annuity greater than the income. The fund has been as a rule invested in Consols or India Stock at a time when the income was higher. The deficiency produced by the conversion of the stock has to be met by a gradual sale of the fund. In such cases it is necessary to deduct from the gross value of the reversion to the fund at the time of purchase the single premium for an increasing assurance on the life of the life tenant for the approximate amounts of the fund which will be realized yearly to supply the deficiency in the income.

**Funds subject to annuities after death of life tenant.** Where the fund is subject to the payment of an annuity after the death of the life tenant it will in the ordinary way be sufficient to value part of the fund, the income of which on say a  $2\frac{1}{2}$  per-cent or 3 per-cent basis will pay the annuity, as a last survivor reversion. But enquiry should be made as to whether the trustees are specially directed or authorized to retain the whole of the fund until the death of the annuitant.

**Reversion to a reversion.** A case not frequently met with is that of a fund which includes a reversion receivable on the death of another life. The estate of the life tenant of the original fund

may be entitled to part of the proceeds of the subsidiary reversion in lieu of the income which has been lost. Whether this rule applies in any particular case depends on the wording of the document creating the reversion, and cannot generally be determined without taking legal advice.

**Reversion to a policy.** If the fund in reversion includes a life policy subject to a periodic premium, it should be ascertained whether there is any provision for the payment of the premium out of the income of the fund, and whether such payment is compulsory on the trustees or merely permissive.

**Advances to beneficiaries.** Among the items of the fund will frequently be included advances to beneficiaries to be deducted from their shares when the reversion falls into possession. If the advance is rolling up at interest (which is unusual) an enquiry should be made as to whether the life tenant's estate is entitled to so much of the advance as represents interest. It is also important to see that the advance does not exceed the value of the share on which it is secured. This point is not likely to arise unless the advance is accumulating at interest.

**Duty.** The rate of duty payable on the fund cannot in the ordinary way be ascertained without investigating the title; and in many cases the vendor is unable to furnish definite information on this point. In such cases it is important to state clearly what duty, if any, has been assumed as the basis of the quotation, and to warn the solicitors as to this when referring the matter to them for investigation of title. When there are advances to be deducted from the share of the fund offered for purchase, the duty will in the ordinary way be chargeable on the gross fund. In the case of a reversion to the residue of a fund subject to prior charges, it should be ascertained whether these charges were created free of duty, in which case the duty on the whole fund will fall on the residue. Where the fund is liable to estate duty allowance must be made for a higher rate than that chargeable on the fund regarded as a separate estate, if there is likely to be other property passing on the same death.

**Effect of Finance Act, 1894.** Under the provisions of the Finance Act of 1894, reversions dealt with before the date of the Act are subject to the same scale of duty when they fall into possession, as if the Act had not been passed. But a case has come under my notice where the reversion had been dealt with before the date of the Act, but did not fall into possession until the death of the survivor of three life tenants. On the death of

one of the life tenants a claim for estate duty was made by the Inland Revenue authorities, and it was contended on their behalf that the exemption from estate duty conferred by the Act did not apply to such a case because the fund did not "fall into possession" on the death of the first life tenant. As the amount at stake was small the estate duty claimed was paid, although there seems little doubt that the exemption was intended to apply just as much to reversions depending on two or more lives as to those depending on one life.

**Trustees.** It is important that there should be at least two trustees in a responsible position, more especially if the life tenant is a trustee, or if the investments are such that distringages cannot be placed on them.

**Reversioner a trustee.** When the reversion is to a share of a fund, and the reversioner whose share is being dealt with is one of the trustees, his share is liable to make good any claims that may be made against him on account of any breach of trust. It is therefore desirable to get him to retire in favour of a new trustee. This cannot, of course, always be arranged, but in many cases the risk is more technical than real, and may safely be ignored.

**Powers of investment.** It is desirable that the powers of investment should be well restricted, but too much stress need not be attached to wide powers where the trust has been in existence some time and the fund has been allowed to remain invested in first-class securities.

**Statement from trustees before purchase.** Before completion of the purchase a statement will be obtained by the purchasers' solicitors from the trustees verifying the particulars of the reversion and stating what notices, if any, have been received affecting it.

**Notice to trustees after completion.** On completion, notice must be served on all the trustees individually and not on their solicitors. Notice to the solicitors of the trustees will protect a purchaser against subsequent encumbrancers who served notice on the trustees themselves only in the quite exceptional case of the solicitors having been appointed by the trustees as their agents for the purpose of accepting notices. In this connection it may be pointed out that if the fund consists of land of any tenure held in trust for persons in succession, and *not* subject to a trust for conversion, priority of notice will not protect the purchaser. If there are conflicting claimants, priority will be determined by the dates of the assignments.



**Auction sales.** In the case of sales by auction, it will not be possible to obtain completion of a form of application, and it is therefore important to clear up any points on which the particulars do not supply full information before the sale. It is generally possible to obtain the usual particulars as to the life tenant, &c., from the auctioneers.

**Special conditions.** The special conditions of sale, if any, should be carefully examined, as they are not infrequently drawn in such a form that a purchaser would be compelled to accept an insufficient title or be precluded from making requisitions on some points of importance. A common but objectionable condition is one providing that no compensation shall be paid in the event of any error being discovered in the particulars. This is a usual condition in sales of properties in possession where the purchaser has the opportunity of satisfying himself as to the accuracy of the particulars before the sale. But in the case of a reversionary security he has no such opportunity, and should therefore insist on the omission of this condition. There is sometimes a condition that a statement from the solicitor of the trustees should be accepted as evidence as to the particulars of the fund and the notices received. It is more satisfactory that the statements should be made by the trustees themselves. It is

**Is hostility of trustees good ground for rescinding contract?** generally assumed that in the event of there being no special condition covering the point, a purchaser can refuse to complete, if the trustees will not give information as to the notices they have received; and it is quite usual for contracts to be rescinded on this ground without protest on the part of the vendor. But I have heard on good authority that the vendor in such a case is legally under no obligation to prove that no notices have been received affecting his reversion, and that he would probably be successful if he brought an action for "specific performance." Perhaps some of the members who take part in the discussion will be able to throw some light on this point.

**Flaws in title sometimes covered by special conditions.** If the special conditions contain anything unusual, it will be prudent to take legal advice on them before deciding to bid for the reversion. The following case may serve as an example of the dangerous defects in title sometimes covered by special conditions of sale: A testator bequeathed an estate on trust to pay the income to his daughter for life, and after her decease the capital for such one or more of her children or remoter issue as she should by deed or will

appoint. The daughter in pursuance of this power appointed part of the fund upon trust to pay the income to her son, and after his death to such one or more of his children as he should appoint. Two important points arose out of this case :

- (1) Whether the law against perpetuities had not been contravened.
- (2) Whether the power of appointment given to the daughter covered a right to appoint a life interest and delegate the power of appointment to her son.

These points were not shewn *explicitly* in the conditions of sale, which were however drawn in such a form that no requisitions could have been made on them.

Auctioneer's  
commissions.

It is the practice at London and Scottish sales for the Auctioneer's commission to be paid by the vendor. But this is not universal, and special enquiry should be made before bidding at country or Irish sales.

Costs of  
realization.

The question is sometimes raised as to what provision, if any, should be made at the time of purchase to cover the costs of realization when the reversion falls in. Although the income accrues in favour of the holder of the reversion from the date of death, it is usual to value by annual and not continuous functions and thus assume that the reversion will not on the average be received until six months after death. It will be found in practice that interest thus ignored will about cover the costs of realization.

Purchase of  
policies; letters  
of reference.

It is usual in connection with the purchase of a policy to obtain one or two letters from personal friends of the assured agreeing to give information as to his address and to certify as to his identity when the policy becomes a claim. It is of course important that these letters should be given by people in a responsible position who may be expected to carry out their undertaking to the best of their ability. In this connection it should be noted that, as these letters are given without any consideration, they cannot be legally enforced. The purchaser is therefore dependent on the good faith of the parties giving them.

Contingent  
reversions;  
risk of  
reversioner  
disappearing.

Although in most cases the trustees are in touch with the reversioner, it is a useful precaution in the case of a contingent reversion to obtain the same means of keeping in touch with the reversioner as in the case of a life assured under a policy. Two cases have recently come under my notice that illustrate possible dangers arising out of the

disappearance of the reversioner. In the one case the reversioner was last heard of in Rhodesia in 1901. His body has never been found, and he is supposed to have been eaten by lions. But the holders of the reversion have not yet been able to establish his death to the satisfaction of the office which granted the covering contingent policy. In the other case the reversioner was a beneficed clergyman. His clothes were found on the bank of a river and he was supposed to have been drowned. Memorial services were held and obituary notices appeared in the papers. It was afterwards conclusively proved that he was alive after the date of his disappearance and he is believed to have gone abroad under an assumed name. In either of these cases the holders of the reversions would be in a rather difficult position if the life tenant were to die now. The trustees would probably refuse to pay over the fund without evidence of the reversioner's existence, and the life office would refuse to admit a claim under the contingent policy without proof of his death.

**After purchase.** After a reversion or policy has been purchased, the most important questions that arise are in connection with the tracing of the life tenants and lives assured, and obtaining information from time to time as to any changes in the fund or the trustees.

**Necessity for periodic enquiries.** Dealing in the first case with reversions, the purchaser's attention will be drawn to any proposed change in cases where distringas have been placed on all the securities included in the fund by the receipt of a notice warning off the distringas. This will give him an opportunity of ascertaining what changes are being made, and whether any proposed re-investment of the fund is within the powers of the trustees. But in many cases it is not possible to lodge a distringas. It is therefore necessary to take every precaution against any improper re-investment and against the possibility of the fund being distributed and the share purchased paid over to some other party either through carelessness or fraud on the part of the trustees. It is therefore usual to trace periodically the existence of the life tenants and ascertain if there have been any changes in the trustees or the investments. In the case of a life office this tracing can be conveniently carried out in the year preceding the distribution of profits, though some reversionary companies find it worth while to make enquiries every year.

The advantages of systematic tracing on the lines suggested far outweigh the comparatively small trouble involved. Apart

from the possibility of the distribution of the fund, it is very desirable that any changes in the investments should be brought to light, so that if any action is necessary it can be taken as soon as possible; and if any changes in the trustees are discovered, although from a strictly legal point of view it is unnecessary to serve notice on the new trustees, it is very desirable to do so. It not only makes it less likely that the new trustees will distribute the funds without any knowledge of the purchaser's interest, but also may prevent any complication arising from their receiving notice of a conflicting claim. Apart from these special points, a system of periodic enquiries serves to remind the trustees of the purchaser's interest in the fund, and that they cannot commit any breach of trust with impunity.

**Fund in Court.**

When the fund is in Court a certificate of it can be obtained on payment of a small fee. If there are no accumulations of income, this may be taken as sufficient evidence of the existence of the life tenant. If there are accumulations it is probable that the life tenant is dead, and an attempt should at once be made to trace him and prove his death.

**Procedure suggested for enquiries.**

When the fund is in the hands of trustees, a letter of enquiry should be sent to one of the trustees giving particulars of the fund, the life tenant, and the trustees, and asking him to verify or correct these particulars. To avoid incurring costs the enquiry should be addressed, when possible, to a trustee who is not a solicitor.

**Results of an enquiry on lines suggested.**

It may be of interest to mention the results of a tracing recently made on the lines suggested. An enquiry was sent to the trustees in seventy-one cases. In sixty of these replies were received direct from the trustees, and in the remaining eleven cases the trustees referred the enquiry to their solicitors, who gave the information required. In twelve cases, most of them involving complicated funds, a fee was required, and the total costs amounted to £18. In eighteen cases changes in the funds were brought to light, and in twenty cases there were changes in the trustees or in the addresses of the life tenants.

**Tracing of lives assured.**

In the case of policies purchased, a different procedure must be adopted. The importance of keeping in close touch with the life is greater than in the case of reversions. There are no trustees through whom enquiry can be made; and in the event of death, any delay in making a claim will involve

loss of interest on the policy monies. It is therefore desirable to trace systematically all the lives assured at intervals of not less than one year. An enquiry addressed to one of the parties giving letters of reference at the time of the purchase is the most convenient method of tracing the life; but these letters of reference cannot be obtained in all cases; and it sometimes happens that the referee predeceases the life assured or has lost sight of him. It is therefore a useful precaution to record in a book the names and addresses of all parties who may be of use in tracing the life, as, *e.g.*, the vendor, any intermediate assignees or mortgagees, other offices who have issued policies on the life, and the holders of such policies, the agent through whom the premium is paid, &c. It is convenient to record in this book the results of each attempt to trace the life, and to note any change of address.

In addition to enquiries sent to the referees, the following methods of keeping in touch with the life assured will be frequently found useful.

- (1) If the purchaser is an office with an agency organization, an arrangement should be made with an agent residing in the same district as the life to keep in touch with him.
- (2) Lists should be kept of all the lives who are clergymen, lawyers, &c., as these can be traced annually through the Clergy List, Law List, &c.
- (3) Considerable assistance can be obtained through enquiry agencies, who for a nominal fee will ascertain if the life is still living at the last known address.
- (4) A method found useful by some purchasers is to send a circular letter to the life assured enclosing a post card and asking him to report if he is still living at the same address.
- (5) A daily search of the deaths' column in *The Times* will bring to light a large proportion of the deaths among the lives assured.

Merging  
operations.

When the reversion to the whole of the fund has been purchased, a profit can generally be made if the life interest in the same fund is purchased at its market value; and in case of funds invested in Consols and other low interest bearing securities it is often a profitable operation to grant the life tenant an annuity of the present income in exchange for his life interest and thus wind up the trust estate. In this



connection it is of course important to obtain evidence of the health of the life tenant before making a cash payment for the life interest.

**Treatment in accounts.**

Several methods of treating reversionary securities in the accounts have been suggested. The question is merely one of machinery and is therefore of secondary importance. But in the case of an office making periodic distributions of profits, the most convenient method is to re-value the securities in the year when the distribution is made and bring into the revenue account for that year the increase in value which has accrued by effluxion of time. In the intervening years the securities will be written up by the amount of any premiums paid and the profit on any that fall into possession will be brought into the revenue account.

**Realization.**

When the death of a life tenant is reported, it will generally be desirable for the purchaser to instruct his solicitors to prove his title and deal with the requisitions of the solicitors to the trustees. When the title has been proved, the trustees will submit an account, showing the proceeds of the fund and the apportionment of the income between the life tenant and the holders of the reversion. This account should be carefully examined; trustees sometimes put in a claim for expenses to which they are not entitled, or which should have been paid out of the income. There is also a tendency on the part of the solicitors to the trustees to charge their costs on a high scale.

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#### ABSTRACT OF THE DISCUSSION.

MR. J. D. WATSON said that the paper was a practical one, and would, he thought, prove very useful to students, and possibly to legal advisers of life offices, who often were not familiar with all the points which arose in connection with reversionary securities. A difficulty met with by actuaries in considering reversions from the legal point of view, was occasioned by the absence of any text-book dealing with the actual questions arising in practice. For that reason he wished that the author had made his paper somewhat longer and fuller, and had quoted his authorities for the various statements he made as to the legal position. Looking at the paper in detail with a view to discussing it, he found it was a little difficult, because one was bound to agree with nearly all the statements made, and he was not prepared to suggest amendments in the law which would get over the difficulties pointed out by the author. Possibly some other speaker would discuss the paper from that point of view. Confining himself, therefore, to the discussion of what the existing law was on some of the points mentioned, there seemed to be uncertainty as to

what the exact position was with regard to distringas, and, as far as he could see, the advantage of placing a distringas on each item in the fund was very largely illusive. In normal cases, of course, the distringas had a great advantage in keeping the office in touch with the trustees, and informed when changes of investment were in contemplation. If, however, fraud were contemplated he did not suppose a distringas would be of very much use, because there would be sure to be a plausible explanation of the reason of the change, and he had never heard of a successful application to the court to restrain any contemplated action by trustees arising in this way. In practice he thought one distringas on a fund was perhaps sufficient; notice would then be received of any contemplated change of trustees, and it would be possible to serve upon the new trustees fresh notices if it were thought necessary. That brought him naturally to the question of notices to trustees, and in that respect also he had difficulty in ascertaining exactly what was the position. As far as he could see, the law books were apt to state on one page that notice to one trustee was sufficient, and on another that, if notice was only given to one trustee, a purchaser was very apt to find himself behind someone who subsequently gave notice to all the trustees. One particular question, to which it would be interesting to have an answer, was the following. Supposing a man bought, or lent upon, a reversion, duly gave notice to all the trustees, and subsequently they all died. If the reversioner again purported to sell his reversion, the second purchaser gave notice to all the new trustees; the fund was subsequently distributed and handed over to the second purchaser. He should very much like to know what was the exact position of the first purchaser, and against whom was his remedy. He presumed he could not follow the fund, and it was desirable to know what his remedy was against the first set of trustees, or against the second set, if any. That referred to notices to trustees of personal property. As regarded real property, which the author dismissed in a short paragraph, he thought there was a grave risk in dealing with legal remainders. As a rule, of course, there were no trustees to whom notice could be given, and even if notice were given to the trustees of the settlement under the Settled Lands Act, it presumably had no effect. There was a point, however, which might conveniently be elucidated. Supposing real property had been sold compulsorily or otherwise under a settlement, and the proceeds had been paid to the trustees of the settlement, it then took the form of personalty, and the question arose as to whether notice could be given to the trustees, or a stop-order obtained on that personal property. In practice, when a legal remainder was offered, he always felt bound to advise that it was not a transaction that could safely be carried out, as he did not see how the difficulty of there being no trustees or title deeds could be got over. He understood there were purchasers who dealt in those securities, and it would be interesting to know how they satisfied themselves that they were legally secure. Another point in the paper was in connection with answers by trustees. The author said, and he thought rightly, that trustees were not, in the absence of express stipulation, bound to answer enquiries as to previous notices. He

could not remember having seen auction conditions containing the statement that the trustees would answer enquiries, and he sometimes wondered how that point was overcome in practice. With regard to the cost of making enquiries as to changes of investments, he should have thought that the purchaser of a reversion stood in the position of a *cestui que trust*, and could therefore compel the trustees to give information as to investments of the fund, without their being in a position to charge costs on that account. The author suggested that in cases where there were bearer bonds, it was sometimes possible and desirable to deposit bonds in the joint names of the purchaser and the trustees. He did not know whether the author had special grounds for saying that that was a good plan, but he imagined such an arrangement would be in all cases a breach of trust, and that on application to the Court the trustees could compel the bonds to be given up.

He was disappointed when he came to the paragraph on the duty question, as he had hoped that there would be a short and clear account of the duties payable in connection with reversions. In the reference to buying reversions where the reversioner was one of the trustees, he noticed it was suggested that he should be invited to retire. He did not quite see where the benefit of that would arise, because it would not cure a past breach of trust on the part of the trustee, and having satisfied oneself that there was no past breach of trust, if the reversioner were an honourable man he might be just as good a trustee as any other person, and perhaps better than any other, because he would be familiar with the trust. As regards treating reversions in the accounts of life offices, it was suggested that the best plan was to deal with them at the date of valuation. He thought there were practical objections to that course, and in the office with which he was connected the reversions were re-valued annually, the increase in the values being included as interest in the revenue account. That was very much more convenient, from the point of view of the accounts. He quite agreed as to the unsatisfactory nature of contingent reversions. He only remembered buying one, and that was in the case of a young man highly spoken of, who had just been insured by a first-class life office, one of his references being an official at the head office of a life company. A few months after the reversion was bought the young man disappeared, and could not be traced for some time. He was ultimately discovered to be in prison; but on his release he disappeared again.

MR. A. LEVINE said that the author had dealt with the question of the vitality of life-tenants, and if he (the speaker) was not mistaken, the usual practice now in purchasing reversions in the open market was still to use the old Carlisle Table as the standard basis of mortality. He thought the experience of most of those who had anything to do with reversions was that the Carlisle Table under-estimated the vitality of life-tenants. One point to be remembered in connection with reversions was that, speaking generally, the more comfortable one's position was, the easier it was to obtain the medical and other assistance necessary to prolong life, and therefore it was not unlikely that the larger reversions might

show lower rates of mortality among the life-tenants. That being the case, a much more stringent table than the Carlisle should be adopted, even after making the usual reduction of three years for female lives, and purchasers of reversions should if possible endeavour to make the standard table the British Offices' Select Annuity Tables. In connection with the question of mortality, there had to be considered the mortality of reversioners under contingent reversions and reversionary life interests. He did not think he was wrong in saying that in the case of most offices the mortality they had experienced under contingent assurances which were effected chiefly in connection with contingent reversions had been decidedly bad, and if a large profit was sometimes shown in an office under a contingent reversion it not infrequently happened that such profit was really the result of a very bad claim. He should like to know whether that point had been taken into account in the financial investigation referred to by the author. He was not altogether convinced by the argument of the author, drawn from the experience of reversionary companies, as to the high rate of interest earned by reversionary securities. It seemed to him that the more speculative the reversion the higher would be the profit, if a profit was made at all. Naturally, competition for reversions of the very highest class was keener than the competition for second-class reversions. Some reversionary companies might more freely use their funds than life offices, and might invest in second-class reversions, which life offices would not be justified in purchasing. Therefore the fact that reversionary companies—who bought as a rule fairly small reversions and not necessarily of the best class—found their business profitable, did not necessarily prove that life offices buying larger reversions of a better class would find their business equally profitable.

MR. R. R. TILT said it was his daily lot to come in contact with reversionary securities, and he desired to thank the author for some very valuable hints. There was one point in the paper which Mr. Levine had alluded to—"Perhaps the most direct and convincing evidence of the high rate of interest earned by reversionary securities, notwithstanding the doubt as to the mortality and the various risks referred to above, is the general prosperity enjoyed by reversionary companies." He did not agree with the author in that statement. The principal reversionary companies had been established for a great many years. He was recently reading, in connection with another subject, a paper published in the *Journal*, thirty-five years ago (vol. xvi, p. 279), in which an eminent actuary said that reversionary companies settled the prices of reversions at the auction mart, and insurance offices had nothing to do with it. Things had altered since then. Reversionary companies had now to a large extent to follow their leaders in the matter. Studying the accounts of reversionary companies, it would be found that they had, as a rule, larger funds (accumulated from past years) earning interest than the capital on which they paid dividends. The rate of dividend was therefore higher (and in some cases the difference was considerable) than the rate of interest at which the funds were invested. Reversionary companies had very little money to invest, as compared with insurance companies, and to a great



extent they lived on the crumbs that fell from the rich man's table. They devoted the whole of their energy to this class of business, and they looked into securities that insurance companies rejected. They did not take second-class securities at all, nor did they go in for speculative business. Sometimes amongst the crumbs they found rich plums which amply repaid them, and with their limited funds they could almost live on these. He thought that these reasons accounted for the dividends paid by reversionary companies. It would be a mistake to suppose that because reversionary companies paid about 5 per-cent, reversions could be bought in the market to pay 5 per-cent. It was quite certain that they could not. With regard to the mortality amongst tenants-for-life, the best and most suitable table in his opinion was the British Offices' Annuitant Experience, which was very close to the experience of tenants-for-life.

MR. S. G. WARNER said that at intervals of a few years it was always profitable to have a paper upon some aspect of reversionary interests, because they came constantly, in practice as well as theory, before every actuary, and new points were from time to time occurring on which it was useful to have an interchange of opinion. A distinction had been drawn in the paper between two classes of security which were dealt with in reversion, personalty and real property. Personal property consisted chiefly of trust investments in Stock Exchange securities, and real property consisted of houses or lands, which need not necessarily pass in the precise legal manner which Mr. Watson had mentioned, but sometimes occurred as part of the estate under wills, settlements, or like instruments, and had to be considered in that way. It seemed a sound rule to take the broad view that what one would not buy in possession one should not buy in reversion, because it was only a roundabout way of doing the same thing. From that point of view he thought it would be found that to buy reversions to real property, especially undivided shares, would seldom be satisfactory, and that in the end one would find oneself probably in possession of the property. If there were a trust for sale the position might be even worse, because of the possibility of a forced sale at an unpropitious time. There were other complications which might arise. On the whole, he thought no harm would be done in confining transactions pretty strictly to reversions to personalty.

A closer connection might have been observed throughout the paper between lending and purchasing. It was not possible to draw a hard and fast line of distinction between the two, and the same principle should guide one either in lending on reversions or in buying them, because the one frequently merged into the other. The process was very familiar. First a loan was made well within the value, then more was asked for, and then the office was asked to buy the equity; or else the property came into possession, because the borrower could not pay. With regard to the interesting question of purchasing policies, it was gratifying to have it on record that the mortality in that class of investment was high. It was long an accepted actuarial principle that a man who surrendered his policy was in exceptionally good health, and exercised what was known as "selection against the office." But he believed a good many actuaries had thought for



some time that the principle was subject to very grave modification, and the experience mentioned that evening certainly tended in that direction. Many causes, financial pressure above all, tended to the sale or surrender of policies, and when these were put into the scale against the more or less theoretical "selection against the company", it would be found there was not so much in the latter as was supposed. On the question of buying policies, he confessed he had hardly become a convert to the practice, on the strength of what he had heard from Mr. Coutts. Perhaps one was temperamentally disposed to magnify certain possible difficulties in connection with investments, but to him the difficulty of tracing the assured—and the same thing applied to contingent reversions as regarded tracing the reversioner—was very serious, and likely to lead to considerable trouble and complication in the future. He hardly knew that anyone yet was in a position to measure those difficulties fully. Extensive investment in policies had not lasted for a sufficient number of years. What occurred to him with regard to the various ingenious devices suggested in the paper as safeguards was that they were things that would work admirably so long as matters were all right, and the man was easily traceable; but in the very event against which it was desired to provide, that of the impecunious man deliberately selling his reversion so as to lay his hands on all ready cash possible, and having no further interest in letting anybody know his whereabouts, those safeguards would be of no value whatever. It would be almost impossible to do anything in face of the practical difficulty of being unable to trace such a man, and that might lead to considerable trouble, delay, and resultant financial loss. On the whole, he regarded with considerable doubt the wisdom of buying securities of that kind. It might be urged in the light of what he had said about the close connection between lending and buying, that these criticisms would apply with equal logical force to lending on a contingent reversion. That was true in a sense, because it was always open to the purchaser to sell his equity; but there was the safeguard, which could not but be considerable, of someone else standing between the office and the ultimate difficulty. The chances were that the buyer would be even more keenly interested in tracing the missing man, and might have some means of doing so.

MR. W. P. PULLEY said it appeared to him that there was a reason for the mortality being different under policies that were actually surrendered to the office from that under policies bought by other parties, in that the policyholders who came to the office and took their surrender value naturally looked upon themselves as good lives; whereas those who knew they were under the average would often sell their policies outside the company, and get a better price for them, and naturally the mortality from such policies would be higher.

MR. F. E. COLENZO, in closing the discussion, joined with other speakers in expressing gratitude to the author for his paper. Although it contained many points upon which all agreed, some of those points might nevertheless provide occasion for interesting discussions. He was not surprised, having regard to the scope of the paper, that the author had not thought it necessary to lay before the Institute a complete disquisition on the effect of notices, and

upon the subject of duties. Each case of duty must be dealt with upon its own basis, and there was no authority that would render anyone independent of his legal advisers in such matters. He felt inclined to doubt whether the difference of opinion on the part of offices as to purchasing reversions turned so much upon difficulties relating to mortality, as on the feeling on the part of some offices that the prices which they were expected to pay were hardly sufficiently remunerative. The considerations which led some offices to regard annuity business as unprofitable where money was paid down in the present, might of course guide others to reject investments where the money was supposed to be paid in the future. With regard to mortality, he thought there was no reason to distinguish closely between the mortality to be expected in the case of life-tenants and that experienced in connection with annuity business. There was a general consensus of opinion as to the employment of the most recent annuity tables for measuring that particular type of risk. If there was to be an investigation into the experience of reversions, he would like to see a discrimination made with regard to amounts. In the past, reversionary business had offered great temptations to the speculative mind. Looking at more or less ancient accounts, one saw evidence that the speculation had in some instances turned out very much to the advantage of the offices purchasing the reversions. How far large reversions, as a class, had been found to be profitable might be an interesting question to consider. There seemed to be a difficulty in the way of sharing purchases of that sort, and on that point there was room for an interesting investigation, from a theoretical point of view, into the question of limitation of risks—how far the annuity transaction which was essentially involved in the purchase of a reversion should be re-assured. Within his experience there had been cases of sharing reversions where everything had gone on very smoothly, and no particular difficulties had arisen from the side of the lawyers. He did not himself see why all large reversions should not, as a matter of course, be divided amongst offices as large insurances were. However, he admitted that one did not come across large reversions very often. With regard to the question of bearer bonds, when one had had some experience of the danger of leaving bearer bonds in the hands of trustees, one could appreciate the effect of the precaution of having them deposited under joint control. He remembered a case a good many years ago where there were two trustees, one being the tenant-for-life. The other trustee handed his co-trustee some bearer bonds, for the purpose of placing them in another type of security. The gentleman who received them converted them to his own use. Whatever might be said as to the fact that the Courts would regard it as a breach of trust on the part of the trustees to enter into such an engagement, one result would be to secure that the bonds should not be dealt with, except by the joint action of the trustees. He supposed everyone agreed that reversions to legal remainders were, as a rule, dangerous to deal with, but at the same time in all such matters there was the practical aspect of the case. There were undoubtedly in this country individuals whose statutory declarations on the question of encumbrances might with safety be relied upon.

There were special cases where a reasonable charge standing as first mortgage upon large expectations might be considered well secured, in spite of the fact that reliance was placed upon statutory declarations. Otherwise there was an agreement that reversions to legal remainders were not satisfactory things.

The PRESIDENT said the members were always glad to welcome a new contributor, especially when he was one of the younger Fellows. Mr. Coutts had certainly brought together in a comprehensive and useful way a number of points relating to the subject of reversionary interests, which would be of interest, not only to those who were accustomed to deal in such matters, but also to younger members. With regard to a table of mortality for life-tenants, the British Offices' Tables at the present time fairly represented all that was needful, and a further investigation was hardly called for at present. On the other hand, he had a little doubt whether the number of reversions that were obtainable as investments was quite sufficient, so to speak, to go round, and he was afraid that if all the companies were to be moved by the author to endeavour to get the high rate of interest spoken of, they might outbid each other, and disappointment would ensue; in fact, it was already thought by some people that in certain cases very high prices indeed had been given for reversionary interests. With regard to the practical point touched upon by Mr. Colenso, the case of a large reversion out of proportion to all the others on the books of a particular company, that difficulty had, in a number of cases with which he was acquainted, been got over by reducing the transaction to one of a simple annuity purchase, the primary office retaining so much of that annuity as it was accustomed to retain, and simply re-insuring the rest. That plan had worked exceedingly well; and he knew many cases where four or six or eight companies were equally concerned in one transaction. He concluded by moving a very hearty vote of thanks to the author for his interesting and important paper.

The motion having been unanimously adopted,

MR. C. R. V. COUTTS, in responding, said Mr. Watson had suggested that he should make a disquisition on the subject of duty, but that was such a highly technical matter that even solicitors and counsel sometimes failed to deal successfully with it, and he should certainly have some hesitation in dealing with it himself. Mr. Watson also said that he did not see much use in a distringas, because it could be warned off, but that was just the benefit obtained from placing a distringas on a stock. It at once brought to notice any attempt to deal with the stock, not necessarily due to a change in trustees, but to the trustees themselves wishing to sell out and invest in some inferior security, and it put the purchaser on his guard. Further enquiries could be made, and if it was outside the trustees' powers, action could be taken in the Courts to restrain them. Mr. Levine had referred to the question of the most suitable mortality table to adopt. He did not venture to suggest any table in the paper, because he did not think there had been any attempt to collect the experience which would justify one in saying that any particular table was suitable. Probably the Carlisle Table was

not suitable, certainly not for the typical case of a reversion depending on the life of a lady in easy circumstances; although in cases of reversions, where there were four or five life-tenants, there was nothing like the same selection, and probably the mortality would be found very different. As to whether the British Offices' Table was a suitable one, he supposed the chief reason it was suggested was because it was the most stringent table in existence. Mr. Levine also raised the question as to whether, in the investigation referred to in the paper, the yield was obtained by including profit on contingent policies. Speaking from memory, he did not think any contingent reversions were included, at any rate none fell into possession, or produced any profit from the policies becoming claims. Mr. Tilt had answered Mr. Levine's further suggestion that the profit made by reversionary companies was obtained chiefly from their investing in inferior reversions at low prices. Reversionary companies were rather more conservative, he thought, than some life offices in their ideas as to the kind of reversions they should invest in.

Mr. Warner seemed unduly alarmed as to the possible difficulty of keeping in touch with the lives assured under purchased policies. Out of a large number of lives, undoubtedly some would go to Canada or Australia, or cause trouble in various ways, but as a set-off against that, there was the additional profit on the fund so invested. He had come across four or five cases where the lives assured had got into prison, but he regarded that rather as a safeguard, as one need not worry about them until they were released, and they were not likely to be first-class lives then. In one case a man died in prison, a year after purchase. Mr. Colenso had suggested that large reversions should be split up among different offices, and the President had pointed out that that could be done by setting up annuities and re-insuring them. But he had come across cases where it had been done without going through that machinery. If an office had a larger reversion than it cared to entertain, it was easy to apply to some friends, who might be willing to take up a part. With regard to accepting declarations and buying reversions depending entirely on the good faith of such declarations, he felt it was too serious a risk to run, for the sake of the investment obtained. He had come across the case of an heir to a peerage, who made a solemn declaration that there had been no previous dealings with his reversionary interest at all, but it came to light afterwards there had been two mortgages created on it. It was true that those had been discharged, and that there was not an attempt at fraud, but it illustrated the light way in which people signed declarations drawn up for them by their solicitors, without taking the trouble to read them.

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MR. G. J. LIDSTONE asks us to publish the following remarks, which he found no opportunity of making in the discussion:—

I should like to support strongly Mr. Coutts' suggestion that a proper investigation should be made into the mortality of life-



tenants—not because I agree with him that the absence of any reliable table of mortality has operated to restrict dealings with reversionary securities, but rather because, in the absence of any standard table derived from the experience of life-tenants themselves, the tendency appears to be to under-estimate the vitality of the life-tenant, and therefore to give unduly high prices. While the President held out no hope of an official investigation by the Institute, there could be little doubt that if the matter were taken up by an individual actuary, or by a small committee, the life and reversionary offices would willingly contribute their experience, which in combination would be of sufficient extent to yield results of great value. Even the investigation by Mr. Neil Campbell (vide *Transactions of the Faculty of Actuaries*, volume i, pages 91–5) to which Mr. Coutts refers, was sufficient, limited though it was in extent, to raise considerable doubt as to whether the  $O^a$  Tables do not over-estimate the mortality. It may be remembered that in that experience the total actual deaths between ages 60 and 89 were 102, and the expected deaths by the  $O^a$  experience 112, and while the deviation of 10 deaths is not greatly in excess of the “expected deviation”, the results were so consistent for different age groups that it is difficult to suppose the deviation was purely accidental.

Some idea of the extent to which the rate of interest earned is affected by a light mortality may be formed as follows:—If the reversions stand in the books on a  $4\frac{1}{2}$  per-cent basis, the mere increase in the values of the reversions from year to year (owing to the increase of age) will represent interest at from  $1\frac{1}{2}$  per-cent to 2 per-cent, and the balance of  $2\frac{1}{2}$  per-cent to 3 per-cent will be represented by the profit on reversions falling in. If the amount so falling in be, say, 10 per-cent under the amount expected, the amount of interest derived under this second head will, roughly speaking, be reduced in the same proportion, *i.e.*, by say one-tenth of  $2\frac{1}{2}$  per-cent, or about 5s. per-cent. This result is, of course, only roughly correct, as the actual difference must depend on the incidence of the mortality, and the distribution of the ages, but it is sufficient to show that a comparatively slight difference in the mortality may very seriously affect the rate earned.

It may be of some slight theoretical interest to point out that if absolute reversions stand in the books on the basis of interest at rate  $i$ , the interest annually yielded by the mere increase of value, in the case of reversions which do not fall in, is  $i \cdot \frac{{}_1V}{(1+i)P}$ . Thus:

$$\frac{{}_1A - A}{A} = \frac{d[a - {}_1a]}{A} = \frac{d[a - {}_1a]}{Pa} = \frac{d}{P} {}_1V = i \cdot \frac{{}_1V}{(1+i)P}$$

The many difficulties and dangers referred to, but not disposed of, in the paper and in the subsequent discussion, raise the question whether the offices might not take some collective action with the view of improving the position. In particular I would refer to the following points:

(1) *Conditions of Sale*.—It is generally felt and admitted that in many cases conditions of sale are unreasonable, and involve distinct



risk to purchasers. It is true that in some cases these conditions are relaxed, on application being made before the sale, but seeing that life and reversionary offices constitute such an important part of the market, they are surely in a position to bring to bear such pressure on the principal auctioneers as to secure general conditions of sale which would override, and not as at present be overridden by, special conditions of an unreasonable nature.

(2) *Distringas Notices*.—It ought to be possible to simplify the process of placing *distringas* notices on stocks, and to reduce the very considerable expense. There seems no reason why the present cumbersome process, requiring an *alidavit* to be filed in court as a preliminary to the lodgment of a *distringas* notice, should not be dispensed with, and the companies obliged to register a notice (in a simple form which could be easily settled) on payment of a comparatively small fee.

(3) *Notices to Trustees*.—The risks arising from the trustees failing to register, preserve or disclose notices, or from notices not being handed on to new trustees, when these are appointed, would be very greatly reduced if the following amendments in the law could be secured:

- a. Trustees to be obliged to endorse a Register of Notices (in common form) on the settlement, will, or other document, by which the trust is constituted or evidenced, and for the future all charges to rank according to the date of entry in such register.
- b. The document bearing the register of notices to be produced for inspection by the *cestuis que trustent*, or persons claiming under them, if and when required.
- c. Trustees to be obliged to give a written acknowledgment of receipt of notice on payment of a small fee—as in the case of notices served on life offices under the Policies of Assurance Act, 1867.

The alterations suggested under (2) and (3) could only be effected by legislation, and might not be secured at the first attempt; but it would surely be worth while for the offices to use their great collective weight—at present, perhaps, insufficiently utilized for the common welfare—to press for the inclusion, in the next conveyancing statute, of clauses to something like the above effect, with the view of protecting transactions which in modern times are both extensive and important, as well as perfectly straightforward and legitimate.

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*French Assurance Law, 1905.*

WE publish, for the information of our readers, the accompanying translation of the text of the French Law, passed in March 1905, relative to the superintendence of Life Assurance Companies and kindred institutions; and of the Administrative Decrees, and General Regulation, issued in January and June 1906, formulating in detail the requirements of the Law.

These requirements may be briefly summarized as under :

- (a) Compulsory registration of all French companies, and all foreign companies operating in France and Algeria (Arts. 1 to 3) ; with proof of legal authorization in the country of origin and in France, and copy of the constitution of the society; also scales of net and gross premium rates, in respect of existing and of new business, with technical explanations of such rates; and lodgment of a deposit, varying from £2,000 to £20,000, in cash or in specified French securities; with provisions for return of such deposit, and for withdrawal of the registration (Decrees A\*, B).
- (b) Regulations limiting the establishment expenses of French societies, in the case of mutual or tontine societies, to the amount of the original establishment fund, and in the case of other societies, to one quarter of the original capital (Decree C).
- (c) The provision of a guarantee reserve fund, by an annual payment equivalent to 3 per 1,000 of the receipts from single or periodical premiums; such payment to be reduced by one-half when the guarantee fund is equal to 5 (or 6) per-cent of the mathematical reserves, and to be no longer obligatory, when equal to 10 per-cent of such reserves (Art. 5 and Decree D).
- (d) The provision of mathematical reserves, and for annual returns as to expected and actual mortality, and interest assumed and realized; with power to the Minister to rectify the bases of the reserves and the premium rates, once every five years (Art. 6). Decree E sets out in detail the minimum bases as to interest, mortality, and loading, upon which the premium rates and

\* The translations of the several Administrative Decrees, and the General Regulation, have been distinctively marked by the letters (A) to (K); the order of the Decrees following that of the sub-sections of Art. 9 of the Law.—[Ed., J.I.A.]

mathematical reserves must be computed; interest being taken at  $3\frac{1}{2}$  per-cent, and mortality according to the several French tables of mortality specified. Specific regulations are made as to the minimum proportionate loadings, in respect of different classes of assurance and annuity business, to provide for expenses of (a) management; (b) collection; (c) acquisition of business; (d) disbursement of annuities. The mathematical reserves are not to be less than those brought out by means of valuation premiums (*primes d'inventaire*) equal to the gross premiums, less the portions of the loadings for (b) collection and (c) acquisition of business; and with due allowance for the incidence, and fractional payments, of premiums and annuities.\*

- (e) The setting apart, in the case of French companies, of the mathematical reserves, the guarantee reserve, and the amounts held on account of deferred profits, as a provision for the assurance liabilities; with annual notification to individual assured of the share allocated to each assured respectively, in respect of profits declared as payable after a deferred period. The securities representing these reserves, in respect of the French and Algerian policyholders of foreign companies, to be deposited at the "Caisse des Dépôts et Consignations", as a charge in favour of such policyholders (Art. 7); and to be subject to annual increment and adjustment, in respect of each year's transactions and accounts (Decree F).
- (f) The assets representing the amounts thus set apart, or placed on deposit, to be invested in certain specified classes of securities, as defined in detail in Decree K. These investments may be;—*without limit*, in securities guaranteed by the French Government, or in bonds secured by local French or Algerian Governments; in policy loans; or mortgages of city property in France, up to 50 per-cent of its value;—*to the extent of two-fifths at most*, in loans to French Departments, Communes or Colonies; in purchase, or mortgage to the extent of 50 per-cent, of real estate in France or Algeria;—*to the extent of one-quarter at most*, in any

\* See Note on p. 358 [ED. J.I.A.]

security whatever quoted on the French Bourse, and approved by the shareholders ; or, in the case of foreign societies, and until the maturing of policies existing, in any securities allowed by legislation in their own land. Securities are to be valued at cost price, except where such value exceeds in the aggregate by more than 5 per-cent that deduced from the current quotation on the Paris Bourse, in which case it shall be determined by Decree as to the reduction of the value to the current quotation ; and provisions are added as to the valuation of mortgages, loans, house property, and reversions and life interests. Foreign societies are allowed to replace their existing securities by those specified, in a period of 5 years, at the rate of one-fifth per annum.

- (g) Regulations governing the conduct of tontine societies, and of survivorship and assurance associations established by such societies (Decree G).
- (h) Provisions as to the record of policies in special registers, according to class, right to participation, &c. ; and as to modifications in, or cancelment of such policies (Decree H).
- (j) Provisions regulating the affairs of societies established for the management of life assurance (*Entreprises de gestion d'assurances sur la vie*) (Decree J).
- (k) The appointment of a Consultative Committee on Life Insurance, including Senators, Deputies and Government Officials ; three qualified Actuaries ; four Managers or Directors of Companies ; four Experts on life insurance matters ; and a Professor on Law ; to confer with the Minister of Commerce as to the execution of the provisions of the law (Art. 10).
- (l) Provisions for publication in French, and official submission of Annual Reports by the companies, with schedules and tables in an approved form, to be sent on application (and payment) to any member or assured ; also for the appointment of Commissioner-Comptrollers to examine into the companies' affairs at their offices (Art. 11).
- (m) Provisions for a legal office and agent, and a special system of accounts, in respect of foreign companies operating in France and Algeria (Art. 12).

- (n) Provisions for an Annual Report from the Minister of Commerce, as to the working of the Law, and the position of the companies regulated thereby; a rateable charge to be made on the companies within a fixed limit, for all expenses of supervision and control under the Law (Art. 13).
- (o) Provisions for penalties, on non-compliance with requirements of the Law (Arts. 14 to 18).
- (p) Application for registration to be made within two months of the promulgation of the administrative regulations and Decrees referred to in the Act (Art. 19).
- (q) Provisions modifying in certain respects the requirements of the Law, in the case of French companies duly authorized under existing laws, as to changes in their bye-laws, amount of capital, existing investments, &c. (Art. 20).

We are indebted to the Rev. J. P. S. R. Gibson, F.I.A., for a careful and scholarly translation of the French text of the Law and the Decrees; and also to our esteemed Corresponding Member, M. Léon Marie, for elucidation of some doubtful points arising on the translation and interpretation of the provisions.—ED. J.I.A.

*Law relating to the superintendence of Life Insurance Companies, and of all Societies whose transactions are connected with the duration of Human Life.*

SECTION I.—REGISTRATION OF SOCIETIES.

*Article 1.*—All French or foreign societies of every kind which contract liabilities whose carrying out is dependent upon the duration of human life are subject to the present law.

This law shall not refer to the societies defined by the law of 1 April 1898, *re* friendly societies and provident institutions, either public or private, which are governed by special laws.

*Article 2.*—The said societies must limit their transactions to one or more of those which are the subject of the present law. They shall not arrange for nor carry out contracts which provide for profits to be apportioned by lot.

They shall not begin operations before being registered at their request by the Minister of Commerce. Within six months of the notification of such request, the Minister of Commerce must notify the registration of such company in the *Official Journal*, or inform those interested of his refusal to do so.

No modification, either of the bye-laws, or of the tables of premiums or rates, shall be put into force by any such company until a new registration has been obtained under the same conditions.



*Article 3.*—The refusal to register must be based, either on an infringement of the laws, particularly of those regulating the societies, or of the decrees provided for in Article 9 of the present law.

The interested parties can lodge an appeal, on the grounds of incompetency of the Court, before the Council of State, which must give its verdict within three months.

## SECTION II.—GUARANTEES.

*Article 4.*—The bye-laws of French joint stock (*anonyme*) or limited (*commandite*) companies must provide for compulsory dissolution of the company, in case of a loss of one-half of the original capital of the company.

The bye-laws of mutual and tontine societies must determine the rules governing the use to be made of the amounts paid in, as well as the proportion of such amounts intended to cover expenses of management of the society.

*Article 5.*—French joint stock or limited companies must have an original capital of at least two million francs.

French mutual or tontine companies must constitute an initial guarantee fund, which must not be less than 50,000 francs, and which must be written off within a period not exceeding fifteen years.

All societies must also set up, under the conditions set forth in Article 9, paragraph 4, a guarantee reserve, which shall take the place of the amounts to be set aside as provided by Article 36 of the law of 24 July 1867. This reserve is not compulsory, however, for tontine transactions.

*Article 6.*—All societies which make contracts with fixed liabilities must set up mathematical reserves for these, equal to the difference between the amount of the liabilities of the said companies and of their policyholders respectively, under the conditions determined by the decree provided for in Article 9, paragraph 5. This shall not apply to foreign companies, except as to their contracts entered upon or carried out in France and Algeria.

The said societies must submit annually, at such time and in such form as may be determined by the Minister, after having the advice of the Consulting Committee on Life Insurance provided for in Article 10, a comparison: (1) Between the actual mortality among their insured, and the expected mortality, according to the tables upon which the calculations of their mathematical reserves and their premium rates have been based. (2) Between the actual rate of interest realized on their investments, and the rate used in the calculations above mentioned.

In case of important or persistent divergencies in either of these elements, a rectification of the bases of the calculation of the reserves of existing policies, and of the premium or rates, may be demanded by Ministerial decree, but not oftener than every five years.

These decrees will be made in conformity with the advice given by the Consulting Committee on Life Insurance, after the representatives of the societies have been heard, who must submit their observations in writing within one month. The said decrees

will fix the time within which the rectification must be made. The amounts of the payments due to the rectification of the mathematical reserves must, at the end of each such examination, take into account the time taken in making the rectification.

Tontine societies must at once invest all the premiums paid in, after deduction of the statutory expenses of management, under the conditions stated by the decree as set forth in Article 9, paragraph 7.

*Article 7.*—When profits due to the insured are not immediately payable at the close of the business period within which they have been earned, a special account must be sent each year to the insured, showing the share of profits apportionable to each policy applied for or undertaken in France and Algeria.

The following assets of French societies are reserved for the settlement of their insurance liabilities, as provided by paragraph 6 of Article 2101 of the Civil Code:—namely, the mathematical reserve, the guarantee reserve, and the amount of the account specified in the preceding paragraph.

The securities representing the corresponding portion of the assets of foreign societies must, with the exception of real estate, be deposited with the “Caisse des Dépôts et Consignations”, under the conditions prescribed by paragraph 6 of Article 9. The act of making this deposit confers upon the insured a lien on the said securities, as regards all contracts entered upon or carried out in France and Algeria.

*Article 8.*—A General Regulation, decided upon by the suggestions of the Ministers of Commerce and of Finance, will determine the class of securities and real estates in which assets of French societies and the proportion of the assets of foreign societies which is to be regarded as belonging to their contracts in France and Algeria, must be invested, also the bases of the annual valuation of the different classes of investments, and the guarantees to be given for securities not represented by nominative certificates.

The societies must file with the Minister, in the forms and within the period prescribed by him upon the advice of the Consulting Committee, periodical statements of alterations that have occurred in the nature of their assets.

*Article 9.*—The Decrees, issued on the advice of the Consulting Committee on Life Insurance provided for by the next Article, will determine:

1. The documents and proofs to be filed in support of the request for registration, and also the amount of the deposit required to be previously made at the “Caisse des Dépôts et Consignations” by the various classes of societies, also the conditions under which the said deposit can be charged or withdrawn.
2. The time beyond which the registration of a society, which has not commenced operations, shall not be valid.
3. The maximum amount allowed for the initial establishment expenses for the various classes of French societies, and the time limit within which these must be written off.

4. The fixing of the guarantee reserve for each class of societies.
5. The different mortality tables, the rate of interest, and the loading, by which the minimum premiums or rates for business to be done, as also the mathematical reserves, must be calculated. These figures must be published in the *Official Journal*, at least six months before the beginning of the accounts of the first period to which they apply.
6. The conditions of deposit and withdrawal of securities representing, for foreign societies, the proportion of their assets as set out in Article 7.
7. The conditions under which Tontine societies shall be carried on.
8. The forms and special registers in which societies must keep the policies entered on or carried out in France and Algeria.
9. The forms under which societies for the management of life insurance business\* must be conducted, and the fixed maximum charges that may be incurred for expenses of management. Such societies must deposit with the "Caisse des Dépôts et Consignations" a guarantee fund of 100,000 francs. These shall not secure an initial right of management for a longer period than twenty years, at the expiration of which time a renewal for a period of not longer than ten years may be made; each renewal can only be made during the year preceding the expiration of the right exercised.

### SECTION III.—SUPERVISION AND CONTROL.

*Article 10.*—There is to be a Consulting Committee on Life Insurance in communication with the Minister of Commerce, which shall consist of 21 members, as follows: Two Senators and Three Deputies, elected by their colleagues; the Director of Insurance and Social Providence (Prévoyance Sociale) under the Ministry of Commerce; the Director-General of the "Caisse des Dépôts et Consignations"; a representative of the Minister of Finance; three duly qualified Fellows of the French Institute of Actuaries; the President of the Chamber of Commerce, or a member of the Chamber delegated by him; a Professor of the Faculty of Law of Paris; two Managers or Directors of mutual or tontine insurance companies; two Managers or Directors of joint-stock or limited insurance companies; and four experts on life insurance questions.

A decree shall determine the method of nomination of members and of appointment to vacancies in the said Committee.† It shall also decide who is to be President, Vice-President and Secretary.

The Committee must be consulted concerning all requests for registration provided for by Article 2, and in all other cases provided

\* "*Entreprises de gestion d'assurances sur la vie.*" We understand that the societies referred to are those which, in consideration of a specified remuneration, undertake the management of French mutual assurance societies and kindred institutions.

† This Decree, published on 17 March 1905, is not included with those here translated.—[ED. J.I.A.]

for in this law. The Minister may consult the Committee upon all other questions relating to the carrying out of this law.

To render valid any decisions on the points specified in the third paragraph of Article 6, and in Articles 18 and 21, at least nine members must be present.

*Article 11.*—Every society shall: (1) Publish in French an annual report of all its operations, with statements and tables annexed. (2) Submit the said report to the Minister of Commerce, and deposit it at the offices of the Civil Courts Registrars, and the Commercial Tribunal Registrars, both of the Department of the Seine, and of that Department in which the Head Office of the Society happens to be. (3) Supply a copy of the said report, to any member or person insured who may demand one on payment of a sum not exceeding one franc. (4) Publish in the *Official Journal* every year, at its own cost, a summarized report which shall include the general profit and loss account, the general balance sheet, and the general movement of business in force.

Ministerial decrees, issued on the advice of the Consulting Committee on Life Insurance, shall determine, at least three months before the beginning of the financial period, the forms of schedules and tables to be annexed to the published report, the date when the report must be published and deposited, and the form and time within which the publication of the said report must be made in the *Official Journal*.

The societies shall, moreover, at any time, and in the forms and within the periods stipulated by him, furnish the Minister with all documents and explanations that he may consider necessary.

The societies shall be subject to the supervision of sworn Commissioner-Comptrollers, who shall be appointed as determined by the decrees, on the advice of the Consulting Committee on Life Insurance, and they shall have the right at any time to examine all the companies' business in its own offices, independently of other persons who may be specially delegated for this purpose by the Minister.

*Article 12.*—Foreign societies, in so far as concerns their transactions under the present law, must have in France and Algeria a special office, and a special system of accounts, for all contracts entered on in France and Algeria, and must have an accredited agent who shall represent them before the Minister of Commerce and who shall manage the transactions referred to above. Such Agent must be domiciled in France. He alone shall represent the society before the Minister, as regards holders of policies issued in France and Algeria, and especially with regard to signing policies, amendments, receipts, and other documents relating to business transacted.

Every society must, when required, place before the Minister of Commerce, within the period specified by him, a French translation duly certified, of all documents issued in a foreign language, which relate to its operations.

The general and special conditions of policies, amendments, and other documents relating to the carrying out of contracts, must be either written in or translated into French. In the latter case, the French text is alone binding upon French insured.



*Article 13.*—The Minister of Commerce shall each year lay before the President of the Republic, and publish in the *Official Journal*, a general report relating to the working of the present law, and the position of all societies under its control.

Expenses of every nature resulting from the supervision and control are chargeable to the societies. At the end of each examination a Ministerial decree shall determine the liability of each society, which shall be in proportion to the aggregate premiums and rates of every nature collected by them during the year, with the exception of the business transacted out of France and Algeria by foreign societies. The contribution of each of the societies shall not, however, in any case exceed 1 per thousand of the said amount.

A detailed account of the receipts and expenses incurred in connection with the supervision and control of the undertakings shall be attached to the decree.

#### SECTION IV.—PENALTIES.

*Article 14.*—The Minister of Commerce may in his own right and without the assistance of law, impose on the said societies fines as follows, which shall be payable in the same manner as registration fees :

1. 20 francs a day, for delay in the production of documents mentioned in the third paragraph of Article 11, and in the second paragraph of Article 12.
2. 100 francs a day, for any delay regarding the issue or publication of documents mentioned in the second paragraph of Article 6, and in paragraphs 1, 2 and 4 of Article 11.

*Article 15.*—Infringements of the first and third paragraphs of Article 6, of the first and third paragraphs of Article 7, of Article 8, of Article 20, of Article 21, of the public administrative regulation prescribed in Article 8, and of the decrees provided by paragraphs 3 to 8 of Article 9, are to be notified by official reports of the Commissioner-Comptrollers, which shall be considered accurate, until proof to the contrary is forthcoming, without prejudice to the right of proceedings provided by Common Law. The societies shall be prosecuted before a Criminal Court, at the request of the Public Prosecutor, and be punishable by a fine of 100 to 5,000 francs, and in case of repetition of the offence to a fine of 500 to 10,000 francs.

*Article 16.*—Any person who may propose or offer to enter on policies of insurance, and especially each of the directors or managers of societies who transact business coming within the scope of the present law, before the publication in the *Official Journal* of the registration provided for under Article 2, or who undertake new business after the publication of the decree provided for by Article 18, or after registration has been refused according to Article 19, shall be prosecuted before the Criminal Court, and be liable to a fine of 16 to 100 francs. The said fine shall be payable for each transaction carried out by the offender, who may, also, in case of repetition of the offence, be liable to not more than one month's imprisonment.



Under the same penalties, all prospectuses, advertisements, circulars, and other documents intended for distribution or published by a society, subject to the present law, must, in every case, bear in addition to the name or title of the society, the following words in similar characters: "*Entreprise privée, assujettie au contrôle de l'Etat*" (private society subject to State control), and must not include any misleading statement as to the true nature or the actual scope of its transactions, or as to the character of the control.

Every fraudulent declaration or dissimulation, either in a report or in any other document laid before the Minister of Commerce, or communicated to the public, is subject to the penalties provided for by Article 405 of the Penal Code.

Article 464 of the Penal Code shall be applied to all acts punishable under the present and preceding Article.

*Article 17.*—Judgments pronounced against societies or against their representatives, in the carrying out of the foregoing Article and of Article 15, must be published at the expense of the offenders or of the societies legally responsible, in the *Official Journal*, and in at least two other papers named by the Court.

*Article 18.*—The registration of a society, effected under Article 2 of the present law, ceases to be valid from the issue of a decree, stating that the society is no longer working in conformity with its bye-laws, or with the present law, or with the decrees or regulations which this law prescribes. Such a decree will be issued, on the advice of the Consulting Committee on Life Insurance, and after due notice has been given to the representatives of the societies in question, either to submit explanations and comments in writing, or to arrange to be heard personally, within one month of their receiving notice of the irregularities in question. The Committee must, within the following month, give the reasons upon which it bases its opinion.

On the ground of incompetence of the Court, the society may, within eight days of the notification of the decree, appeal to the Council of State, and the latter must give its verdict within one month. The appeal nullifies the said decree, which shall not be published in the *Official Journal*, unless the Council of State rejects the appeal.

#### SECTION V.—TEMPORARY PROVISIONS.

*Article 19.*—French or foreign societies, subject to the present law, and transacting business in France or Algeria at the time of its promulgation, shall immediately conform to its provisions, and make a request for registration, as specified in Article 2, within two months of the setting forth of the public administrative regulations set forth in Articles 8 and 22, and of the decrees mentioned in Article 9.

They may, however, provisionally continue their operations until the said request has been dealt with.

*Article 20.*—French societies authorized in accordance with existing laws, may, after registration has been granted them as specified in Article 2, modify their approved bye-laws, without

obtaining the sanction of the Government, so long as they conform to the legislation in force respecting such societies.

Notwithstanding the provisions of Article 5, they need not increase their original capital to the minimum stated in the said Article. Moreover, if they obtain registration as specified in the preceding Article, they may keep the investments previously made by them in conformity with their bye-laws, without reference to the limitations prescribed by the public administrative regulation set forth in Article 8, but they shall not from the date of the promulgation of the said regulation, make any new investments in those classes whose maximums, as fixed by this present law, have been attained or exceeded, until the statutory proportion has been re-established.

Nevertheless, investments in first mortgages, at most to the extent of one-half of the estimated value of the property, may, within a period not exceeding twenty-five years, be renewed so long as the total amount does not exceed that invested in this way prior to 1 July 1904.

*Article 21.*—A Ministerial decree, to be issued in accordance with the advice of the Consulting Committee on Life Insurance, shall settle under the conditions specified in the last paragraph but one of Article 6 for every one of the societies registered according to the provisions of Article 19, the basis for the calculation of the mathematical reserves for the business done, prior to the enforcement of the decree provided for by paragraph 5 of Article 9.

*Article 22.*—The first paragraph of Article 66 of the law of 24 July 1867 is hereby cancelled, as are also all other provisions regarding Tontine and Life Insurance companies.

A General Regulation shall determine the conditions under which mutual Life Insurance companies or Tontine associations may be organized.\*

*Article 23.*—The present law is applicable to Algeria, to the colonies of Réunion, Martinique, Guadeloupe, Guiana, the French Indies and New Caledonia.

The present law has been deliberated upon and adopted by the Senate and by the Chamber of Deputies, and shall be carried out as a State law.

PARIS, 17 March 1905.

#### ADMINISTRATIVE DECREE

under § (1) of Article 9 of the Assurance Law of 17 March 1905.

(A.)—AS TO REGISTRATION AND DEPOSIT.

*Article 1.*—The requests for registration, referred to in Articles 2 and 19 of the law of 17 March 1905, will not be considered unless they are duly accompanied by the following documents and substantiations:

- i. The receipt for the previous deposit at the "Caisse des Dépôts et Consignations" of the amount referred to later;
- ii. An original or copy of the Articles of the society;

\* This General Regulation, published on 12 May 1906, is not included with those here translated.—[ED. J.I.A.]

- iii. The complete text of the bye-laws ;
- iv. The complete scale of gross premiums, pure premiums, and if necessary, valuation premiums (*primes d'inventaire*), in connection with all the transactions of the society ;
- v. In the case of tontine operations, the rates and reckonings (*barèmes*) referring thereto ;
- vi. A technical memorandum explaining how these rates are arrived at, and the bases used in the calculation of the various classes of premiums or rates.

*Article 2.*—The societies, referred to in Article 19 of the law of 17 March 1905, must produce in addition :

- i. A notification of the legal régime under which the company works ;
- ii. The rates referring to the transactions carried out prior to the registration, also an explanatory technical note as specified in numbers iv, v and vi of the preceding Article ;
- iii. The guarantee that the society possesses, as regards its contracts and rates in force before registration, mathematical reserves equal to the difference between the values of its liabilities and those of the assured.

*Article 3.*—Foreign societies must produce, besides the documents and qualifications respectively provided for above :

- i. The customary certificates, attestations and documents necessary to establish the legality of the society in its own land ;
- ii. A specification of the Head Office of the society, with regard to the transactions referred to in Articles 12 and 23 of the law of 17 March 1905 ;
- iii. The document accrediting to the Minister of Commerce an agent specially appointed for the management of the said transactions.

*Article 4.*—The deposit which the societies must previously make at the “Caisse des Dépôts et Consignations” shall be equal to :

- i. For mutual or tontine French societies, a quarter of the original capital ; this deposit must not be less than 50,000 francs, nor greater than 500,000 francs ;
- ii. For all other societies, whether French or foreign, 500,000 francs.

*Article 5.*—This deposit must be made, either in cash or in State bonds or in bonds guaranteed by the State, in negotiable and paid-up stock of the Departments, of the Communes and of the Chambers of Commerce, or in real estate bonds of the Crédit Foncier. The stock shall be valued at the mean quotation of the Bourse of Paris on the day preceding the deposit, and if there be no quotation on that date, at the value of the last quotation made.

*Article 6.*—The deposit is returnable to the societies on the decision of the Minister of Commerce, and within ten days of the notification. This notification shall be addressed to the society, and to the “Caisse des Dépôts et Consignations” :

- i. In the case of registration, within a month of the notification of the registration in the *Official Journal*;
- ii. In the case of refusal of registration, within a month either of the acquiescence in the refusal by the society, or of the rejection of its appeal for incompetency of the Court before the State Council.

*Article 7.*—The Minister of Industry and of Public Works is responsible for the carrying out of the present decree, which shall be published in the *Official Journal*, and inserted in the Bulletin of Laws.

PARIS, 22 June 1906.

#### ADMINISTRATIVE DECREE

under § (2) of *Article 9 of the Assurance Law of 17 March 1905.*

##### (B.)—AS TO LAPSE OF REGISTRATION.

*Article 1.*—The registration, prescribed by Art. 2 of the law of 17 March 1905, will cease to be valid, if the society has not commenced business within one year, dating from the notification of the registration in the *Official Journal*.

*Article 2.*—Every society which, before the expiration of the said period, has not thus commenced business, is definitely deprived of the benefit of registration, and can only take up business after a fresh registration. The Minister of Commerce shall notify such cancelling of registration in the *Official Journal*.

*Article 3.*—The Minister of Commerce, Industry, Posts and Telegraphs is responsible for the carrying out of the present decree, which shall be published in the *Official Journal* of the French Republic, and included in the Bulletin of Laws.

PARIS, 20 January 1906.

#### ADMINISTRATIVE DECREE

under § (3) of *Article 9 of the Assurance Law of 17 March 1905.*

##### (C.)—AS TO ESTABLISHMENT EXPENSES.

*Article 1.*—The preliminary establishment expenses of French societies shall be limited as follows :

- i. For mutual or tontine societies, to the amount of the original establishment fund ;
- ii. For other societies, to one-quarter of the original capital.

*Article 2.*—These expenses must be completely written off within a period not exceeding fifteen years from the date of registration.

*Article 3.*—The Minister of Commerce, Industry, Posts and Telegraphs is responsible for the carrying out of the present decree, which shall be published in the *Official Journal* of the French Republic, and included in the Bulletin of Laws.

PARIS, 20 January 1906.

## ADMINISTRATIVE DECREE

under § (4) of Article 9 of the Assurance Law of 17 March 1905.

## (D.)—AS TO GUARANTEE RESERVES.

*Article 1.*—The guarantee reserve which the societies must set up to comply with the third paragraph of Article 5 of the law of 17 March 1905, must be provided :

- i. For French societies, joint stock or limited, by the annual payment from their receipts of an amount equal to at least 3 per 1,000 of the total of their single or periodical premium receipts during the period. This payment shall be reduced by one-half when the guarantee reserve becomes equal to 5 per-cent of the mathematical reserves, and it shall no longer be obligatory when it becomes equal to 10 per-cent of the mathematical reserves.
- ii. For French societies other than those referred to in the preceding paragraph, except for tontine societies, as well as for foreign societies as regards transactions undertaken or carried out in France, Algeria, or the Colonies referred to in Article 23 of the law of 17 March 1905, by the annual payment from their receipts of a sum equal to at least 3 per 1,000 of the total of the premiums or rates received during the period. This payment shall be reduced to half when the guarantee reserve becomes equal to 6 per-cent of the mathematical reserves, and shall cease to be obligatory when this reserve becomes equal to 10 per-cent of the mathematical reserves.

*Article 2.*—The Minister of Commerce, of Industry, and of Public Works is responsible for the carrying out of this decree, which shall be published in the *Official Journal* of the French Republic, and inserted in the Bulletin of Laws.

PARIS, 22 June 1906.

## ADMINISTRATIVE DECREE

under § (5) of Article 9 of the Assurance Law of 17 March 1905.

## (E.)—AS TO CALCULATION OF PREMIUMS AND RESERVES.

*Article 1.*—For the time being, and without prejudice to the application of surplus or profits which may be due or granted to the assured at the close of the periodical accounts, the premiums or rates for business to be transacted, as also the corresponding mathematical reserves, shall be calculated on the following bases as a minimum :

- I. For mutual assurance societies which by agreement in their bye-laws neither pay commission nor expenses, in any form, for the acquisition of new business :



- i. A rate of interest of  $3\frac{1}{2}$  per-cent;
  - ii. The AF (a) Table of Mortality for whole-life assurances; the CR (b) Table for endowments and life annuities;
  - iii. A loading of 6 per-cent of the gross premium or rate, for expenses of management, and of 1 per-cent for expenses of collection.
- II. For all other insurance societies, whether mutual, joint-stock, limited, or others, who carry on insurance business:
- i. A rate of interest of  $3\frac{1}{2}$  per-cent;
  - ii. The AF Table of Mortality for whole-life assurances; the RF (c) Table for endowments and life annuities;
  - iii. A uniform loading in respect of whole-life assurances, endowment assurances, and fixed term assurances, equal to  $3\frac{1}{2}$  per 1,000 (d) of the sum assured, on each annual premium payable during the entire length of the assurance, for expenses of management; 6 per-cent on each of the gross premiums, for expenses of collection; 1 per-cent on the sum assured for expenses of acquisition. However, in the case of temporary assurances, this last loading is fixed at  $\frac{1}{25}$  per-cent (e) on the sum assured, for each year of duration, up to a maximum of 1 per-cent; and, in the case of assurances securing reversionary annuities, the loading for expenses of management is  $3\frac{1}{2}$  per 100 of the annuity until the death of the survivor, and the loading for expenses of acquisition is  $\frac{1}{25}$  per-cent (f) of the annuity for each year of duration, when such annuity is temporary, up to a maximum of 10 per-cent.
  - iv. A uniform loading, for endowments, of 1 per 1,000 of the sum assured, on each annual premium payable during the entire length of the assurance, for expenses of management;  $2\frac{1}{2}$  per 100 on each of the gross premiums, for expenses of collection;  $\frac{1}{2}$  per 100 on the gross single premium, for expenses of acquisition.
  - v. A uniform loading, on life annuities, of 4 per-cent of the annuity assured, for expenses of management; 1 per-cent of the annuity assured, for expenses of disbursement; 3 per-cent of the gross single premium, for expenses of acquisition. Provided that, in the case of deferred annuities, the loadings

(a) Experience of French Assured Lives: see *J.I.A.*, vol. xxxii, page 498-9.

(b) "Caisse des retraites pour la vieillesse":—deduced from the experience of the French Government.

(c) Experience of French Annuitants: see *J.I.A.*, vol. xxxiii, page 500-1.

(d) Printed as "3.5 p. 100" (in the copy of the decree from which our translation is made. The corrections have been made, after confirmation by M. Léon Marie.—[Ed. *J.I.A.*])

(e) Printed as "1.25 p. 100"

(f) Printed as "10.25 p. 100"

shall be the same as those payable on a deferred capital, the amount of which is equal to the capital required at the end of the deferred period to provide the annuity, together with a sum corresponding to the present value of the expenses of management, and of disbursement of the said annuity.

*Article 2.*—The bases to be applied to transactions involving a combination of different simple benefits shall be determined by analogy.

*Article 3.*—The mathematical reserves must not be less than those which would be obtained by means of valuation premiums (*primes d'inventaire*), equal to the gross premiums, calculated in accordance with Articles 1 and 2, less that portion of the loading which is provided to cover expenses of collection and of acquisition.\* Allowance must be made, in the calculation of such reserves, for the incidence, and the fractional payments, of the premiums or rates; and, as regards immediate life annuities, for the incidence of the instalments.

*Article 4.*—The premiums and the mathematical reserves, in respect of assurances with participation in profits, where the amount of bonus is determined in advance, must be increased accordingly.

*Article 5.*—The Minister of Commerce, Industry, Posts and Telegraphs is responsible for the carrying out of the present decree, which shall be published in the *Official Journal* of the French Republic, and included in the Bulletin of Laws.

PARIS, 20 January 1906.

#### ADMINISTRATIVE DECREE

under § (6) of Article 9 of the Assurance Law of 17 March 1905.

(F.)—AS TO DEPOSIT OF RESERVES BY FOREIGN COMPANIES.

*Article 1.*—Every year, before 31 May, foreign societies must prove that they have deposited in the “Caisse des Dépôts et Consignations”, as demanded by the law of 28 July 1875, securities representing the portion of their assets referred to in Article 7, paragraph 3, of the law of 17 March 1905, and the amount shall be settled by the balance-sheets of the preceding 31 December.

*Article 2.*—This proof shall consist in producing before the Minister of Commerce a certificate of deposit given by the “Caisse des Dépôts et Consignations”, enumerating, according to the declarations of the society, approved of by the Minister of Commerce or his delegate:

\* We understand from Mr. H. Bearman, F.I.A. (who has kindly perused the proofs of this translation) that the usual practice of the French companies is to value by the use of *primes d'inventaire*, which include the provision for expenses of management; and that the single premium by which the sum assured is valued is deduced from these valuation premiums, so that  $A'_x = (\pi_x + X)(1 + a_x)$ , where  $A'$  is the single premium employed in the valuation,  $X$  is the provision for expenses of management, and  $\pi$  and  $a$  are the usual net values of the premium and annuity. The reserve for an ordinary whole-life policy, after  $n$  completed years' duration, is thus equal to  $(\pi_{x+n} + X)(1 + a_{x+n}) - (\pi_x + X)(1 + a_{x+n}) = (\pi_{x+n} - \pi_x)(1 + a_{x+n})$ ; an expression equivalent to the normal net reserve value. In the case, however, where the premiums are fully paid up, or where the period over which they are payable is less than the whole duration of the policy (as in limited premium cases), the reserve made is no longer equivalent to the normal net reserve; but an additional provision is made (as it were automatically) by the formula, in respect of the expenses of management arising after the cessation of premiums.—[ED. J.I.A.]

- i. The securities included in that part of the assets which corresponds (a) to the mathematical reserves for transactions previous to registration; (b) to bonuses allotted on the said contracts under the conditions of Article 7, paragraph 1, of the above law.
- ii. The securities included in that part of the assets which corresponds (a) to the mathematical reserves of transactions entered upon after registration; (b) to the bonuses allotted to the said transactions under the conditions of Article 7, paragraph 1, of the said law; (c) to the guarantee reserve.

The declaration of the society shall specify the nature of the securities, their number, and their value, as calculated either by the quotation of the Paris Bourse the day previous to the declaration, or, for securities not then quoted at the said Bourse, at the last known quotation at the said Bourse; or, in default of this, at the quotation of one of the principal towns of the country issuing the security. The declaration must, as regards foreign securities referred to in Article 6 of the Public Administrative Decree (K) of 9 June 1906, be accompanied by proofs that each of the securities is held by the Society in accordance with the law relative to Life Insurance in its own land.

*Article 3.*—Foreign societies undertaking tontine business must prove, before 31 May of each year, under the conditions of Article 2 above, that they have deposited securities representing the funds of the survivorship associations and the assurance associations as at the previous 31 December. The certificate of deposit shall reproduce the Society's declaration, which must enumerate separately:

- i. The securities representing the funds of each of the survivorship associations which was not fully liquidated on the preceding 31 December.
- ii. The securities representing the funds of each of the assurance associations which was not fully liquidated on the previous 31 December.

*Article 4.*—The securities thus deposited cannot be withdrawn except in the case of:

- i. Re-investment of funds previously realized as certified by the "Caisse des Dépôts et Consignations", and at least equivalent to the value of the securities withdrawn, according to the Bourse quotation on the day of re-investment.
- ii. A reduction in the reserves, or of the bonuses allotted, due to the fluctuations in the Life Insurance business.
- iii. Liquidation of tontine associations.

The withdrawal of securities representing the funds of liquidated tontine associations can only take place after the liquidation has been verified by the Minister of Commerce.

The withdrawal of securities representing the mathematical reserves or the bonus allotted, as provided for above, can only take place at most once in three months, and only after proof that at

least equivalent reduction has taken place in the liabilities of the society.

The withdrawal of securities shall not take place without the previous consent of the Minister of Commerce or his delegate.

*Article 5.*—The Minister of Commerce, of Industry and of Public Works, and the Minister of Finance, are responsible each in his own department for the carrying out of the present Decree, which shall be published in the *Official Journal* of the French Republic and inserted in the Bulletin of Laws.

PARIS, 25 June 1906.

#### ADMINISTRATIVE DECREE

*under § (7) of the Assurance Law of 17 March 1905.*

(G.)—AS TO TONTINE SOCIETIES.

*Article 1.*—Tontine societies must fix in their bye-laws, according to the Public Administrative Decree referred to in Article 22 of the law of 17 March 1905, and the present Decree, the conditions of the formation and duration of survivorship and assurance associations.

#### *Class I.—Survivorship Associations.*

*Article 2.*—The opening and the formation of each survivorship association, as also the closing of the subscription lists for the said association, must be verified by the administrative council of the society.

*Article 3.*—Each association must be liquidated as soon as it matures.

*Article 4.*—The funds arising from subscriptions must be put *in toto* to the account of the associations, after deduction only of the statutory management expenses.

The funds of each association must be managed separately, and cannot in any case be pooled with those of other associations.

*Article 5.*—The funds of the associations must be invested at the latest within a month of their reception.

The date of the purchase and the price of the securities shall be proved by the lists of the broker, who must, moreover, mention the associations on whose account the securities have been bought.

Interest and arrears (*arrérages*), as well as reimbursements, premiums and lots (*lots*), must be invested under the same conditions.

*Article 6.*—The securities belonging to the associations formed by French societies must be at once deposited, either at the “Caisse des Dépôts et Consignations” or at the Bank of France, in the name of the society, mentioning the associations to which they belong, which shall also appear on the receipt for the deposit.

These securities can only be realized when the association is liquidated, in order that they may be distributed among the beneficiaries. Re-investments may only take place after the previous consent of the Minister of Commerce or his delegate.

This consent shall only be given after production of a resolution of the Board of the Society, which shall indicate the number and the nature of the securities to be withdrawn, as also the nature of the proposed re-investments. The value of the re-invested funds must be at least equal to the value of the funds withdrawn.



The re-invested securities must be deposited, as soon as they are obtained, as indicated above.

*Article 7.*—At the maturing of each association the Board shall decide by resolution on the distribution amongst the beneficiaries. A copy of such resolution, certified by the Manager of the Society, and by two members of the Board, specially appointed for this purpose by the Board, shall be sent in duplicate to the Minister of Commerce, with details of the distribution.

*Article 8.*—All the funds of the association must be distributed. The beneficiaries shall share in proportion to the payments made by them. However, the beneficiaries whose rights shall have been reduced, through the non-payment of the premiums due by the subscribers, shall only participate in the distribution as provided for by the bye-laws of the Society.

The rights of the beneficiaries are brought to a proportional equality by means of distribution rates (*barèmes*) based on a mortality table, and, if necessary, on a rate of interest specified by the bye-laws, taking into consideration the age of the members of the association, as also the method and times of their payments.

*Article 9.*—The distribution provided for in Article 7 can only be decreed, on presentation of the certificates of survival of the surviving members, or of the death certificates of the said members, if they died after the date fixed in the contracts for the maturing of the association. These certificates must be provided within the times fixed by the bye-laws for their production.

*Class II.—Assurance Associations.*

*Article 10.*—Assurance associations must be liquidated at the end of each year.

*Article 11.*—The assurance association for each society must be the only association; a second association may, however, be formed, called “return of premium” (*contre-assurance*) association, with the exclusive object of compensating any loss, arising from the death of the members, to the subscribers to survivorship associations formed by the society.

*Article 12.*—The payments shall be calculated by taking account of the ages of the members at the time the premiums are due, and according to a table based on a mortality table specified by the bye-laws. They shall be proportional to the probable sum to be obtained, as reached by means of the said table, when the distribution takes place.

*Article 13.*—Articles 4, 5 and 6 apply also to assurance associations.

*Article 14.*—At the end of each year, the whole of the funds of each association shall be distributed to the legal heirs of the members who have died during the year, with the deduction only of those sums which may be specified by the bye-laws in conformity with the public administrative regulation of Article 22 of the law of 17 March 1905.

Article 7 applies also to the distribution of the funds of assurance associations.



*Article 15.*—The distribution shall be proportional to the amounts of each payment, as provided for by Article 12 above.

In the case of associations called "return of premium" associations, the distribution shall be proportional to the sums paid on the premiums of survivorship associations.

*Article 16.*—The distribution can only be decreed, after production of documents proving decease of the members, within the period fixed by the bye-laws for such production.

*Article 17.*—The Minister of Commerce, of Industry, and of Public Works is responsible for the carrying out of this decree, which shall be published in the *Official Journal* of the French Republic, and inserted in the Bulletin of Laws.

PARIS, 22 June 1906.

#### ADMINISTRATIVE DECREE

under § (8) of Article 9 of the Assurance Law of 17 March 1905.

(H.)—AS TO THE RECORD OF POLICIES IN SPECIAL REGISTERS.

*Article 1.*—The societies shall, immediately after the acceptance of contracts, whether directly or by reinsurance, enter the same in special registers, according to the following conditions:

*Article 2.*—A separate register shall be kept for each distinct class of assurance, and especially for each of the undermentioned classes:

- i. Whole-life assurances, subject to whole-life premiums;
- ii. Whole-life assurances, subject to temporary premiums;
- iii. Temporary life assurances;
- iv. Whole-life assurances on two or more lives;
- v. Endowment assurances;
- vi. Fixed term assurances;
- vii. Contingent assurances;
- viii. Reversionary annuities;
- ix. Endowments (*g*);
- x. "Combined" assurances (*h*);
- xi. Return of premiums (*j*);
- xii. Pure endowments, without return of premiums;
- xiii. Pure endowments, with return of premiums;
- xiv. Deferred annuities;
- xv. Immediate annuities on single lives;
- xvi. Immediate annuities on two or more lives.

Participating Policies must be separated from Non-Participating Policies, and must be further classified according as the profits are, or are not, payable immediately on the expiration of the period in which they arise. In this latter case, a special classification shall be made in respect of each system of participation.

(*g*) "*Assurances dotales*", or dowry assurances, similar to our educational endowments.

(*h*) A special form of assurance granted by French companies, securing *guaranteed* sums, additional to the amount assured, and payable, with alternative benefits, at the end of a stipulated period.

(*j*) "*Contre-assurances*": the counter-assurance, or return of premium benefit, being usually valued by French companies as a distinct benefit.—[Ed. J.I.A.]

*Article 3.*—The entries in each class shall be consecutively numbered, irrespective of the Policy number. The particulars entered shall include:

- i. The date of the Policy, and the date on which it comes into force;
- ii. The surname, Christian names, and date of birth of each person assured;
- iii. In the case of a contingent assurance, the surname, Christian names, and date of birth of the life on whose survivorship the assurance depends;
- iv. In the case of a pure endowment, the surname, Christian names, and date of birth of the life assured;
- v. The amount of the capital, or annuity, assured by the Policy;
- vi. The amount of single, or annual premium, payable under the Policy;
- vii. The date on which the first premium, or the first instalment of the annual premium is payable;
- viii. The number of annual premiums payable under the Policy;
- ix. In the case of fixed-term assurances, the date when the Policy matures;
- x. In the case of annuities, the date on which the first instalment of the annuity is payable.

When the payment of premiums or annuities is to be other than annual, the mode of payment prescribed in the Policy shall be indicated.

*Article 4.*—The proportion of the sums assured, or annuities, re-assured, shall be indicated in respect of each Policy, also the name of the re-assuring company.

*Article 5.*—All cancelments arising from death-claim, maturity, surrender, or other cause, shall be at once entered up in the register. Any reduction made in the Policy shall be clearly noted against the entry, together with a statement of the amount of capital, or annuity, remaining assured.

*Article 6.*—Modifications in the Policies shall be clearly noted both in the original register, and in that in which they now appear; and deferred and reversionary annuities shall be similarly dealt with when they mature.

*Article 7.*—In cases where the bonus, instead of being paid immediately in cash, is applied to increase the sum or annuity assured, the successive additions to the sum assured or annuity shall be entered up, under their respective dates, opposite the amount of capital or annuity originally assured.

Reductions in the original premium, arising either from bonus or from payments made during the currency of the Policy must be similarly noted under their respective dates.

*Article 8.*—Tontine undertakings shall keep separate registers for each group involving survival, as also for those involving death, notifying the duration of the transaction and the date of its expiration.

The entry under each Policy in such registers shall be consecutively numbered, irrespective of the Policy number. The particulars entered shall include :

- i. The name and Christian names of the subscriber (*i.e.*, contracting party) ;
- ii. The name, Christian names, and date of birth of the assured (*i.e.*, the life on whom the tontine transaction depends) ;
- iii. The name and Christian names of the beneficiary (*i.e.*, the life in whose favour the subscription is made) ;
- iv. The gross amount of total subscription specified in the Policy, without deduction for management expenses ;
- v. As regards groups involving death, the probable amount of the sum payable to the beneficiaries ;
- vi. The mode of payment, and the date, on which the annual premiums or rates are payable. Cancellments, and reductions in policies, arising by non-payment of annual premiums or rates, shall be immediately entered under the particular Policy.

*Article 9.*—The above provisions, in so far as they concern transactions entered on prior to registration, are not binding on societies which give satisfactory proof that the entries made in their books respecting such transactions sufficiently comply with the spirit of the said provisions.

*Article 10.*—The Minister of Commerce, Industry, Posts and Telegraphs is responsible for the carrying out of the present decree, which shall be published in the *Official Journal* of the French Republic, and included in the Bulletin of Laws.

PARIS, 20 January 1906.

#### ADMINISTRATIVE DECREE

*under § (9) of Article 9 of the Assurance Law of 17 March 1905.*

(J.)—AS TO SOCIETIES ESTABLISHED FOR THE MANAGEMENT OF LIFE ASSURANCE.

*Article 1.*—No Society undertaking the management of a Life Insurance Company can carry on business as such unless the responsibility is taken by the society whose business it manages. The following documents must also be produced before the Minister of Commerce :

- i. The receipt for the deposit at the “Caisse des Dépôts et Consignations” of the guarantee capital provided for in Article 9, paragraph 9, of the Law of 17 March 1905 ;
- ii. The Articles of Association of the society managing the business ;
- iii. The complete text of its bye-laws ;
- iv. The complete text of the management agreement between the two societies.

*Article 2.*—The management agreement mentioned in paragraph iv of the preceding article must specify :

- i. The object, the name, and the head office of the managing society ;
- ii. The object, the name, and the head office of the society under management ;
- iii. The date at which the management begins, and the length of the first period of management ;
- iv. The powers of the managing society ;
- v. The conditions under which the society under management may control the method under which it is to be managed ;
- vi. The conditions under which the original society may resume its own management, either at the end of the management agreement, or, in the intervening period, should cause for this arise from any reason or other ;
- vii. The measures to be adopted in the case of the withdrawal of the registration of the society under management.

*Article 3.*—In case of renewal, the new management agreement must be produced before the Minister of Commerce, as provided for in Article 2, at least three months before the maturing of the agreement in force.

*Article 4.*—Management societies can only take the remuneration for the work they do as fixed by the agreement ; in no case shall this exceed more than :

- i. As regards assurances with fixed premiums, the amount of the loadings as decided by the bye-laws and the tables of the Society under management, less, however, that part of the said loadings which may be necessary for the setting up of the guarantee reserve ;
- ii. As regards tontine transactions, the amount of the payments for management expenses fixed by the bye-laws of the society under management.

*Article 5.*—Management societies shall in no case have conceded to them by the society under management the powers which have reference to assurance transactions, and especially to the undertaking of contracts, to the carrying out and closing of liabilities arising therefrom, to the investing of funds destined to secure these liabilities, to the opening, to the setting up, to the closing and to the liquidation of tontine associations.

*Article 6.*—The deposit prescribed in Article 9, paragraph 9, of the law of 17 March 1905, shall be returned to the societies when the management lapses, on the approval of the Minister of Commerce or his delegate, after proof that all the undertakings in the management agreement have been completely carried out, and after the production of an affidavit by the representatives of the Society under management that these undertakings have been carried out.

*Article 7.*—The Minister of Commerce, of Industry, and of Public Works is responsible for the carrying out of the present decree, which shall be published in the *Official Journal* of the French Republic, and inserted in the Bulletin of Laws.

PARIS, 22 June 1906.

#### GENERAL REGULATION

*under § (8) of the Assurance Law of 17 March 1905.*

#### (K.)—AS TO INVESTMENTS.

*Article 1.*—The assets of French societies, except for the deductions referred to in Article 2 hereafter, and that part of the assets of foreign societies referred to in Article 7, paragraph 3, of the law of 17 March 1905, must be invested as follows:

- i. *Without limit:* In securities issued by the French Government, or in securities whose capital or interest is guaranteed by it, in paid-up or negotiable bonds of the Departments, Communes, or Chambers of Commerce of France and Algeria, and in bonds of real estate issued by the Cr dit Foncier. In loans on all the above securities up to 75 per-cent of their value; in loans on the policies of the Society; or in mortgage loans on city property in France, so long as these loans, including loans already made, do not exceed 50 per-cent of the value of the property;
- ii. *To the extent of two-fifths at most:* In loans to the Departments, the Communes, or the Chambers of Commerce in France and Algeria, in French Colonies or in Protectorate Countries: in real property situated in France and in Algeria, in mortgage loans on this property up to 50 per-cent of its value, as conditioned in the previous paragraph;
- iii. *To the extent of one quarter at most:* In any security whatever, French or foreign, appearing in the official quotations of the Paris Bourse, and inscribed on a list previously approved by the general meeting of the shareholders; in loans on these securities up to 75 per-cent of their value; in real estate in French Colonies or Protectorate Countries; in mortgage loans on this real estate up to 50 per-cent of its value, as conditioned above.

In each of the classes enumerated above must be respectively included, not only investments in the whole of such properties, but also reversions and life interests in corresponding securities.

*Article 2.*—Notwithstanding the limitations fixed in the preceding article, French societies may invest the part of their assets corresponding to the mathematical reserves respectively belonging to their transactions in each foreign country in which they do business, also that part required as guarantee by the said countries, in securities approved by foreign life assurance legislation.



They may also, as representing the said proportion of their assets, acquire buildings for the carrying on of their work in each foreign country where they do business.

*Article 3.*—In the balance sheets the securities making up the assets shall be valued as follows :

- i. Securities at the purchase price, except when for the total of these securities this value exceeds by more than 5 per-cent that arrived at by taking the Paris Bourse quotation, or, failing this, the quotation of one of the principal towns in the country of issue, at the date of the closing of the balance sheet. In this latter case, a ministerial decree, issued on the advice of the consulting committee on life insurance, shall determine the conditions and the period within which the estimated value must be reduced by the difference between the purchase price and its present value ;
- ii. Mortgage loans, loans on securities, loans to the Departments, to the Communes, to the Chambers of Commerce, to the Colonies and to Protectorate Countries, as also loans on policies ; according to the agreements concerning them, taking account in each balance sheet of the amounts paid off ;
- iii. House property, either at the purchase price, or at cost price, with the outlays for building or improvements added, but not including the expenses of maintenance.  
The verification of the value of house property may be made at any time by the Minister of Commerce, on the advice of the consulting committee on Life Insurance.
- iv. Reversions and life interests, according to the general rules fixed by a Ministerial Decree on the advice of the Consulting Committee on Life Insurance.

*Article 4.*—The assets of tontine societies must, subject to the provisions of foreign legislation as to subscriptions received abroad, be invested in securities issued by the French Government or whose capital or interest is guaranteed by it in paid-up and negotiable bonds of the Departments, Communes and the Chambers of Commerce of France and Algeria, and in bonds of the *Crédit Foncier of France*.

*Article 5.*—The securities of French societies must be represented by certificates or nominative deeds.

Securities which have no certificates or nominative deeds must be represented by receipts from the Bank of France.

*Article 6.*—Foreign societies are allowed a period of five years for replacing, to the extent of at least one-fifth per annum, the securities which they must deposit at the “*Caisse des Dépôts et Consignations*” as guarantee for the transactions carried out in France or Algeria prior to their registration. This shall be effected by an account set up in accordance with Article 1 of the present decree. Changes in the securities shall only take place in order to replace securities, temporarily allowed, by those specified in Article 1.

However, up to the proportion of a quarter, allowed in the third paragraph of the said article, foreign securities not quoted at the Paris Bourse may be kept until the maturing of the contracts in force, so long as these securities are allowed to Life Insurance Companies by the laws of the Company's own land.

*Article 7.*—French societies that did not need licenses, and those which carried on transactions defined in Article 1 of the Law and have now to be registered, shall also be allowed a period of five years within which to make their investments, in accordance with Article 1 above. At least one-fifth of their securities account must annually be brought under these conditions.

*Article 8.*—The Minister of Commerce, of Industry and Public Works and the Minister of Finance are held responsible, each in his department, for the carrying out of this decree, which shall be published in the *Official Journal* of the French Republic, and inserted in the Bulletin of Laws.

PARIS, 9 June 1906.

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## ORIGINAL TABLES.

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### *Continuous Temporary Annuities. O<sup>M</sup> 2 $\frac{3}{4}$ per-cent.*

[We are indebted to Mr. A. LEVINE for the appended table of Continuous Temporary Annuities, according to the O<sup>M</sup> Table at 2 $\frac{3}{4}$  per-cent interest, which will be useful, where these bases as to mortality and interest are adopted, in the valuation of Endowment Assurance policies by Mr. G. J. LIDSTONE's approximate method. The table follows the general plan of that given by Mr. Lidstone in his paper read in December 1897 (*J.I.A.*, vol. xxxiv, pp. 82–84), but the limits are somewhat extended. The values have been calculated by the approximate formula  $\bar{a}_{x:\overline{n}|} = \frac{1}{2} [a_{x:\overline{n}|} + a_{x:\overline{n}|}]$ . The values of  $n$  are given at the side, and the values of Mr. Lidstone's  $M$  (which is equal to  $x+n+1$ ) at the head of the columns. The column headed "Diff." gives the difference between two successive values *in the same row*, corresponding to an increase in  $M$ ,  $n$  remaining constant. The values of  $a_{x:\overline{n}|}$ , from which the interpolation was made, were kindly furnished to Mr. Levine by Mr. A. G. Hemming.—ED. *J.I.A.*]

## Continuous Temporary Annuities.

 $0^M 2\frac{3}{4}$  PER-CENT. $\bar{a}_{M-n+1:n}$  $0^M 2\frac{3}{4}$  PER-CENT.

| MATURITY-AGE, i.e., M |        |       |        |       |        |       |        |       |        |       |        |       | n  |
|-----------------------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|----|
| n                     | 35     | Diff. | 36     | Diff. | 37     | Diff. | 38     | Diff. | 39     | Diff. | 40     | Diff. |    |
| 1                     | '984   | 1     | '983   | ...   | '983   | ...   | '983   | 1     | '982   | ...   | '982   | ...   | 1  |
| 2                     | 1'934  | ...   | 1'934  | 1     | 1'933  | 1     | 1'932  | ...   | 1'932  | 1     | 1'931  | 1     | 2  |
| 3                     | 2'854  | ...   | 2'854  | 2     | 2'852  | 2     | 2'850  | ...   | 2'850  | 2     | 2'848  | 2     | 3  |
| 4                     | 3'745  | 2     | 3'743  | 2     | 3'741  | 2     | 3'739  | 3     | 3'736  | 2     | 3'734  | 2     | 4  |
| 5                     | 4'608  | 3     | 4'605  | 3     | 4'602  | 4     | 4'598  | 2     | 4'596  | 4     | 4'592  | 4     | 5  |
| 6                     | 5'444  | 4     | 5'440  | 4     | 5'436  | 4     | 5'432  | 5     | 5'427  | 5     | 5'422  | 5     | 6  |
| 7                     | 6'255  | 5     | 6'250  | 6     | 6'244  | 6     | 6'238  | 6     | 6'232  | 6     | 6'226  | 6     | 7  |
| 8                     | 7'041  | 6     | 7'035  | 7     | 7'028  | 8     | 7'020  | 6     | 7'014  | 8     | 7'006  | 8     | 8  |
| 9                     | 7'804  | 8     | 7'796  | 8     | 7'788  | 8     | 7'780  | 10    | 7'770  | 9     | 7'761  | 9     | 9  |
| 10                    | 8'544  | 8     | 8'536  | 10    | 8'526  | 10    | 8'516  | 10    | 8'506  | 12    | 8'494  | 12    | 10 |
| 11                    | 9'263  | 10    | 9'253  | 11    | 9'242  | 12    | 9'230  | 12    | 9'218  | 12    | 9'206  | 13    | 11 |
| 12                    | 9'961  | 11    | 9'950  | 12    | 9'938  | 14    | 9'924  | 13    | 9'911  | 14    | 9'897  | 15    | 12 |
| 13                    | 10'638 | 12    | 10'626 | 13    | 10'613 | 14    | 10'599 | 15    | 10'584 | 16    | 10'568 | 16    | 13 |
| 14                    | 11'296 | 13    | 11'283 | 14    | 11'269 | 15    | 11'254 | 16    | 11'238 | 18    | 11'220 | 18    | 14 |
| 15                    | ...    | ...   | 11'922 | 16    | 11'906 | 16    | 11'890 | 18    | 11'872 | 18    | 11'854 | 20    | 15 |
| 16                    | ...    | ...   | ...    | ...   | 12'526 | 18    | 12'508 | 18    | 12'490 | 21    | 12'469 | 21    | 16 |
| 17                    | ...    | ...   | ...    | ...   | ...    | ...   | 13'108 | 20    | 13'088 | 20    | 13'068 | 23    | 17 |
| 18                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 13'672 | 22    | 13'650 | 24    | 18 |
| 19                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 14'215 | 25    | 19 |
| 20                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 20 |
| 21                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 21 |
| 22                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 22 |
| 23                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 23 |
| 24                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 24 |
| 25                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 25 |
| 26                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 26 |
| 27                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 27 |
| 28                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 28 |
| 29                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 29 |
| 30                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 30 |
| 31                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 31 |
| 32                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 32 |
| 33                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 33 |
| 34                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 34 |
| 35                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 35 |
|                       | 35     | Diff. | 36     | Diff. | 37     | Diff. | 38     | Diff. | 39     | Diff. | 40     | Diff. |    |

## Continuous Temporary Annuities—(continued).

 $0^M 2\frac{3}{4}$  PER-CENT. $\bar{a}_{M-n+1:n}$  $0^M 2\frac{3}{4}$  PER-CENT.

| n  | MATURITY-AGE, i.e., M |       |        |       |        |       |        |       |        |       |        |       | n  |
|----|-----------------------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|----|
|    | 41                    | Diff. | 42     | Diff. | 43     | Diff. | 44     | Diff. | 45     | Diff. | 46     | Diff. |    |
| 1  | '982                  | ...   | '982   | ...   | '982   | ...   | '982   | ...   | '982   | ...   | '982   | 1     | 1  |
| 2  | 1'930                 | ...   | 1'930  | 1     | 1'929  | 1     | 1'928  | 1     | 1'927  | 1     | 1'926  | ...   | 2  |
| 3  | 2'846                 | 1     | 2'845  | 1     | 2'844  | 2     | 2'842  | 2     | 2'840  | 2     | 2'838  | 2     | 3  |
| 4  | 3'732                 | 2     | 3'730  | 4     | 3'726  | 2     | 3'724  | 4     | 3'720  | 2     | 3'718  | 4     | 4  |
| 5  | 4'588                 | 4     | 4'584  | 4     | 4'580  | 4     | 4'576  | 4     | 4'572  | 5     | 4'567  | 5     | 5  |
| 6  | 5'417                 | 5     | 5'412  | 6     | 5'406  | 6     | 5'400  | 6     | 5'394  | 6     | 5'388  | 6     | 6  |
| 7  | 6'220                 | 7     | 6'213  | 7     | 6'206  | 8     | 6'198  | 8     | 6'190  | 8     | 6'182  | 8     | 7  |
| 8  | 6'998                 | 9     | 6'989  | 9     | 6'980  | 8     | 6'972  | 10    | 6'962  | 10    | 6'952  | 11    | 8  |
| 9  | 7'752                 | 11    | 7'741  | 10    | 7'731  | 11    | 7'720  | 12    | 7'708  | 12    | 7'696  | 12    | 9  |
| 10 | 8'482                 | 11    | 8'471  | 13    | 8'458  | 12    | 8'446  | 14    | 8'432  | 14    | 8'418  | 15    | 10 |
| 11 | 9'193                 | 14    | 9'179  | 15    | 9'164  | 14    | 9'150  | 16    | 9'134  | 16    | 9'118  | 17    | 11 |
| 12 | 9'882                 | 16    | 9'866  | 16    | 9'850  | 17    | 9'833  | 17    | 9'816  | 19    | 9'797  | 19    | 12 |
| 13 | 10'552                | 18    | 10'534 | 18    | 10'516 | 20    | 10'496 | 19    | 10'477 | 21    | 10'456 | 21    | 13 |
| 14 | 11'202                | 20    | 11'182 | 20    | 11'162 | 20    | 11'142 | 22    | 11'120 | 23    | 11'097 | 23    | 14 |
| 15 | 11'834                | 21    | 11'813 | 22    | 11'791 | 23    | 11'768 | 24    | 11'744 | 24    | 11'720 | 26    | 15 |
| 16 | 12'448                | 22    | 12'426 | 24    | 12'402 | 24    | 12'378 | 26    | 12'352 | 28    | 12'324 | 27    | 16 |
| 17 | 13'045                | 23    | 13'022 | 26    | 12'996 | 26    | 12'970 | 28    | 12'942 | 28    | 12'914 | 31    | 17 |
| 18 | 13'626                | 25    | 13'601 | 27    | 13'574 | 28    | 13'546 | 28    | 13'518 | 32    | 13'486 | 32    | 18 |
| 19 | 14'190                | 26    | 14'164 | 27    | 14'137 | 29    | 14'108 | 32    | 14'076 | 32    | 14'044 | 34    | 19 |
| 20 | 14'740                | 28    | 14'712 | 28    | 14'684 | 31    | 14'653 | 32    | 14'621 | 35    | 14'586 | 36    | 20 |
| 21 | ...                   | ...   | 15'246 | 30    | 15'216 | 32    | 15'184 | 34    | 15'150 | 34    | 15'116 | 38    | 21 |
| 22 | ...                   | ...   | ...    | ...   | 15'734 | 34    | 15'700 | 34    | 15'666 | 37    | 15'629 | 38    | 22 |
| 23 | ...                   | ...   | ...    | ...   | ...    | ...   | 16'204 | 36    | 16'168 | 38    | 16'130 | 40    | 23 |
| 24 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 16'656 | 38    | 16'618 | 41    | 24 |
| 25 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 17'092 | 42    | 25 |
| 26 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 26 |
| 27 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 27 |
| 28 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 28 |
| 29 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 29 |
| 30 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 30 |
| 31 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 31 |
| 32 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 32 |
| 33 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 33 |
| 34 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 34 |
| 35 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 35 |
|    | 41                    | Diff. | 42     | Diff. | 43     | Diff. | 44     | Diff. | 45     | Diff. | 46     | Diff. |    |

## Continuous Temporary Annuities—(continued).

 $OM\ 2\frac{3}{4}\%$  PER-CENT. $\bar{a}_{M-n+1:n}$  $OM\ 2\frac{3}{4}\%$  PER-CENT.

| n  | MATURITY-AGE, i.e., M |       |        |       |        |       |        |       |        |       |        |       | n  |
|----|-----------------------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|----|
|    | 47                    | Diff. | 48     | Diff. | 49     | Diff. | 50     | Diff. | 51     | Diff. | 52     | Diff. |    |
| 1  | '981                  | 1     | '980   | ...   | '980   | ...   | '980   | ...   | '980   | ...   | '980   | 1     | 1  |
| 2  | 1'926                 | 2     | 1'924  | 1     | 1'923  | 1     | 1'922  | 2     | 1'920  | 1     | 1'919  | 1     | 2  |
| 3  | 2'836                 | 2     | 2'834  | 2     | 2'832  | 4     | 2'828  | 2     | 2'826  | 4     | 2'822  | 2     | 3  |
| 4  | 3'714                 | 4     | 3'710  | 4     | 3'706  | 4     | 3'702  | 4     | 3'698  | 6     | 3'692  | 5     | 4  |
| 5  | 4'562                 | 5     | 4'557  | 6     | 4'551  | 7     | 4'544  | 6     | 4'538  | 7     | 4'531  | 8     | 5  |
| 6  | 5'382                 | 8     | 5'374  | 8     | 5'366  | 8     | 5'358  | 9     | 5'349  | 9     | 5'340  | 10    | 6  |
| 7  | 6'174                 | 9     | 6'165  | 9     | 6'156  | 12    | 6'144  | 10    | 6'134  | 13    | 6'121  | 13    | 7  |
| 8  | 6'941                 | 11    | 6'930  | 12    | 6'918  | 13    | 6'905  | 14    | 6'891  | 15    | 6'876  | 16    | 8  |
| 9  | 7'684                 | 14    | 7'670  | 14    | 7'656  | 16    | 7'640  | 16    | 7'624  | 17    | 7'607  | 19    | 9  |
| 10 | 8'403                 | 15    | 8'388  | 17    | 8'371  | 17    | 8'354  | 20    | 8'334  | 20    | 8'314  | 21    | 10 |
| 11 | 9'101                 | 18    | 9'083  | 19    | 9'064  | 20    | 9'044  | 21    | 9'023  | 23    | 9'000  | 24    | 11 |
| 12 | 9'778                 | 20    | 9'758  | 22    | 9'736  | 22    | 9'714  | 24    | 9'690  | 25    | 9'665  | 27    | 12 |
| 13 | 10'435                | 23    | 10'412 | 23    | 10'389 | 25    | 10'364 | 26    | 10'338 | 28    | 10'310 | 30    | 13 |
| 14 | 11'074                | 26    | 11'048 | 26    | 11'022 | 27    | 10'995 | 29    | 10'966 | 30    | 10'936 | 32    | 14 |
| 15 | 11'694                | 28    | 11'666 | 28    | 11'638 | 30    | 11'608 | 31    | 11'577 | 33    | 11'544 | 34    | 15 |
| 16 | 12'297                | 30    | 12'267 | 31    | 12'236 | 32    | 12'204 | 34    | 12'170 | 35    | 12'135 | 37    | 16 |
| 17 | 12'883                | 31    | 12'852 | 34    | 12'818 | 34    | 12'784 | 36    | 12'748 | 38    | 12'710 | 40    | 17 |
| 18 | 13'454                | 34    | 13'420 | 34    | 13'386 | 38    | 13'348 | 38    | 13'310 | 40    | 13'270 | 43    | 18 |
| 19 | 14'010                | 36    | 13'974 | 37    | 13'937 | 39    | 13'898 | 41    | 13'857 | 43    | 13'814 | 44    | 19 |
| 20 | 14'550                | 36    | 14'514 | 40    | 14'474 | 41    | 14'433 | 43    | 14'390 | 46    | 14'344 | 47    | 20 |
| 21 | 15'078                | 40    | 15'038 | 41    | 14'997 | 43    | 14'954 | 45    | 14'909 | 48    | 14'861 | 49    | 21 |
| 22 | 15'591                | 41    | 15'550 | 44    | 15'506 | 44    | 15'462 | 48    | 15'414 | 49    | 15'365 | 52    | 22 |
| 23 | 16'090                | 42    | 16'048 | 44    | 16'004 | 48    | 15'956 | 48    | 15'908 | 52    | 15'856 | 54    | 23 |
| 24 | 16'577                | 44    | 16'533 | 45    | 16'488 | 48    | 16'440 | 52    | 16'388 | 52    | 16'336 | 56    | 24 |
| 25 | 17'050                | 44    | 17'006 | 47    | 16'959 | 49    | 16'910 | 52    | 16'858 | 55    | 16'803 | 57    | 25 |
| 26 | 17'512                | 46    | 17'466 | 47    | 17'419 | 51    | 17'368 | 52    | 17'316 | 58    | 17'258 | 58    | 26 |
| 27 | ...                   | ...   | 17'915 | 49    | 17'866 | 51    | 17'815 | 54    | 17'761 | 57    | 17'704 | 61    | 27 |
| 28 | ...                   | ...   | ...    | ...   | 18'303 | 53    | 18'250 | 54    | 18'196 | 58    | 18'138 | 62    | 28 |
| 29 | ...                   | ...   | ...    | ...   | ...    | ...   | 18'675 | 55    | 18'620 | 60    | 18'560 | 62    | 29 |
| 30 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 19'032 | 59    | 18'973 | 63    | 30 |
| 31 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 19'376 | 64    | 31 |
| 32 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 32 |
| 33 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 33 |
| 34 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 34 |
| 35 | ...                   | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 35 |
|    | 47                    | Diff. | 48     | Diff. | 49     | Diff. | 50     | Diff. | 51     | Diff. | 52     | Diff. |    |



## Continuous Temporary Annuities—(continued).

O<sup>M</sup> 2<sup>3</sup>/<sub>4</sub> PER-CENT. $\bar{a}_{M-n+1:n}$ O<sup>M</sup> 2<sup>3</sup>/<sub>4</sub> PER-CENT.

| MATURITY-AGE, i.e., M |        |       |        |       |        |       |        |       |        |       |        |       | n  |
|-----------------------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|----|
| n                     | 53     | Diff. | 54     | Diff. | 55     | Diff. | 56     | Diff. | 57     | Diff. | 58     | Diff. | n  |
| 1                     | '979   | 1     | '978   | ...   | '978   | ...   | '978   | 2     | '976   | ...   | '976   | 1     | 1  |
| 2                     | 1'918  | 2     | 1'916  | 2     | 1'914  | 2     | 1'912  | 2     | 1'910  | 3     | 1'907  | 3     | 2  |
| 3                     | 2'820  | 4     | 2'816  | 4     | 2'812  | 4     | 2'808  | 5     | 2'803  | 5     | 2'798  | 6     | 3  |
| 4                     | 3'687  | 5     | 3'682  | 8     | 3'674  | 6     | 3'668  | 8     | 3'660  | 8     | 3'652  | 10    | 4  |
| 5                     | 4'523  | 9     | 4'514  | 8     | 4'506  | 11    | 4'495  | 11    | 4'484  | 12    | 4'472  | 12    | 5  |
| 6                     | 5'330  | 12    | 5'318  | 12    | 5'306  | 14    | 5'292  | 14    | 5'278  | 16    | 5'262  | 18    | 6  |
| 7                     | 6'108  | 14    | 6'094  | 16    | 6'078  | 16    | 6'062  | 18    | 6'044  | 20    | 6'024  | 22    | 7  |
| 8                     | 6'860  | 16    | 6'844  | 20    | 6'824  | 20    | 6'804  | 22    | 6'782  | 23    | 6'759  | 27    | 8  |
| 9                     | 7'588  | 20    | 7'568  | 22    | 7'546  | 24    | 7'522  | 25    | 7'497  | 28    | 7'469  | 29    | 9  |
| 10                    | 8'293  | 23    | 8'270  | 25    | 8'245  | 27    | 8'218  | 30    | 8'188  | 31    | 8'157  | 35    | 10 |
| 11                    | 8'976  | 26    | 8'950  | 28    | 8'922  | 30    | 8'892  | 34    | 8'858  | 36    | 8'822  | 37    | 11 |
| 12                    | 9'638  | 30    | 9'608  | 30    | 9'578  | 34    | 9'544  | 36    | 9'508  | 39    | 9'469  | 43    | 12 |
| 13                    | 10'280 | 32    | 10'248 | 34    | 10'214 | 36    | 10'178 | 40    | 10'138 | 42    | 10'096 | 46    | 13 |
| 14                    | 10'904 | 35    | 10'869 | 37    | 10'832 | 40    | 10'792 | 42    | 10'750 | 46    | 10'704 | 50    | 14 |
| 15                    | 11'510 | 38    | 11'472 | 40    | 11'432 | 42    | 11'390 | 46    | 11'344 | 49    | 11'295 | 53    | 15 |
| 16                    | 12'098 | 40    | 12'058 | 42    | 12'016 | 46    | 11'970 | 48    | 11'922 | 52    | 11'870 | 56    | 16 |
| 17                    | 12'670 | 42    | 12'628 | 46    | 12'582 | 48    | 12'534 | 50    | 12'484 | 56    | 12'428 | 58    | 17 |
| 18                    | 13'227 | 45    | 13'182 | 48    | 13'134 | 50    | 13'084 | 54    | 13'030 | 58    | 12'972 | 62    | 18 |
| 19                    | 13'770 | 48    | 13'722 | 51    | 13'671 | 53    | 13'618 | 56    | 13'562 | 61    | 13'501 | 64    | 19 |
| 20                    | 14'297 | 49    | 14'248 | 54    | 14'194 | 56    | 14'138 | 58    | 14'080 | 63    | 14'017 | 67    | 20 |
| 21                    | 14'812 | 52    | 14'760 | 56    | 14'704 | 58    | 14'646 | 62    | 14'584 | 64    | 14'520 | 70    | 21 |
| 22                    | 15'313 | 55    | 15'258 | 56    | 15'202 | 60    | 15'142 | 64    | 15'078 | 68    | 15'010 | 72    | 22 |
| 23                    | 15'802 | 56    | 15'746 | 60    | 15'686 | 62    | 15'624 | 66    | 15'558 | 70    | 15'488 | 74    | 23 |
| 24                    | 16'280 | 59    | 16'221 | 61    | 16'160 | 66    | 16'094 | 68    | 16'026 | 72    | 15'954 | 76    | 24 |
| 25                    | 16'746 | 61    | 16'685 | 63    | 16'622 | 67    | 16'555 | 71    | 16'484 | 74    | 16'410 | 78    | 25 |
| 26                    | 17'200 | 62    | 17'138 | 66    | 17'072 | 68    | 17'004 | 72    | 16'932 | 77    | 16'855 | 79    | 26 |
| 27                    | 17'643 | 63    | 17'580 | 67    | 17'513 | 70    | 17'443 | 75    | 17'368 | 77    | 17'291 | 83    | 27 |
| 28                    | 18'076 | 65    | 18'011 | 68    | 17'943 | 72    | 17'871 | 75    | 17'796 | 80    | 17'716 | 84    | 28 |
| 29                    | 18'498 | 66    | 18'432 | 69    | 18'363 | 73    | 18'290 | 77    | 18'213 | 81    | 18'132 | 86    | 29 |
| 30                    | 18'910 | 66    | 18'844 | 70    | 18'774 | 75    | 18'699 | 78    | 18'621 | 82    | 18'539 | 87    | 30 |
| 31                    | 19'312 | 68    | 19'244 | 70    | 19'174 | 75    | 19'099 | 79    | 19'020 | 84    | 18'936 | 87    | 31 |
| 32                    | 19'704 | 68    | 19'636 | 72    | 19'564 | 75    | 19'489 | 79    | 19'410 | 85    | 19'325 | 89    | 32 |
| 33                    | ...    | ...   | 20'018 | 72    | 19'946 | 76    | 19'870 | 80    | 19'790 | 84    | 19'706 | 90    | 33 |
| 34                    | ...    | ...   | ...    | ...   | 20'319 | 77    | 20'242 | 80    | 20'162 | 85    | 20'077 | 90    | 34 |
| 35                    | ...    | ...   | ...    | ...   | ...    | ...   | 20'606 | 81    | 20'525 | 85    | 20'440 | 90    | 35 |
| 36                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 20'880 | 86    | 20'794 | 90    | 36 |
| 37                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 21'141 | 91    | 37 |
| 38                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 38 |
| 39                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 39 |
| 40                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 40 |
| 41                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 41 |
| 42                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 42 |
| 43                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 43 |
| 44                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 44 |
| 45                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 45 |
| 46                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 46 |
| 47                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 47 |
| 48                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 48 |
| 49                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 49 |
|                       | 53     | Diff. | 54     | Diff. | 55     | Diff. | 56     | Diff. | 57     | Diff. | 58     | Diff. |    |

## Continuous Temporary Annuities—(continued).

 $O^M 2\frac{3}{4}$  PER-CENT. $\bar{a}_{M-n+1:n}$  $O^M 2\frac{3}{4}$  PER-CENT.

| MATURITY-AGE, i.e., M |        |       |        |       |        |       |        |       |        |       |        |       |    |
|-----------------------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|----|
| n                     | 59     | Diff. | 60     | Diff. | 61     | Diff. | 62     | Diff. | 63     | Diff. | 64     | Diff. | n  |
| 1                     | '975   | 1     | '974   | ...   | '974   | 2     | '972   | ...   | '972   | 2     | '970   | 1     | 1  |
| 2                     | 1'904  | 3     | 1'901  | 3     | 1'898  | 4     | 1'894  | 4     | 1'890  | 4     | 1'886  | 4     | 2  |
| 3                     | 2'792  | 6     | 2'786  | 7     | 2'779  | 7     | 2'772  | 8     | 2'764  | 8     | 2'756  | 10    | 3  |
| 4                     | 3'642  | 9     | 3'633  | 11    | 3'622  | 12    | 3'610  | 12    | 3'598  | 15    | 3'583  | 15    | 4  |
| 5                     | 4'460  | 15    | 4'445  | 15    | 4'430  | 17    | 4'413  | 19    | 4'394  | 20    | 4'374  | 22    | 5  |
| 6                     | 5'244  | 18    | 5'226  | 20    | 5'206  | 22    | 5'184  | 24    | 5'160  | 27    | 5'133  | 29    | 6  |
| 7                     | 6'002  | 24    | 5'978  | 25    | 5'953  | 27    | 5'926  | 30    | 5'896  | 33    | 5'863  | 36    | 7  |
| 8                     | 6'732  | 27    | 6'705  | 31    | 6'674  | 33    | 6'641  | 37    | 6'604  | 38    | 6'566  | 42    | 8  |
| 9                     | 7'440  | 34    | 7'406  | 35    | 7'371  | 39    | 7'332  | 42    | 7'290  | 46    | 7'244  | 48    | 9  |
| 10                    | 8'122  | 36    | 8'086  | 42    | 8'044  | 43    | 8'001  | 47    | 7'954  | 52    | 7'902  | 56    | 10 |
| 11                    | 8'785  | 42    | 8'743  | 45    | 8'698  | 50    | 8'648  | 52    | 8'596  | 58    | 8'538  | 62    | 11 |
| 12                    | 9'426  | 45    | 9'381  | 50    | 9'331  | 53    | 9'278  | 59    | 9'219  | 63    | 9'156  | 68    | 12 |
| 13                    | 10'050 | 50    | 10'000 | 54    | 9'946  | 58    | 9'888  | 64    | 9'824  | 68    | 9'756  | 73    | 13 |
| 14                    | 10'654 | 53    | 10'601 | 58    | 10'543 | 62    | 10'481 | 68    | 10'413 | 73    | 10'340 | 79    | 14 |
| 15                    | 11'242 | 56    | 11'186 | 62    | 11'124 | 67    | 11'057 | 71    | 10'986 | 78    | 10'908 | 84    | 15 |
| 16                    | 11'814 | 61    | 11'753 | 65    | 11'688 | 70    | 11'618 | 76    | 11'542 | 80    | 11'462 | 89    | 16 |
| 17                    | 12'370 | 64    | 12'306 | 68    | 12'238 | 74    | 12'164 | 78    | 12'086 | 86    | 12'000 | 92    | 17 |
| 18                    | 12'910 | 66    | 12'844 | 71    | 12'773 | 77    | 12'696 | 82    | 12'614 | 89    | 12'525 | 96    | 18 |
| 19                    | 13'437 | 69    | 13'368 | 74    | 13'294 | 80    | 13'214 | 85    | 13'129 | 92    | 13'037 | 99    | 19 |
| 20                    | 13'950 | 71    | 13'879 | 77    | 13'802 | 82    | 13'720 | 88    | 13'632 | 96    | 13'536 | 102   | 20 |
| 21                    | 14'450 | 74    | 14'376 | 78    | 14'298 | 86    | 14'212 | 90    | 14'122 | 98    | 14'024 | 104   | 21 |
| 22                    | 14'938 | 76    | 14'862 | 82    | 14'780 | 86    | 14'694 | 94    | 14'600 | 100   | 14'500 | 106   | 22 |
| 23                    | 15'414 | 78    | 15'336 | 84    | 15'252 | 90    | 15'162 | 94    | 15'068 | 103   | 14'965 | 109   | 23 |
| 24                    | 15'878 | 80    | 15'798 | 86    | 15'712 | 91    | 15'621 | 97    | 15'524 | 104   | 15'420 | 112   | 24 |
| 25                    | 16'332 | 82    | 16'250 | 88    | 16'162 | 94    | 16'068 | 99    | 15'969 | 105   | 15'864 | 114   | 25 |
| 26                    | 16'776 | 85    | 16'691 | 90    | 16'601 | 95    | 16'506 | 101   | 16'405 | 107   | 16'298 | 116   | 26 |
| 27                    | 17'208 | 86    | 17'122 | 91    | 17'031 | 97    | 16'934 | 103   | 16'831 | 109   | 16'722 | 116   | 27 |
| 28                    | 17'632 | 88    | 17'544 | 93    | 17'451 | 99    | 17'352 | 104   | 17'248 | 111   | 17'137 | 117   | 28 |
| 29                    | 18'046 | 89    | 17'957 | 95    | 17'862 | 100   | 17'762 | 106   | 17'656 | 112   | 17'544 | 120   | 29 |
| 30                    | 18'452 | 92    | 18'360 | 96    | 18'264 | 102   | 18'162 | 106   | 18'056 | 114   | 17'942 | 120   | 30 |
| 31                    | 18'849 | 93    | 18'756 | 98    | 18'658 | 102   | 18'556 | 110   | 18'446 | 114   | 18'332 | 121   | 31 |
| 32                    | 19'236 | 93    | 19'143 | 99    | 19'044 | 104   | 18'940 | 110   | 18'830 | 116   | 18'714 | 122   | 32 |
| 33                    | 19'616 | 94    | 19'522 | 100   | 19'422 | 106   | 19'316 | 110   | 19'206 | 118   | 19'088 | 122   | 33 |
| 34                    | 19'987 | 95    | 19'892 | 100   | 19'792 | 106   | 19'686 | 112   | 19'574 | 118   | 19'456 | 124   | 34 |
| 35                    | 20'350 | 96    | 20'254 | 101   | 20'153 | 106   | 20'047 | 112   | 19'935 | 119   | 19'816 | 125   | 35 |
| 36                    | 20'704 | 96    | 20'608 | 100   | 20'508 | 108   | 20'400 | 112   | 20'288 | 118   | 20'170 | 126   | 36 |
| 37                    | 21'050 | 95    | 20'955 | 101   | 20'854 | 106   | 20'748 | 114   | 20'634 | 119   | 20'515 | 125   | 37 |
| 38                    | 21'389 | 95    | 21'294 | 101   | 21'193 | 107   | 21'086 | 112   | 20'974 | 120   | 20'854 | 126   | 38 |
| 39                    | ...    | ...   | 21'625 | 101   | 21'524 | 106   | 21'418 | 112   | 21'306 | 119   | 21'187 | 126   | 39 |
| 40                    | ...    | ...   | ...    | ...   | 21'848 | 106   | 21'742 | 111   | 21'631 | 119   | 21'512 | 125   | 40 |
| 41                    | ...    | ...   | ...    | ...   | ...    | ...   | 22'060 | 112   | 21'948 | 117   | 21'831 | 125   | 41 |
| 42                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 22'260 | 118   | 22'142 | 123   | 42 |
| 43                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 22'448 | 124   | 43 |
| 44                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 44 |
| 45                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 45 |
| 46                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 46 |
| 47                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 47 |
| 48                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 48 |
| 49                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 49 |
|                       | 59     | Diff. | 60     | Diff. | 61     | Diff. | 62     | Diff. | 63     | Diff. | 64     | Diff. |    |

## Continuous Temporary Annuities—(continued).

OM  $2\frac{3}{4}$  PER-CENT. $\bar{a}_{M-n+1:n}$ OM  $2\frac{3}{4}$  PER-CENT.

| MATURITY-AGE, i.e., M |        |       |        |       |        |       |        |       |        |       |        |       |
|-----------------------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|
| n                     | 65     | Diff. | 66     | Diff. | 67     | Diff. | 68     | Diff. | 69     | Diff. | 70     | Diff. |
| 1                     | '969   | 1     | '968   | 2     | '966   | 2     | '964   | 1     | '963   | 2     | '961   | ...   |
| 2                     | 1'882  | 5     | 1'877  | 5     | 1'872  | 6     | 1'866  | 7     | 1'859  | 7     | 1'852  | ...   |
| 3                     | 2'746  | 10    | 2'736  | 11    | 2'725  | 12    | 2'713  | 13    | 2'700  | 14    | 2'686  | ...   |
| 4                     | 3'568  | 16    | 3'552  | 18    | 3'534  | 20    | 3'514  | 21    | 3'493  | 23    | 3'470  | ...   |
| 5                     | 4'352  | 23    | 4'329  | 25    | 4'304  | 28    | 4'276  | 30    | 4'246  | 32    | 4'214  | ...   |
| 6                     | 5'104  | 30    | 5'074  | 34    | 5'040  | 36    | 5'004  | 40    | 4'964  | 42    | 4'922  | ...   |
| 7                     | 5'827  | 39    | 5'788  | 41    | 5'747  | 45    | 5'702  | 49    | 5'653  | 53    | 5'600  | ...   |
| 8                     | 6'524  | 47    | 6'477  | 49    | 6'428  | 54    | 6'374  | 58    | 6'316  | 63    | 6'253  | ...   |
| 9                     | 7'196  | 54    | 7'142  | 58    | 7'084  | 62    | 7'022  | 67    | 6'955  | 73    | 6'882  | ...   |
| 10                    | 7'846  | 60    | 7'786  | 66    | 7'720  | 70    | 7'650  | 76    | 7'574  | 82    | 7'492  | ...   |
| 11                    | 8'476  | 67    | 8'409  | 72    | 8'337  | 79    | 8'258  | 84    | 8'174  | 91    | 8'083  | ...   |
| 12                    | 9'088  | 74    | 9'014  | 79    | 8'935  | 85    | 8'850  | 92    | 8'758  | 100   | 8'658  | ...   |
| 13                    | 9'683  | 80    | 9'603  | 85    | 9'518  | 94    | 9'424  | 98    | 9'326  | 108   | 9'218  | ...   |
| 14                    | 10'261 | 85    | 10'176 | 92    | 10'084 | 98    | 9'986  | 108   | 9'878  | 113   | 9'765  | ...   |
| 15                    | 10'824 | 90    | 10'734 | 97    | 10'637 | 105   | 10'532 | 112   | 10'420 | 122   | 10'298 | ...   |
| 16                    | 11'373 | 95    | 11'278 | 102   | 11'176 | 110   | 11'066 | 118   | 10'948 | 128   | 10'820 | ...   |
| 17                    | 11'908 | 100   | 11'808 | 106   | 11'702 | 115   | 11'587 | 123   | 11'464 | 133   | 11'331 | ...   |
| 18                    | 12'429 | 103   | 12'326 | 110   | 12'216 | 120   | 12'096 | 128   | 11'968 | 137   | 11'831 | ...   |
| 19                    | 12'938 | 106   | 12'832 | 114   | 12'718 | 124   | 12'594 | 132   | 12'462 | 142   | 12'320 | ...   |
| 20                    | 13'434 | 109   | 13'325 | 118   | 13'207 | 126   | 13'081 | 136   | 12'945 | 145   | 12'800 | ...   |
| 21                    | 13'920 | 113   | 13'807 | 121   | 13'686 | 130   | 13'556 | 138   | 13'418 | 148   | 13'270 | ...   |
| 22                    | 14'394 | 116   | 14'278 | 123   | 14'155 | 132   | 14'023 | 142   | 13'881 | 151   | 13'730 | ...   |
| 23                    | 14'856 | 117   | 14'739 | 125   | 14'614 | 135   | 14'479 | 144   | 14'335 | 155   | 14'180 | ...   |
| 24                    | 15'308 | 119   | 15'189 | 127   | 15'062 | 136   | 14'926 | 146   | 14'780 | 156   | 14'624 | ...   |
| 25                    | 15'750 | 121   | 15'629 | 129   | 15'500 | 138   | 15'362 | 147   | 15'215 | 158   | 15'057 | ...   |
| 26                    | 16'182 | 122   | 16'060 | 131   | 15'929 | 139   | 15'790 | 149   | 15'641 | 159   | 15'482 | ...   |
| 27                    | 16'606 | 124   | 16'482 | 132   | 16'350 | 142   | 16'208 | 149   | 16'059 | 161   | 15'898 | ...   |
| 28                    | 17'020 | 126   | 16'894 | 133   | 16'761 | 141   | 16'620 | 152   | 16'468 | 161   | 16'307 | ...   |
| 29                    | 17'424 | 126   | 17'298 | 134   | 17'164 | 142   | 17'022 | 153   | 16'869 | 162   | 16'707 | ...   |
| 30                    | 17'822 | 128   | 17'694 | 135   | 17'559 | 144   | 17'415 | 153   | 17'262 | 162   | 17'100 | ...   |
| 31                    | 18'211 | 129   | 18'082 | 136   | 17'946 | 144   | 17'802 | 154   | 17'648 | 163   | 17'485 | ...   |
| 32                    | 18'592 | 130   | 18'462 | 136   | 18'326 | 146   | 18'180 | 154   | 18'026 | 163   | 17'863 | ...   |
| 33                    | 18'966 | 131   | 18'835 | 137   | 18'698 | 146   | 18'552 | 154   | 18'398 | 164   | 18'234 | ...   |
| 34                    | 19'332 | 132   | 19'200 | 138   | 19'062 | 146   | 18'916 | 154   | 18'762 | 164   | 18'598 | ...   |
| 35                    | 19'691 | 131   | 19'560 | 140   | 19'420 | 146   | 19'274 | 154   | 19'120 | 165   | 18'955 | ...   |
| 36                    | 20'044 | 132   | 19'912 | 140   | 19'772 | 147   | 19'625 | 155   | 19'470 | 163   | 19'307 | ...   |
| 37                    | 20'390 | 133   | 20'257 | 139   | 20'118 | 148   | 19'970 | 155   | 19'815 | 164   | 19'651 | ...   |
| 38                    | 20'728 | 132   | 20'596 | 140   | 20'456 | 147   | 20'309 | 155   | 20'154 | 164   | 19'990 | ...   |
| 39                    | 21'061 | 133   | 20'928 | 139   | 20'789 | 147   | 20'642 | 156   | 20'486 | 162   | 20'324 | ...   |
| 40                    | 21'387 | 133   | 21'254 | 138   | 21'116 | 148   | 20'968 | 154   | 20'814 | 164   | 20'650 | ...   |
| 41                    | 21'706 | 131   | 21'575 | 139   | 21'436 | 146   | 21'290 | 155   | 21'135 | 162   | 20'973 | ...   |
| 42                    | 22'019 | 131   | 21'888 | 138   | 21'750 | 146   | 21'604 | 154   | 21'450 | 161   | 21'289 | ...   |
| 43                    | 22'324 | 129   | 22'195 | 137   | 22'058 | 144   | 21'914 | 154   | 21'760 | 160   | 21'600 | ...   |
| 44                    | 22'624 | 128   | 22'496 | 136   | 22'360 | 144   | 22'216 | 152   | 22'064 | 160   | 21'904 | ...   |
| 45                    | ...    | ...   | 22'790 | 135   | 22'655 | 142   | 22'513 | 151   | 22'362 | 158   | 22'204 | ...   |
| 46                    | ...    | ...   | ...    | ...   | 22'944 | 140   | 22'804 | 149   | 22'655 | 157   | 22'498 | ...   |
| 47                    | ...    | ...   | ...    | ...   | ...    | ...   | 23'088 | 147   | 22'941 | 155   | 22'786 | ...   |
| 48                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 23'222 | 154   | 23'068 | ...   |
| 49                    | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | ...    | ...   | 23'345 | ...   |
|                       | 65     | Diff. | 66     | Diff. | 67     | Diff. | 68     | Diff. | 69     | Diff. | 70     | Diff. |

## THE INSTITUTE OF ACTUARIES.

## EXAMINATIONS OF THE INSTITUTE, APRIL 1906.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE  
(PART I).*Examiner*—PROF. S. L. LONEY, M.A.*Supervisors*—MESSRS. A. D. BESANT, B.A., and A. G. HEMMING.*First Paper.*

1. A bill for £789. 10s. is drawn on March 16 for 3 months, and is discounted on April 10, the rate of commercial discount being 5 per-cent; allowing for the three days of grace, find the amount, to the nearest penny, which the bill produces.

2. A train leaves London for Exeter at 3 p.m., and travels at a uniform rate of 45 miles per hour; when and where does it meet a train which leaves Exeter at 2.36 p.m., and travels at a uniform rate of 25 miles an hour, the distance between London and Exeter being 171 miles?

3. Solve the equations

$$(1) \quad \frac{yz}{y+z} = 1, \quad \frac{zx}{z+x} = 2, \quad \frac{xy}{x+y} = 3;$$

$$(2) \quad \sqrt{x+10} + \sqrt{x-5} = \sqrt{3x+7}.$$

4. Prove that there is no value of  $x + \frac{1}{x}$  between +2 and -2.

Show that the series

$$1 + \frac{x}{1+x^2} + \left(\frac{x}{1+x^2}\right)^2 + \left(\frac{x}{1+x^2}\right)^3 + \dots \text{ad inf.}$$

can be always summed, and that its sum is never greater than 2 or less than  $\frac{2}{3}$ .

5. Find the sum of the squares of the first  $n$  natural numbers.

6.  $n$  quantities form an arithmetic progression whose first term is  $a$  and whose common difference is  $d$ ; find the sum of the squares of these quantities.

7. In an election there are  $m$  candidates and  $n$  voters, each having one vote; in how many ways can the votes be distributed?

8. Prove the truth of the binomial theorem for any exponent.

Neglecting cubes and higher powers of  $x$ , prove that

$$\frac{(1+2x)^3}{(2-x)^2} = \frac{1}{4} \left[ 1 + 7x + \frac{75x^2}{4} \right].$$

8. Define a logarithm, and show that  $\log_a m^n = n \log_a m$ .

In each year the number of births in a certain country was 3 per-cent, and the number of deaths 2 per-cent, of the population at the commencement of that year; find the total number of births in that country during 20 years from the time when the population was forty millions.

9. Obtain the term of the  $n$ th degree in the product

$$\left[1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \text{ad. inf.}\right] \times \left[1 + y + \frac{y^2}{2} + \frac{y^3}{3} + \dots \text{ad. inf.}\right].$$

10. There are two independent events the respective probabilities of the happening of each being known; find the probability that both will happen.

Two men, A and B, each draw a card from a well-shuffled pack of playing cards, find that they are of the same value, and replace the cards; they do this four times in succession. Find the chance that this would happen, and show that it is approximately

$$\frac{1}{84,000}.$$

11. Given the chance of a certain event happening in one particular trial, find the chance of its happening at least  $r$  times in  $n$  trials.

A man tosses 20 pennies and removes all that fall head up; he then tosses the remainder and removes all that fall head up, and so on. How many times ought he to be allowed to repeat this operation if he is to have an even chance of removing all the pennies before he has finished?

12. Given the values of a function  $u$  of a variable  $x$  for any  $n$  given values of  $x$ , find the general value of  $u_x$ .

Given  $u_0=98203$ ,  $u_1=97843$ ,  $u_2=97459$  and  $u_3=97034$ , find  $u_{2.25}$ .

### Second Paper.

13. A grocer makes a profit of 25 per-cent by selling tea at 2s. per lb. when the import duty is 8d. per lb.; if the duty were reduced by 4d. per lb. and the grocer reduced his price by 4d. also, what rate per-cent of profit would he then make?

14. A cask is filled with 40 gallons of water and another with 30 gallons of brandy;  $x$  gallons are drawn from each cask and put into the other. This operation is repeated and there are now

$14\frac{1}{6}$  gallons of brandy in the first cask. Find  $x$ .

15. Divide

$$(a^2 + 2ab - 3b^2)^2 - (a^2 - 4ab + 3b^2)^2 \text{ by } (a-b)^2,$$

and simplify the expression

$$\frac{(x+y)^2 - z^2}{(x-y)^2 - z^2} \times \frac{(y+z)^2 - x^2}{(y-z)^2 - x^2} \div \frac{(x+y+z)^2}{z^2 - (y-x)^2}.$$



16. Find the condition that the roots of the quadratic equation  $ax^2+bx+c=0$  may be imaginary.

Find the greatest and least values of  $x$  which satisfy the equation  $x^2+y^2=4x+5$ .

17. Insert two harmonical means between two quantities,  $a$  and  $b$ , and show that their product is less than  $ab$ .

18. Prove the formula for the number of permutations of  $n$  things taken  $r$  together.

In how many ways can 6 men and 4 ladies be arranged in a row, so that no two of the ladies may be next one another?

19. Making use of the Tables, find the value of

(1)  $x$ , where  $(2.73)^{-x}=5.82$ ;

(2)  $l$ , given that  $t=2\pi\sqrt{\frac{l}{g}}$ , and that  $t=1.275$ ,  $\pi=3.1416$ , and  $g=32.2$ .

20. Prove the truth of the Exponential Theorem.

Prove that

$$\log_e(1+x-2x^2)=x-\frac{5}{2}x^2+\frac{7}{3}x^3-\frac{17}{4}x^4+\dots$$

and find the form of the general term.

21. Given one solution in positive integers of the equation  $ax+by=c$ , find the general solution.

A man's age in 1905 was equal to the sum of the digits in the number of the year of his birth; find the year in which he was born

22. Explain the mathematical meaning of the expressions *Probable Value* and *Expectation*.

A man throws a six-faced die until he gets an ace; he is to receive £1 if he succeeds at the first throw, £ $\frac{1}{2}$  if he succeeds at the second throw, £ $\frac{1}{3}$  if he succeeds at the third throw, and so on; given that  $\log_e 6=1.79176$ , find the value of his expectation to the nearest penny.

23. Show that the  $(n+1)$ th difference of a function of the  $n$ th degree vanishes, and that the  $r$ th differences of a series of terms forming a geometrical progression form a geometrical progression with the same common ratio.

Find the  $n$ th term of the series 2, 12, 36, 98, 270, 768... and the sum of  $n$  terms of it.

24. Prove that

$$\Sigma 2^x x^3 = 2^x [x^3 - 6x^2 + 18x - 26] + C.$$


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EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE  
(PART II).

Examiners—MESSRS. H. J. BAKER, W. H. HODGSON, J. N. LEWIS, and  
A. C. THORNE.

First Paper.

✓ 1. The sum of £3,500 was paid on 1 April 1906 for a yearly annuity-certain of £250, the first payment of which falls due on 1 July 1906 and the last payment on 1 July 1930. Find the rate of interest realized by the purchaser, assuming that the sinking fund is accumulated at  $3\frac{1}{2}$  per-cent.

2. Obtain expressions for the present value and amount of (1) an annuity payable  $m$  times a year with interest convertible yearly and (2) an annuity payable momentarily with interest convertible yearly, and prove the correctness of the formulæ by general reasoning.

3. Of the three lives  $(x)$ ,  $(y)$ , and  $(z)$ , find the probability :

- (a) That one and one only will die in the  $t$ th year ;
- (b) That the last survivor will die in the  $t$ th year ;
- (c) That  $(x)$  will die in the  $t$ th year, leaving  $(y)$  surviving him, and having survived  $(z)$  by at least  $n$  years.

4. Obtain the following formulæ and give verbal explanations :

- (a)  $A_x = v - da_x$  ;
- (b)  $A_{x:\overline{n}|}^1 = v(1 - v^n p_x) - (1 - v)a_{x:\overline{n}|}$  ;
- (c)  $A_{x:\overline{n}|} = v(1 + a_{x:\overline{n-1}|}) - a_{x:\overline{n-1}|}$  .

5. Interpret and obtain expressions not involving the differential or integral calculus for the values of :

$$a_{xy:\overline{n}|}^1 ; \quad A_{xy:\overline{n}|}^1 ; \quad A_{xy:\overline{n}|}^{\left(\frac{n}{2}\right)}_{1.3}$$

6. The first payment of a complete annuity is to be made one year after the attainment of age  $x+n$  by a life now aged  $x$ , and in the event of death before that age all premiums paid are to be returned with  $2\frac{1}{2}$  per-cent simple interest. If death occur after age  $x+n$  the representatives of the annuitant are to receive the difference, if any, between the premiums accumulated with  $2\frac{1}{2}$  per-cent simple interest up to age  $x+n$ , and the total annuity payments received by the annuitant. Find an expression for the net annual premium for this benefit. Assume that the office premium equals the net premium loaded with a percentage and a constant.

7. Prove that  $({}_nV_x + P_x)(1+i) = {}_{n+1}V_x + q_{x+n}(1 - {}_{n+1}V_x)$ .

Find analogous expressions in the case of (1) a whole-life policy effected at a single premium and (2) an immediate annuity on a single life, and give verbal explanations of the resulting formulæ.

8. Describe fully the method you would adopt in the construction of tables of single and annual premiums for joint-life endowment assurances.

*Second Paper.*

9. A loan of £4,800 is repayable by 40 equal half-yearly instalments of principal, interest at 4 per-cent per annum being payable half-yearly on the balance of the loan unpaid at the beginning of the half-year. Find the sum for which the remaining payments might be commuted at the time when the 20th is due, taking interest at 3 per-cent per annum.

10. Find an expression for the value of an annuity-certain for  $n$  years, the terms of which are in geometrical progression and represented by  $s, s^2, s^3 \dots s^n$ . Under what conditions could the value of a perpetual annuity of this nature be ascertained, and what would be its value?

11. It is desired to provide pensions for present and future members of a staff on attainment of age 60. The staff consists of, and would be kept constant at, 70 members, varying in ages from 20 to 60. Assuming that vacancies only occur from death or superannuation and that new members enter at age 20, explain how a suitable mortality table may be employed to obtain an idea of the number of pensioners in, say, 5, 10, 15 . . . up to 60 years hence.

12. What does  $\frac{P_x}{d}$  represent?

If used as a single premium for an assurance upon the life of  $(x)$ , what amount of assurance will it provide?

13. State what methods could be adopted in the calculation of the value of an annuity payable so long as at least two lives out of three are living, having given tables of annuity-values for single and two joint lives.

Determine the respective interests of  $(x)$ ,  $(y)$  and  $(z)$  in such an annuity if it is to be equally divided between  $(x)$  and  $(y)$  during their joint lives, and after the death of either to be divided in like proportions between  $(z)$  and the survivor during what may remain of their joint lives.

14. Find the value of a half-yearly annuity on a life  $(y)$ , the first payment to be made six months after the death of  $(x)$ , and the annuity to be payable for  $t$  years certain whether  $(y)$  survive or not.

15. If  $\mu_x = -\lambda s - \lambda g \lambda c \cdot c^x$  and  $\bar{a}'$  denotes an annuity calculated at a rate of interest  $i'$  such that  $\frac{1}{1+i'} = vc$ , prove that

$$\bar{A}_{xyz}^1 = -\lambda s \bar{a}_{xyz} + (\mu_x + \lambda s) \bar{a}'_{xyz}.$$

16. What data would you require in order to construct a table representing the mortality experience of the policyholders in a life assurance company? What policies, if any, would you exclude from the investigation?

## Third Paper.

✓ 17. Explain what is meant by repayment of a loan by accumulative sinking fund. { 20 193

A foreign Government issues bonds for £5,000,000, bearing 4 per-cent interest. An uniform sum is set aside annually out of which interest is paid and the balance applied to the redemption of bonds at a premium of 2 per-cent, the whole issue being thus cancelled in 30 years. What will be the amount of the annual outlay and what nominal amount of bonds will be outstanding immediately after the 15th payment?

18. A whole-life policy for £1,000, effected at age 20, has been thirty years in force, and has accumulated bonus additions of £450. It is proposed to devote part of the bonus to convert the policy (including the remainder of the bonus) into an endowment assurance maturing at age 65. Using 3 per-cent interest, and ignoring the question of loading, find how much bonus will remain attached to the policy after the alteration has been made.

19. If a community consist of  $l_x$  persons aged  $x$ ,  $l_{x+1}$  aged  $x+1$ , and so on down to the extremity of life, show that the total funds to be raised to provide for an assurance of 1 on the death of each member will be  $l_x(a_x + A_x)$ , and for an immediate annuity-due to each person  $l_x\left(1 + e_x + \frac{e_x - a_x}{i}\right)$ .

What would be the uniform contribution of each individual to these several funds?

20. Obtain by the retrospective method a formula in terms of premiums for  ${}_nV_{xt}^1$ , and prove verbally and algebraically that

$${}_nV_x = \frac{P_x - P_{x+n}^1}{P_x^1}.$$

21. Endowment assurances of 1 each, payable at age  $x+n$  or previous death, are effected by  $l_x$  lives, all aged  $x$ . Prove that, theoretically, the amount payable to the survivors at age  $x+n$  equals the accumulated amount of their premiums, less the difference between the accumulated premiums and the accumulated claims in respect of those who die before age  $x+n$ . What conditions are necessary in order that this may be actually the case in practice?

22. Define verbally and algebraically the term "force of mortality", and obtain expressions for the force of mortality when the law of mortality may be expressed by the equations:

$$(1) l_x = l_0 \alpha^x;$$

$$(2) l_x = l_0 \alpha^x b^{x^2} \beta^{c^x}.$$

Show that if a mortality table follow Makeham's law, and complete tables of expectations of life be formed for combinations of lives of equal ages, it would be possible to dispense with tables of the values of annuities.

23. Interpret and obtain integral expressions for

$$\bar{A}_{x:y:z}^2; \bar{A}_{x:yz}^2; \bar{A}_{x:yz}^3; \bar{A}_{x:yz}^1; \bar{A}_{xy:z}^1; \bar{A}_{x:yz}^{1.3}.$$

Give a formula of approximate summation that may be made use of to obtain  $\bar{A}_{x:yz}^2$ , and indicate the steps of the process.

24. Give a concise account of the methods you would adopt in the construction of tables of values of

(a) Single and annual premiums for temporary assurances;

(b) Deferred annuities;

and state how you would verify your results.

### EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

*Examiners*—MESSRS. R. CROSS, W. P. PHELPS, M.A., R. R. TILT and  
H. M. TROUNCER, M.A.

#### *First Paper.*

1. Explain, in detail, what the symbols

$$\sigma_{[x]+t}, \theta_{[x]+t}, w_{[x]+t}, T_{[x]+t}, e_{[x]+t}, \text{ and } G_{[x]+t}$$

used in deducing the numbers exposed to risk for the select assurance tables of the British Offices' Life Tables, 1893, represent, showing exactly what cases are included under duration  $t$ .

Give a formula for the Exposed to Risk.

2. Mention the advantages of Woolhouse's method of graduation.

What simplified method of application was suggested by Mr. G. F. Hardy? Demonstrate the identity of the results obtained with those derived by the direct application of Woolhouse's formula.

3. Give a short account of the materials and methods employed by Dr. Farr in the construction of the English Life Table No. 3.

What variations in the population are assumed?

4. Under what conditions do you consider the simple or the compound reversionary bonus system to be equitable?

Obtain an office premium to provide a compound reversionary bonus, taking  $b$  as the rate of quinquennial bonus and  $b'$  the rate of interim bonus on the sum assured for each year entered upon in the quinquennium.

5. A company proposes to grant annuities on lives. How would you calculate the office rates?

State fully the reasons for the bases and methods selected.



6. Express algebraically the value at the commencement of a policy-year of the gross contribution to surplus from

(1) Interest, (2) Loading, (3) Mortality,

and from the expressions for (1) and (2) show how to ascertain the duration at which a policy will make the same contribution from interest and loading if the valuation rate of interest be reduced a half per-cent.

7. A is entitled to a reversion to a  $\frac{1}{6}$ th share of a fund consisting of

£15,000 West. Australia 4 per-cent Inscribed Stock,  
redeemable 1934.

£650 per annum East India Railway Co. Annuity,  
Class B

expectant on the death of B, aged 60, but subject to the payment out of the income thereof to C, aged 48, of an annuity of £150 and to the payment of £2,500 on the death of C. A is also absolutely entitled during the life of B to half of the balance of the income of the fund after payment to B of an annuity of £600 and payment to C of the above-mentioned annuity of £150.

Show, in detail, the principles on which you would assess the market value of A's interest and the methods you would employ.

8. Estimate the market value of an endowment assurance policy for £5,000 on a male life, born 5 August 1863. The policy will mature on 5 August 1923 or at previous death and is subject to an annual premium of £120 due on 1 July in each year, so long as the policy is in force, up to and inclusive of the year 1922. The bonus additions to 31 December 1905 amount to £1,200. The office values quinquennially on a  $2\frac{1}{2}$  per-cent basis, and on 31 December last (after a substantial provision for depreciation of stocks) it declared a compound reversionary bonus of 25s. per-cent per annum, with an interim bonus at the same rate on the sum assured only.

### *Second Paper.*

9. Describe the method of graduation of the rates of sickness and mortality presented by the experience of the Manchester Unity of Oddfellows (1893-97). Discuss its advantages and disadvantages, in dealing with sickness tables, as compared with other well-known methods of graduation.

10. Indicate briefly the principal changes in the methods of Life Office Valuations and Distributions employed in the last five years as compared with those of twenty-five years ago. Have the changes, in your opinion, been advantageous to policyholders?

11. An office distributes its divisible surplus as a cash percentage of the premiums paid on each policy in the five years immediately preceding the date of valuation and distribution. What are the theoretical and practical objections to this course?

12. Assuming an office to make a  $2\frac{1}{2}$  per-cent net premium valuation, what special reserves should, in your opinion, be provided, and what considerations should guide you in determining their amounts?

13. Criticize Dr. Sprague's modification of the contribution method of distributing surplus, and describe any subsequent improvements that have been suggested.

State which method you consider would most equitably distribute the surplus between policies of different durations in the event of

- (i) A heavy rate of mortality amongst the older lives;
- (ii) A large increase of new business.

14. A, aged 25, is a civil engineer just going out to India, B is his mother, aged 50. Give a formula for a fixed inclusive office annual premium, payable during the joint lives, for an assurance payable at the death of A if he should die in B's lifetime or within 5 years after her death.

What objections in practice might arise, and what alternative method would you suggest?

15. Mention any principles agreed between the Institute of Actuaries and the Estate Duty Department of the Inland Revenue as to the method of assessing the values of reversionary interests for purposes of duty.

Show how to value for Estate Duty purposes the absolute reversion expectant on the death of a lady, aged 64, to £10,000 India  $3\frac{1}{2}$  per-cent Stock.

16. A, aged 46, and B, his wife, aged 47, are entitled, under the Will of A's father and under their Marriage Settlement, to successive reversionary life interests in  $\frac{1}{4}$ th of the following fund (or the proceeds thereof) expectant on the death of A's mother, aged 70:

|                                                                                                                                                  | Assumed<br>Price. |
|--------------------------------------------------------------------------------------------------------------------------------------------------|-------------------|
| £5,000 Consols . . . . .                                                                                                                         | 89 $\frac{1}{2}$  |
| 5,000 Natal 4 per-cent Inscribed Stock, 1937 . . . . .                                                                                           | 110               |
| 250 Great Indian Peninsular Railway B Annuity . . . . .                                                                                          | 21 $\frac{1}{2}$  |
| 2,000 advanced to A, free of interest, to be brought into account on the death of his mother.                                                    |                   |
| 12,000 (viz., £5,000 and £7,000) similarly advanced to two other beneficiaries in respect of their one-fourth shares.                            |                   |
| A leasehold house, occupied by the tenant for life, held for an unexpired term of 30 years at a ground rent of £10; estimated rental value £100. |                   |

Estate Duty at (say)  $4\frac{1}{2}$  per-cent will be payable on the whole fund on the death of the present life tenant.

Show fully how you would find the value of A's and B's interests (if sold together) and how you would apportion that value between them.

*Third Paper.*

17. What were the chief objects to be borne in view in the graduation of the  $O^M$ ,  $O^{M(5)}$  and  $O^{M(1)}$  Tables?

Give a brief outline of the methods adopted.

Given two tables, adjusted by Makeham's first modification of Gompertz's Law of Mortality, in which the value of  $c$  is the same, what conclusions, as regards policy-values, can be drawn from a comparison of the values of the other two constants?

18. What material was made use of in forming the Mortality Experience of Government Life Annuitants published in 1883?

How did the treatment of the material differ from that employed in previous Annuity Tables, and what special features were deduced?

19. How would you proceed to value, for the purpose of determining the amount of the divisible surplus in respect of a quinquennial period, the various securities included in the Balance Sheet of a Life Office, and discuss any special points that might arise.

20. What important considerations have been urged against the net premium method of valuation?

21. An office is proposing to adopt a system of Endowments and Deferred Assurances for Children, without participation in profits, the assurance to commence at age 21, with an option of taking an Endowment at that age; in the event of death before attaining age 21 all premiums paid are to be returned without interest.

On what basis would you calculate the premiums? What Endowment would you grant at age 21, and what surrender values would you allow before and after attaining age 21?

22. On what bases and how would you calculate the office Single and Annual Premiums for the following?

- (i) Sinking Fund Assurances;
- (ii) Temporary Assurances with an option of conversion into Whole-Life Assurances (without fresh medical examination) at any time, except during the last five years of the term of the Assurance, on paying the tabular rate of premium for the age attained at the date of conversion.

23. A, aged 30, is entitled to a Reversionary Life Interest after the death of B, aged 65, in a fund producing upwards of £800 a year; he desires to obtain an advance of £1,000, to be secured by a charge on his Reversionary Life Interest, but he wishes to retain the right of redemption during the first 3 years, on payment of the advance and premiums together with compound interest thereon at 5 per-cent with half-yearly rests; it is proposed to set up at the commencement of the transaction the minimum amount of assurance necessary to protect the advance during the first 3 years and to pay a single option premium for the right to take up further assurances, if required at the end of that period.

Derive formulæ for ascertaining the amount of the annual reversionary charge you would require and the amounts of the policies; state what mortality table and rate of interest you would use.

24. What relation exists between the constants for the Male and Female Tables in Mr. G. F. Hardy's graduation of the British Offices' Life Annuity Tables?

Explain how you would find the 5 per-cent ultimate values of Joint Life Annuities on

- (a) Two males, aged 71 and 77;
- (b) A male, aged 74, and a female, aged 82 (taking advantage of the relationship between the constants);
- (c) Two females, aged 81 and 84.

### EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART IV).

*Examiners*—MESSRS. J. BLAKEY, J. E. FAULKS, B.A., E. C. THOMAS, and R. TODHUNTER, M.A.

#### *First Paper.*

1. A, the owner in fee simple of an estate, placed such estate in strict settlement. Give the probable outlines of the settlement and discuss their effect, with special reference to the position of the eldest son on his father's death.

Explain the necessity for, and the usual incidents of, the periodical re-settlements required to preserve an estate in the same family for several generations.

2. In the case of a life born in the United Kingdom, state shortly what registers or other documents are available in proof of age. Draft, for insertion in a form of policy where age is not admitted, a clause indicating how an understatement of age may be remedied. Discuss, in the light of a recent case, any special importance that may attach to the question of proof of age in the case of endowment assurances.

3. Give the substance of the law relating to abatement of taxable income in respect of life assurance premiums. Discuss shortly any points that arise in the practical application of the law.

4. A widows' fund charges the following marriage fines to its members in addition to annual contributions:

| Age of Member<br>at Marriage | Marriage Fine                                                  |     |     |
|------------------------------|----------------------------------------------------------------|-----|-----|
| 40-50                        | £3 for each year over 5 that his age exceeds that of his wife. |     |     |
| 50-60                        | £4 10s.                                                        | do. | do. |
| Above 60                     | £6                                                             | do. | do. |

How would you estimate the value of these fines in a valuation? Consider the case of first marriages only.

5. Draft a form of book suitable for keeping particulars of *reversions purchased*. In the case of an office doing a large business of this character, what indexes or other additional books should be kept? Give reasons for each record suggested.

6. Describe fully the function of bills in the economy of international trading, and show clearly the means by which the balance of indebtedness between two countries is discharged.

7. What terminable annuities, included in the National Debt, have been created in exchange for stock held by Government departments? Give the reasons for the adoption of this system of exchange and your opinion as to its advisability.

8. In what respects is a life assurance company differently situated, in regard to the selection of investments, from (a) a bank, (b) trustees?

State, with reasons, whether you consider that the position of a life assurance company as regards the investment of its funds is affected by (a) the guarantee of surrender-values, (b) the growth of endowment assurance business, (c) the granting of capital-redemption assurances for large amounts.

#### *Second Paper.*

9. What is the rule against perpetuities? Is there any, and if so what, further provision against accumulation of income?

10. Draft a form of receipt for the amount payable in respect of a claim under a life policy. In paying such a claim what, if any, special requirements or conditions should be made:

- (1) Where the payees are trustees;
- (2) Where the payee is a limited company;
- (3) Where the payee is resident abroad?

Is any, and if so what, stamp necessary on the receipt for a claim? Give reasons for your answer.

11. Draft, for insertion in a policy granted to A on his own life, a clause rendering the policy moneys available for payment of Estate Duty before grant of Probate, and explain an alternative method by which the object in view may be attained.

Is it in any circumstances the duty of a life assurance company to see to the payment of Estate Duty in respect of the proceeds of a policy?

12. A large factory employs the following different classes of workers:

- (1) Clerical staff, male.
- (2) Do. do. female.
- (3) Factory hands, male.
- (4) Do. do. female.

It is proposed to establish a pension fund to include all classes, in which the pension and contributions are calculated as a percentage of the salary received. Discuss the equity of this arrangement. Consider the cases of the pension being based on a uniform percentage for each year of service of (a) average salary, (b) final salary.

How does the Shop Clubs Act, 1902, affect the status of such a fund?



13. In connection with the approaching quinquennial valuation of a life office a committee of directors has been appointed to revise the securities and investments constituting the assets of the office. Assuming that no special enquiries have in ordinary cases been made as to such securities since the last preceding valuation, state, with reasons, what fresh information you would obtain for the committee in the following classes of securities :

- (a) Mortgages on the fee of landed property ;
- (b) Mortgages on freehold house property ;
- (c) Loans on personal security.

14. Describe fully the system of records, accounts and checks you would adopt for collecting the premium income of a life assurance company, and for verifying the fact that the whole of the premiums due in the financial year had been brought into account and accounted for.

15. Explain why the prices of bar gold and bar silver in the London bullion market are of importance, and point out the causes which affect them.

16. By what criterion would you estimate the security afforded by a specified British railway preference or preferred ordinary stock, and by what other considerations would you be guided in forming an opinion of the eligibility of the stock as an investment ?

State (a) in what circumstances a preference stock of a British railway may be quoted at a higher price than an earlier preference stock of the same railway at the same rate of interest ; (b) in what respect the preferred ordinary stocks of the principal Scottish railways differ from those of English railways.

### *Third Paper.*

17. A sum of money due to A is to be paid to him by cheque. Enumerate the various ways in which the cheque may be drawn and crossed, explaining the precise legal effect of each.

Define the expressions "certified cheque", "bankers' draft." What are the legal effects of such documents ? In what classes of transactions are they generally employed, and why ?

Discuss the nature and effect of the form of document commonly described as "cheque with receipt attached."

18. A life office has advanced a sum of money on security of the absolute reversion to a certain trust fund, the advance being only a small proportion of the present value of the reversion. Discuss the remedies open to the office in the event of the interest not being duly paid.

In what circumstances, if any, and subject to what conditions, would it be possible to allow the loan to continue indefinitely, capitalizing the interest as it periodically becomes due ?

If any interest be capitalized, discuss the position as regards income tax on such interest.

19. State the provisions of the Friendly Societies Act, 1896, respecting payments to nominees of members of a registered friendly society. How is a nomination affected if the nominator subsequently makes a will?

20. An invested fund amounting to £30,000 exists for the purpose of providing pensions to certain office bearers *and their successors* in office. The benefit provided is a pension of £300 per annum after 30 years in office. The fund earns an average rate of interest of 3 per-cent on its investments, and is augmented each year by an additional fixed sum of £500 per annum. On the death or resignation of an existing officer his place is filled by a new officer at an average age of 30. The entry and present ages of the existing members, 15 in number, are known. The power of granting the above pensions is a discretionary one, but it is desired to convert this power into a compulsory obligation. How would you decide whether such a proceeding were financially sound, assuming that there would be no resignations before the completion of the stated term of years, and that there are no pensioners at present on the fund?

21. Describe what you consider the most satisfactory method of registering and keeping notices of assignment. What course would you adopt in regard to constructive notices, and why?

Draft a form of letter to be used in reply to enquiries as to the notices affecting a policy.

22. Discuss the principal causes that affect the amount of the profits of a London joint-stock bank.

23. The directors of a life office have ordered the purchase at certain prices of:

- (a) £10,000 debenture-stock of a certain English railway;
- (b) \$50,000 gold bonds of a certain American railroad.

Explain in detail how you would proceed to carry out the order in each case, giving full particulars of all documents that should come under your notice, and stating what examinations or checks should be applied to each.

24. Specify the various methods adopted by life assurance companies for the *temporary* investment of surplus funds which may be required at short notice. Mention their respective recommendations, and state what form of security would be held in each case.

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## PROCEEDINGS OF THE INSTITUTE.—SESSION 1905-1906.

*First Ordinary Meeting, 27 November 1905.*

The first ordinary meeting of the Session 1905-1906 was held at the Hall of the Institute, on the 27th day of November 1905.

The President (Mr. HENRY COCKBURN) in the Chair.

Papers, entitled: (a) "On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables", by Mr. George King; (b) "Notes on an Approximate Method of Valuation of Whole-Life Assurances, with Allowance for Selection", by Mr. T. G. Ackland; (c) "On a Property of the O<sup>M</sup> Select Tables, and its application to the Valuation of Whole-Life Policies", by Mr. O. F. Diver, M.A.; were taken as read.

The following gentlemen took part in the discussion:—Messrs. G. J. Lidstone, R. P. Hardy, H. H. Austin, E. C. Thomas, S. G. Warner, J. Bacon, and W. P. Phelps.

*Second Ordinary Meeting, 18 December 1905.*

The President (Mr. HENRY COCKBURN) in the Chair.

Messrs. William Adamson, F.F.A.; Finlay James Cameron, F.F.A.; and James Murray Laing, F.F.A., were duly elected Associates of the Institute.

A paper entitled "Canadian Vital Statistics: with particular reference to the Province of Ontario", by Mr. M. D. Grant, B.A., of Ottawa, was read in abstract by Mr. Todd, Honorary Secretary.

The following gentlemen took part in the discussion:—Messrs. A. G. Hemming, R. Todhunter, G. King, E. Woods, J. D. Watson, and S. G. Warner.

*Third Ordinary Meeting, 29 January 1906.*

The President (Mr. HENRY COCKBURN) in the Chair.

Mr. Alexander Latta, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "The Variations in Masculinity under different Conditions", by Mr. J. N. and Dr. C. J. Lewis, was read.

The following gentlemen took part in the discussion:—Messrs. T. E. Young, W. P. Elderton, C. D. Higham, and A. W. Watson.

*Fourth Ordinary Meeting, 26 February 1906.*

The President (Mr. HENRY COCKBURN) in the Chair.

Mr. Percy King Paddy, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On a Form of Spurious Selection which may arise when Mortality Tables are amalgamated", by Mr. W. Palin Elderton, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. G. Green, J. Bacon, T. G. Ackland, H. P. Calderon, and S. G. Warner.

*Fifth Ordinary Meeting, 26 March 1906.*

The President (Mr. HENRY COCKBURN) in the Chair.

A paper entitled "Some Aspects of Registration of Title to Land", by Mr. J. R. Hart, was read in abstract by the Author.

The following gentlemen took part in the discussion:—Messrs. W. C. Sharman, J. B. Gillison, A. W. Findlay, J. E. Faulks; and Messrs. J. S. Rubinstein and H. Ross Giles (visitors).

*Sixth Ordinary Meeting, 30 April 1906.*

The President (Mr. HENRY COCKBURN) in the Chair.

Messrs. Albert George Scott and James Charles Wardrop were duly elected Associates of the Institute.

A paper entitled "Reversionary Securities as Investments", by Mr. C. R. V. Coutts, was read by the Author.

The following gentlemen took part in the discussion:—Messrs. J. D. Watson, A. Levine, R. R. Tilt, S. G. Warner, W. P. Pulley, F. E. Colenso, and the President.

*The Fifty-ninth Annual General Meeting, 11 June 1906.*

The President (Mr. HENRY COCKBURN) in the Chair.

The proceedings at the Annual General Meeting will be found on page 397.

## REPORT, 1905–1906.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1905–1906, the fifty-eighth year of its existence.

There has been an *increase* of 41 in the number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

|                   |      |
|-------------------|------|
| On 31 March 1896, | 788, |
| „ 1897,           | 826, |
| „ 1898,           | 860, |
| „ 1899,           | 834, |
| „ 1900,           | 822, |
| „ 1901,           | 818, |
| „ 1902,           | 842, |
| „ 1903,           | 828, |
| „ 1904,           | 856, |
| „ 1905,           | 881, |
| „ 1906,           | 922. |

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last :

*Schedule of Membership, 31 March 1906.*

|                                                             | Honorary<br>Members | Fellows | Associates | Students | Corres-<br>ponding<br>Members | Total |
|-------------------------------------------------------------|---------------------|---------|------------|----------|-------------------------------|-------|
| i. Number of Members<br>in each class on<br>31 March 1905 . | 1                   | 226     | 282        | 350      | 22                            | 881   |
| ii. Withdrawals by                                          |                     |         |            |          |                               |       |
| (1) Death . . .                                             | ...                 | 4       | 4          | 1        | 1<br>... }                    | 50    |
| (2) Resignation or<br>otherwise . . .                       | ...                 | 3       | 13         | 24       |                               |       |
|                                                             | 1                   | 219     | 265        | 325      | 21                            | 831   |
| iii. Additions to Membership                                |                     |         |            |          |                               |       |
| (1) By Election . . .                                       | ...                 | ...     | 6          | ...      | ...<br>...<br>... }           | 91    |
| (2) By Order of Council                                     | ...                 | ...     | ...        | 76       |                               |       |
| (3) By Re-instatement                                       | ...                 | 2       | 1          | 6        |                               |       |
|                                                             | 1                   | 221     | 272        | 407      | 21                            | 922   |
| iv. Transfers                                               |                     |         |            |          |                               |       |
| (1) By Examination:                                         |                     |         |            |          |                               |       |
| <i>from Associates</i>                                      | ...                 | ...     | 11         | ...      | ...                           | ...   |
| <i>to Fellows</i> . . .                                     | ...                 | 11      | ...        | ...      | ...                           | ...   |
|                                                             | 1                   | 232     | 261        | 407      | 21                            | 922   |
| (2) By Examination:                                         |                     |         |            |          |                               |       |
| <i>from Students</i>                                        | ...                 | ...     | ...        | 40       | ...                           | ...   |
| <i>to Associates</i> . .                                    | ...                 | ...     | 40         | ...      | ...                           | ...   |
| v. Number of Members<br>in each class on<br>31 March 1906 . | 1                   | 232     | 301        | 367      | 21                            | 922   |

145 candidates have been admitted as Probationers, and 64 as Students conditionally on their passing Part I of the Examinations. These are not included in the above Schedule of Membership.

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of four Fellows, Mr. H. R. Cockburn, Mr. F. A. Curtis, Mr. A. Hendriks, and Mr. R. Thomson; three Associates, Mr. L. J. Park, Mr. T. J. Pound, and Mr. Edward Woods; and one Student, Mr. A. Macmillan.

Mr. Augustus Hendriks had long been a well-known and prominent Member of the Institute, and his death has caused a widespread feeling of loss among his many friends. He became a Fellow of the Institute in 1864, and was President during 1892-1893.

The Annual Subscriptions, together with admission and other fees, amounted to £2,169. 7s. 6d., as compared with £2,050. 7s. received in the previous year. The total Income for the year was £2,942. 15s. 9d., and the total Expenditure £2,493. 11s. 1d. The Revenue Account and Balance Sheet are given herewith (pp. 392-393).

[Continued on page 394.]



| Dr.                                                                                    | Revenue Account for the |    |    |           |
|----------------------------------------------------------------------------------------|-------------------------|----|----|-----------|
| Amount of Funds at the beginning of the year made up as under—                         | £                       | s. | d. | £ s. d.   |
| General Fund . . . . .                                                                 | 8,247                   | 8  | 4  |           |
| Messenger Legacy Fund . . . . .                                                        | 380                     | 5  | 11 |           |
| Brown Prize Fund . . . . .                                                             | 261                     | 9  | 8  |           |
|                                                                                        | 8,889                   | 3  | 11 |           |
| British Offices Valuation Tables Expenses Fund . . . . .                               | 312                     | 4  | 0  |           |
| Examination Fees in hand . . . . .                                                     | 219                     | 18 | 0  |           |
| Subscriptions—                                                                         |                         |    |    | 9,451 5 1 |
| Fellows . . . . .                                                                      | 660                     | 9  | 0  |           |
| Associates . . . . .                                                                   | 589                     | 1  | 0  |           |
| Students . . . . .                                                                     | 387                     | 9  | 0  |           |
| Probationers . . . . .                                                                 | 80                      | 6  | 6  |           |
|                                                                                        | 1,717                   | 5  | 6  |           |
| Two Annual Subscriptions Compounded for . . . . .                                      | 42                      | 0  | 0  |           |
| Fines for Re-instatement . . . . .                                                     | 5                       | 5  | 0  |           |
| Application Fees—                                                                      |                         |    |    | 1,764 10  |
| Associates . . . . .                                                                   | 12                      | 12 | 0  |           |
| Students . . . . .                                                                     | 57                      | 4  | 6  |           |
| Probationers . . . . .                                                                 | 41                      | 9  | 6  |           |
|                                                                                        |                         |    |    | 111 6     |
| Examination Fees for year 1905 . . . . .                                               |                         |    |    | 26 5      |
| Lecture and Class Fees . . . . .                                                       |                         |    |    | 314 11    |
| Contributions of Companies to British Offices Valuation Tables Expenses Fund . . . . . |                         |    |    | 180 18 1  |
| Sales of Publications—                                                                 |                         |    |    |           |
| Journal . . . . .                                                                      | 182                     | 9  | 2  |           |
| Text-Book, Part I . . . . .                                                            | 39                      | 6  | 0  |           |
| Text-Book, Part II . . . . .                                                           | 133                     | 8  | 5  |           |
| Government Annuity Tables . . . . .                                                    | 1                       | 14 | 4  |           |
| Select Life Tables . . . . .                                                           | 0                       | 14 | 2  |           |
| Short Collection of Actuarial Tables . . . . .                                         | 8                       | 10 | 10 |           |
| Hardy's Friendly Societies . . . . .                                                   | 1                       | 11 | 4  |           |
| Legal, Financial, and Statistical Lectures . . . . .                                   | 5                       | 16 | 9  |           |
| Transactions of Second International Congress . . . . .                                | 8                       | 15 | 0  |           |
| Syllabus and Examination Questions . . . . .                                           | 8                       | 0  | 10 |           |
| British Offices Life Tables . . . . .                                                  | 79                      | 7  | 10 |           |
| Dividends and Interest—                                                                |                         |    |    | 469 14    |
| General Fund . . . . .                                                                 | 246                     | 9  | 1  |           |
| Messenger Legacy Fund . . . . .                                                        | 11                      | 8  | 2  |           |
| Brown Prize Fund . . . . .                                                             | 7                       | 16 | 11 |           |
| British Offices Valuation Tables Expenses Fund . . . . .                               | 7                       | 3  | 7  |           |
|                                                                                        |                         |    |    | 272 17    |
|                                                                                        |                         |    |    | £12,591 9 |

## Balance Sheet

| LIABILITIES.                                             |     |   |    | £ s. d. | £ s. d. | £ s. d.    |
|----------------------------------------------------------|-----|---|----|---------|---------|------------|
| General Fund . . . . .                                   |     |   |    |         | 9,018   | 0 3        |
| Messenger Legacy Fund . . . . .                          | 233 | 9 | 2  |         |         |            |
| Accumulated Dividends . . . . .                          | 158 | 4 | 11 |         |         |            |
|                                                          |     |   |    |         | 391     | 14 1       |
| Brown Prize Fund . . . . .                               | 200 | 0 | 0  |         |         |            |
| Accumulated Dividends . . . . .                          | 69  | 6 | 7  |         |         |            |
|                                                          |     |   |    |         | 269     | 6 7        |
| British Offices Valuation Tables Expenses Fund . . . . . |     |   |    |         | 221     | 6 5        |
|                                                          |     |   |    |         |         | 9,900 7    |
| Examination Fees for year 1906. . . . .                  |     |   |    |         |         | 150 3      |
| Sundry unpaid accounts . . . . .                         |     |   |    |         |         | 65 6 1     |
|                                                          |     |   |    |         |         | £10,115 17 |

year ending 31 March 1906.

£r.

|                                                                                    | £   | s. | d. | £     | s. | d. |
|------------------------------------------------------------------------------------|-----|----|----|-------|----|----|
| Journal—                                                                           |     |    |    |       |    |    |
| Printing of Nos. 220, 221, 222, and 223 . . . . .                                  | 457 | 18 | 11 |       |    |    |
| Clerical assistance . . . . .                                                      | 60  | 0  | 0  |       |    |    |
| Expenditure on account of Index to Forty Volumes . . . . .                         | 26  | 2  | 6  |       |    |    |
|                                                                                    |     |    |    | 544   | 1  | 5  |
| Library—                                                                           |     |    |    |       |    |    |
| Binding, Purchases, and Index Cards . . . . .                                      |     |    |    | 41    | 11 | 3  |
| Publications' Account—Binding . . . . .                                            |     |    |    | 15    | 16 | 8  |
| Meetings . . . . .                                                                 |     |    |    | 47    | 4  | 1  |
| Examination charges . . . . .                                                      |     |    |    | 83    | 3  | 2  |
| Lecturer, and Tutors for classes in Parts I and II . . . . .                       |     |    |    | 356   | 8  | 0  |
| Legal charges . . . . .                                                            |     |    |    | 5     | 18 | 10 |
| Office Expenditure—                                                                |     |    |    |       |    |    |
| Rent . . . . .                                                                     | 600 | 0  | 0  |       |    |    |
| Salaries and Pension . . . . .                                                     | 347 | 7  | 0  |       |    |    |
| House expenses . . . . .                                                           | 76  | 17 | 1  |       |    |    |
| Corporation Duty . . . . .                                                         | 12  | 0  | 6  |       |    |    |
| Fire Insurance . . . . .                                                           | 28  | 12 | 0  |       |    |    |
| Stationery and Printing . . . . .                                                  | 139 | 16 | 4  |       |    |    |
| Postage and Telegrams . . . . .                                                    | 40  | 19 | 3  |       |    |    |
| Furniture and Fittings . . . . .                                                   | 6   | 15 | 2  |       |    |    |
| Sundries . . . . .                                                                 | 6   | 18 | 1  |       |    |    |
|                                                                                    |     |    |    | 1,259 | 5  | 5  |
| Expenditure on Valuations for the Royal Patriotic Fund Corporation . . . . .       |     |    |    | 28    | 13 | 6  |
| Expenditure on account of the Volume of British Offices Valuation Tables . . . . . |     |    |    | 279   | 0  | 0  |
| Amount of Funds at the end of the year, as per Balance Sheet . . . . .             |     |    |    | 9,900 | 7  | 4  |

Examined and found correct 1 May 1906.

W. M. MONILAWS, }  
 GEO. A. BROWN, } *Auditors.*  
 HUGH LUGTON, }

£12,591 9 8

31 March 1906.

## ASSETS.

|                                                                            | £     | s. | d. |
|----------------------------------------------------------------------------|-------|----|----|
| Natal 3 per-cent Inscribed Stock (£3,000), cost . . . . .                  | 2,846 | 6  | 0  |
| Metropolitan Railway 3½ per-cent Debenture Stock (£1,200), cost . . . . .  | 1,185 | 11 | 3  |
| Great Eastern Railway 4 per-cent Debenture Stock (£1,200), cost . . . . .  | 1,546 | 6  | 4  |
| Great Northern Railway Preferred Ordinary Stock (£1,000), cost . . . . .   | 1,142 | 11 | 9  |
| Great Western Railway 4¼ per-cent Debenture Stock (£1,350), cost . . . . . | 1,999 | 15 | 1  |
| Cash on Deposit Account . . . . .                                          | 620   | 2  | 4  |
| Cash on Current Account . . . . .                                          | 775   | 4  | 6  |

The Institute also possesses certain copyrights ]  
 and stocks of publications (see p. 394). ]

Examined and found correct, 1 May 1906.

W. M. MONILAWS, }  
 GEO. A. BROWN, } *Auditors.*  
 HUGH LUGTON, }

£10,115 17 3

The stock in hand of the Institute publications on 31 March was as follows :

| No. of Copies    | Description of Work                                              |
|------------------|------------------------------------------------------------------|
| 12,737 . . . .   | Parts of <i>Journal</i> .                                        |
| 501 . . . .      | Index to Vols. 1 to 10.                                          |
| 980 . . . .      | „ to Vols. 21 to 30.                                             |
| 867 . . . .      | <i>Text-Book</i> , Part I (New Edition).                         |
| 1,238 . . . .    | „ Part II (Second Edition).                                      |
| 683 . . . .      | Government Joint-Life Annuity Tables.                            |
| 766 . . . .      | Select Life Tables.                                              |
| 334 . . . .      | A Short Collection of Actuarial Tables.                          |
| 211 . . . .      | Messenger Prize Essay (Friendly Societies).                      |
| 13 in cloth )    | ( Lectures on Finance and Law (Clare and                         |
| 2,903 in paper ) | ( Wood Hill).                                                    |
| 1,641 . . . .    | Lectures on the Companies Acts (A. C. Clauson).                  |
| 1,589 . . . .    | Lectures on the Law of Mortgage (W. G. Hayter).                  |
| 814 . . . .      | Lectures on the Measurement of Groups and Series (A. L. Bowley). |
| 696 . . . .      | Transactions of the Second International Congress of Actuaries.  |
| 2,347 . . . .    | Syllabus and Examination Questions.                              |

The following papers were submitted at the sessional meetings of the Institute, namely :

27 November 1905.—“On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables.”—Mr. George King.

“Notes on an Approximate Method of Valuation of Whole-Life Assurances, with allowance for Selection.”—Mr. T. G. Ackland.

“On a property of the  $O^M$  Select Tables, and its application to the Valuation of Whole-Life Policies.”—Mr. O. F. Diver, M.A.

18 December 1905.—“Canadian Vital Statistics; with particular reference to the Province of Ontario.”—Mr. M. D. Grant, B.A.

29 January 1906.—“The Variations in Masculinity under Different Conditions.”—Mr. J. N. and Dr. C. J. Lewis.

26 February 1906.—“On a Form of Spurious Selection which may arise when Mortality Tables are amalgamated.”—Mr. W. Palin Elderton.

26 March 1906.—“Some Aspects of Registration of Title to Land.”—Mr. J. R. Hart.

30 April 1906.—“Reversionary Securities as Investments.”—Mr. C. R. V. Coutts.

For the Examinations held in the United Kingdom and the Colonies on 20, 21, 23, and 24 April last, 297 entries were received, namely :

|     |          |      |
|-----|----------|------|
| 131 | for Part | I.   |
| 98  | „        | II.  |
| 49  | „        | III. |
| 19  | „        | IV.  |

The results of the Examinations will be duly announced.\* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

An alteration has been made in the Rules regulating the Examinations, which will permit the Council to grant exemption from the necessity of

\* These results, for the United Kingdom and Colonies combined, are given on pp. 395-7.

passing Part I of the Examinations to graduates in mathematical honours of any University in the British Empire. This privilege has hitherto been confined to graduates of Universities in the United Kingdom, and its extension is justified by the important increase in the number of our Colonial Members.

Mr. George King has completed a course of twenty-four lectures on the actuarial subjects of Parts III and IV of the examinations. The lectures have been well attended, and have proved a valuable and instructive addition to the educational system of the Institute.

The Council have in preparation further Monetary Tables, based on the British Offices' Experience. The calculations, which are well advanced towards completion, have been placed under the supervision of a small committee, consisting of Messrs. R. P. Hardy, T. G. Ackland, and G. J. Lidstone. The thanks of the Institute are due to these gentlemen for kindly undertaking this work, and the result of their labours should form a valuable addition to the volumes already placed in the hands of the Members by the Joint Committee.

The Council have also in hand a new edition of the printed catalogue of the Library, and an index to the first forty volumes of the *Journal*.

### EXAMINATIONS, 1906.

Examinations were held on the 20th, 21st, 23rd, and 24th of April, 1906, in the United Kingdom and the Colonies, at London, Liverpool, Edinburgh, Dublin, Adelaide, Melbourne, Sydney, Montreal, Toronto, Ottawa, with the following results, the names in each class being arranged in alphabetical order:—

#### PART I.

One hundred and thirty-one candidates sent in their names, of whom one hundred and twenty-one presented themselves, and sixty-five passed, namely:—

##### *Class I:*

|                   |                   |
|-------------------|-------------------|
| Anderson, R. D.   | Kine, V. M.       |
| Davis, A. P.      | Lohan, J. J.      |
| Emery, W. S.      | Peter, J. C.      |
| Ferguson, T. R.   | Simmonds, R. C.   |
| Gunningham, S. J. | Stephenson, H. R. |
| Hilbery, R. W.    | Vaughan, H.       |
| Woodall, E. A.    |                   |

##### *Class II:*

|                    |                    |
|--------------------|--------------------|
| Abdul-Ali, S.      | Handford, J. J. W. |
| Bennett, H. G.     | Jerrold, A. L.     |
| Bolt, J. C.        | Johnstone, W. D.   |
| Breeds, A. H.      | Leigh, W. L.       |
| Brown, P. G.       | Lewis, D. H.       |
| Doucet, G. D.      | McCulloch, J. A.   |
| Doyle, J. P.       | Mackenzie, D. R.   |
| Emmerson, W. H. R. | Martin, W. A.      |
| Foster, J.         | Nathan, E. B.      |
| Foster, W. J.      | Priestman, E.      |
| Frost, C. F.       | Tyler, V. W.       |

*Class III:*

|                     |                    |
|---------------------|--------------------|
| Allport, R. N.      | Maunder, H. E.     |
| Brown, B. G. H.     | Naismith, K. E.    |
| Cooper, J. L.       | Needell, B.        |
| Dobbie, J. A.       | Patrick, W. S.     |
| Drake, J. W.        | Rose, G. M.        |
| Fender, W. M.       | Ruddle, F.         |
| Fleming, A. L.      | Selmer, C. F.      |
| Hall, J. V. L.      | Sutton, M. W.      |
| Harrison, R. J.     | Thomson, E. H.     |
| Hinstwitt, W. E.    | Van Homrigh, G. M. |
| Kearns, W. N.       | Warhurst, J.       |
| Kidd, A. B.         | Watson, A. D.      |
| Le Rossignol, L. F. | Watson, J. A.      |
| Lolley, C. F.       | Williamson, W. W.  |
| McCall, R.          | Yeomans, E. C.     |

## PART II.

Ninety-eight candidates sent in their names, of whom eighty-nine presented themselves, and sixteen passed, namely:—

*Class I:*

Thompson, J. S.

*Class II:*

|           |  |                 |
|-----------|--|-----------------|
| Davis, M. |  | Portch, A. G. ✓ |
|-----------|--|-----------------|

*Class III:*

|                   |  |                |
|-------------------|--|----------------|
| Blake, F. S.      |  | Carter, G. S.  |
| Blehl, E. M.      |  | Defries, F.    |
| Boag, H.          |  | Fielder, W. C. |
| Borrajo, E. J. W. |  | Fippard, R. C. |
| Bromby, W.        |  | Latham, P. J.  |
| Burrows, G. E.    |  | Smither, H. B. |

Thomson, F. R. T.

## PART III.

Forty-nine candidates sent in their names, of whom forty-five presented themselves, and twelve passed, namely:—

*Class I:*

None.

*Class II:*

None.

*Class III:*

|                      |  |                 |
|----------------------|--|-----------------|
| Carter, N. J.        |  | Laing, J. M.    |
| †Catchlove, C. H. L. |  | †May, W. T.     |
| Clinton, L. E.       |  | †Symmons, F. P. |
| †Court, A. G. D.     |  | Traversi, A. T. |
| †Denmead, J. C.      |  | †Wares, H. W.   |
| †Ebihara, K.         |  | †Young, A. S.   |

Those marked (†) have now completed the Examination for the Class of Fellow.



## PART IV.

Nineteen candidates sent in their names, of whom seventeen presented themselves, and eleven passed, namely:—

*Class I:*

None.

*Class II:*

†Catchlove, C. H. L.  
†Curtis, W. A.

†Humphreys, H. T.  
†Neill, S.B.

*Class III:*

†Adlard, S.  
Blanchard, N.  
†Culley, A. B.

†Ebihara, K.  
†Kelham, C. S.  
†Spurgeon, E. F.  
†Stewart, L. W.

Those marked (†) have now completed the Examination for the Class of Fellow.

## PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Fifty-Ninth Annual General Meeting of the Institute of Actuaries was held at Staple Inn Hall, Holborn, on Monday evening, 11 June 1906, Mr. Henry Cockburn (the President) in the Chair.

The Report of the Council (given on p. 390) having been taken as read,

THE PRESIDENT, in moving the adoption of the Report and Accounts, said that one of the first things which struck one in reading the report was the large increase in membership, a total of 922 having now been reached, which he thought was the largest figure the Institute had ever attained to. That was due very largely, as would be seen, to the number of students who came in during the last year. Such a large increase, if it was to continue, opened up a somewhat serious question, and taken in conjunction with the results of the examinations, as to which he would have a word to say presently, it was in some ways almost disquieting. He hoped, however, there was a brighter side. Certainly there was no intention on the part of the Council in any way to relax the standards, and all that could be hoped for in relation to the want of success recently shown in the examinations was that the education of the candidates would come up to the required standard. The Institute had to lament the death during the year of several members, including four Fellows, Mr. H. R. Cockburn, Mr. F. A. Curtis, Mr. A. Hendriks, and Mr. R. Thomson. Mr. Curtis, who was known to many of the older members, had been in retirement for some time, and they had not seen him lately, but nevertheless they regretted his loss. The name of Mr. Augustus Hendriks was one which recalled a kindly and able man, who was not only known to the members of the Institute generally, but was the close friend of many of them. His death, last summer, removed one whose presence would, he was sure, long be missed from amongst them, for there was no more kindly nature or man whose advice and opinion could be more relied upon. Turning to the accounts it would be found that the funds continued to increase, the

Institute now possessing rather over £9,000. He knew it was the opinion of some of the members that the funds were increasing more rapidly than was requisite, or even desirable, but at the same time he would like to point out that, in his own opinion at least, it was a creditable thing for a corporation like the Institute of Actuaries to possess a certain amount of money; although the point at which they should endeavour to prevent further increase might be a matter of opinion. The Institute was, at the present time, expending a certain amount of money in, he thought, very useful ways. For one thing, the fortieth volume of the *Journal* would be completed with the issue of the number for October next; and seeing that the existing printed indexes only went as far as the thirtieth volume, the Council had decided to issue, in one volume, a complete index to the first forty volumes, on the lines of that of volumes xxi to xxx. That work was well in hand, and it was hoped to get it finished soon after the publication of volume xl. The Council also proposed to spend some money in producing a new library catalogue, the last edition of which was printed in 1894. That work was also well in hand, and it was hoped that the publication would take place before the end of the year. In such ways the Council were endeavouring to make good use of the funds and the income. There were other ways in which the money might be used which had been suggested by individual members, and he was sure that any suggestions of that sort would be fully considered by the Council. He should pass on to whoever might succeed him in the chair the various proposals which had been made to him personally in that connection. The papers read during the session ranged over a fairly wide list of subjects, and he thought that, on the whole, they had one and all produced good discussions. The Institute was much indebted to the authors of the papers, many of whom appeared in that capacity at the Institute for the first time. Coming to the examinations, the general result as regards the United Kingdom, could not be said to have been strikingly satisfactory, in respect of three, at all events, out of the four examinations. The question of the want of success in the examinations, as measured by the percentage of passes, was one which had been mentioned before in late years, and he assured the Members that it was constantly engaging the attention of the Council. There were, no doubt, various reasons why, as compared with other bodies, the percentage of passes should be relatively small. For one thing, there was nothing to prevent a man going up ten times if he liked for an examination; he was not precluded by one or more failures from trying again, and, therefore, there was always a certain temptation for a man who was not very fully prepared to present himself for examination experimentally. That, perhaps, was not a very satisfactory state of things, but the whole question would be watched. The Institute had the advantage, during the past year, of a course of lectures from Mr. George King, which had been very well attended, and he was glad to think they might hope to have a similar course in the ensuing Session. In connection with the examinations, it would be noticed that the Council had arranged to have power to grant exemption from the necessity of passing Part I of the examinations to graduates in mathematical honours of any University in the British Empire. The

Council thought that, looking at the large number of Colonial members of the Institute, it would be, at least, a graceful act to help their friends across the seas in any reasonable way, and he was very hopeful that that new regulation would be appreciated by their numerous members in the Colonies. He had already mentioned the work which the Library Committee had in hand in relation to two matters—in fact, the Library Committee was, at the present time, exceedingly active. Some new book-cases had been obtained, which were greatly needed, and the whole of the Library was being examined, and sorted out to some extent, with a view to rendering it more useful. The Council were hopeful that by the end of September, some prize essays would be sent in, in response to the circular issued last year. The competition was for a Messenger prize on the subject of “The Methods of ascertaining the rates of mortality amongst the general population of a country, district or town, or amongst different classes of such population, by means of returns of population, births, deaths, and migrations.” There was one matter which came up last summer to which he should like to refer. He then had the pleasure, as President of the Institute, of serving on the Board of Trade Committee in connection with what were called Bond Investment Companies, and he thought that the report of that Committee, following upon the work generally undertaken by it, was likely to be productive, if it had not already been productive, of some good in relation to a certain form of investment which undoubtedly presented some objectionable features. He had great pleasure in moving “That the Report and Accounts for the past year be received and adopted.”

Mr. GEORGE KING said the honour fell to him on the present occasion of seconding the motion for the adoption of the report and accounts. He would like to make a remark on two points. One was the loss the Institute had sustained by the death of their old friend Mr. Augustus Hendriks, in connection with which he desired to add the expression of his personal regret. It was a kind destiny that threw him very much into Mr. Hendriks' society, as during that gentleman's presidency he (Mr. King) had the honour of being Honorary Secretary. It was a very strenuous time, because at that period they were preparing the way and laying the foundation for the British Offices Tables. A great deal of hard work in that connection was shared by the President and Honorary Secretary, and that brought him to know Mr. Hendriks very well indeed, and taught him to esteem and love him. He deeply felt Mr. Hendriks' loss. The other point he wished to mention was with regard to the examinations and the students. It was now about thirty years since he was first an examiner, and through all that time he had taken the greatest interest in the educational work of the Institute, as Tutor, as a member for many years of the Examination Committee, and more recently as Lecturer to the younger members, and he felt considerably troubled that the Institute did not seem to achieve better results in that direction. It appeared to him the difficulty was that the students tried to load their memory with knowledge, and did not give enough attention to thought. It was very difficult to pass the examinations and become a qualified actuary if one only tried to remember the formulas; but he ventured to submit that it was

easy to attain these ends if one thought about general principles. The formulas would in that way take care of themselves, and he wished that the younger generation—not only in the actuarial profession, but in all professions at the present day—would do more thinking and, perhaps, less reading. He believed they would make themselves much more efficient in that way, and would really attain a very much more thorough knowledge. He was very pleased to know that the Council had the matter before them, and was sure that, as they had always tried to do their utmost to bring up the rising generation to be an honour to the profession, the Council would do their very best to find a means of getting over the difficulty.

The resolution was put, and carried unanimously.

#### ELECTION OF OFFICERS.

Messrs. S. J. H. Allin and A. C. R. Cockman were appointed scrutineers of the ballot for the election of officers for the ensuing year.

While the ballot was proceeding, Mr. W. T. MAY said he had very much pleasure in proposing the re-election of Mr. G. A. Brown and Mr. Hugh Lngton as Anditors for the ensuing year, and also the election of Mr. Stanley Hazell. The Institute were very much indebted to the retiring Auditors for their services, considering how arduous and important their work was.

Mr. A. S. YOUNG seconded the motion, which was carried unanimously.

The PRESIDENT said he had great pleasure in informing the members that the balloting list of the names of Fellows recommended by the Council for election had been passed unanimously. He wished his successor, Mr. Wyatt, a very happy and prosperous reign.

The list was as follows:—

#### *President.*

FRANK BERTRAND WYATT.

#### *Vice-Presidents.*

ERNEST WOODS.

FREDERICK SCHOOLING.

THOMAS GANS ACKLAND.

GEORGE TODD, M.A.

#### *Council.*

THOMAS GANS ACKLAND.

HENRY WALSINGHAM ANDRAS.

HENRY JAMES BAKER.

\*THOMAS G. C. BROWNE.

HENRY COCKBURN.

FRANCIS ERNEST COLENZO, M.A.

JOSEPH ERNEST FAULKS, B.A.

DUNCAN CUMMING FRASER, M.A.

NIEL BALLINGAL GUNN.

RALPH PRICE HARDY.

ARTHUR GEORGE HEMMING.

CHARLES DANIEL HIGHAM.

LEWIS FREDERICK HOVIL.

WILLIAM HUTTON.

\*ABRAHAM LEVINE, M.A.

GEORGE JAMES LIDSTONE.

HENRY WILLIAM MANLY.

GEOFFREY MARKS.

WILLIAM PEYTON PHELPS, M.A.

\*EDWARD ARTHUR RUSHER.

GERALD HEMMINGTON RYAN.

FREDERICK SCHOOLING.

\*EDWARD ROBERT STRAKER.

GEORGE TODD, M.A.

\*HAROLD MOLTKE TROUNCER, M.A.

SAMUEL GEORGE WARNER.

JAMES DOUGLAS WATSON.

ERNEST WOODS.

FRANK BERTRAND WYATT.

THOMAS EMLEY YOUNG, B.A.

#### *Treasurer.*

GERALD HEMMINGTON RYAN.

#### *Honorary Secretaries.*

SAMUEL GEORGE WARNER.

| JOSEPH ERNEST FAULKS, B.A.

\* Not Members of the previous Council.



Mr. F. B. WYATT, who received a hearty welcome, said he trusted the members would allow him, on behalf of himself and all the gentlemen who were to be associated with him as colleagues, namely, the Vice-Presidents, the Members of Council, the Treasurer, and the Honorary Secretaries, to thank them most heartily for electing them to their respective posts. It was hardly necessary for him to say that the position of President was not a sinecure, but whatever work was attached to it was very much relieved by the constant assistance and sympathy of those who worked round the Council table. He had had the pleasure of being on the Council for nearly twenty years, and had always noticed that the greatest assistance was accorded to the President. The real work rested with the members of Council, and, if he might say so without offence to the Council, the greatest work of all rested with the Honorary Secretaries. He was sure those who had filled the position of Honorary Secretary would bear him out when he said that that particular work was constantly growing, and required a great deal of time and attention. If he might be allowed to say a few words about himself, he should like to remark that he could scarcely convey to the members the great sense of gratitude he felt at having been promoted to the honour of President. When it was first mentioned to him that he might possibly be nominated for the presidency, he was inclined to stand aside, as he felt there were men who were more worthy than he was to fill the position; but if he could not rank with those who went before him, he could at least try to emulate them. He would like to remind the members that seven of their ex-Presidents—if Mr. Cockburn would allow him to call him an ex-President—were still living. One of the oldest of their ex-Presidents, Mr. Bailey, was present that evening, and they all hoped he would be there for many years to come. He sincerely trusted that the position of the Institute would not suffer during his own term of office. With the assistance of the members of Council he should do everything he possibly could to maintain the prestige of the Institute, and if opportunities arose—and he had reason to hope they would—he would do all he could to aid in the expansion of its sphere of usefulness. He again thanked the members most heartily for his election.

Mr. A. H. BAILEY, who was received with cheers, said that as he happened to be one of the oldest members of the Institute he desired to propose a hearty vote of thanks to the retiring President, the Vice-Presidents, Council, Officers, Examiners, and Supervisors, for their services during the past year. He felt sure that they were all agreed that those gentlemen deserved a hearty vote of thanks, but in proposing it he wished to allude to the question of the accounts, to which the President referred in his opening remarks. He, together with other Fellows, had been struck with the fact that the Institute now possessed a general fund of over £9,000, which was increasing at the rate of about £800 a year. He very much agreed with some of the President's remarks, and did not think it was any part of the function of the Institute of Actuaries to accumulate money. It was necessary that the Institute should have a fund for contingencies, but when that fund continued to increase, as it was doing, he thought it was more than was required. He therefore desired to throw out a suggestion for



the consideration of the Council, whether it might not be expedient to reduce the subscriptions.

Mr. WILLIAM HUGHES, in seconding the vote of thanks, said that he had himself made a suggestion to the President, and as the President had not mentioned it in his remarks, perhaps he might be allowed to do so. He thought it would be very useful to the students if the best papers in past numbers of the *Journal* were reprinted in a convenient form. There might be some difficulty in selecting them, but that difficulty could be overcome. At the present time they were embedded in a great mass of material, which was more or less ephemeral, and seldom referred to. He spoke a little feelingly on the matter, because his own copies of the *Journal* were constantly being borrowed by his student friends, and he found when he turned over the volumes that they were exceedingly worn in certain places, showing him that the students had found particular papers exceedingly valuable in their studies. He thought that if some of those papers could be extracted and re-printed, they would be of great convenience, and handy for general reference. He thought the examiners ought to receive a very large share of the vote of thanks, for theirs was a very onerous and troublesome task. But, at the same time, he ventured to suggest that a great deal of their work was absolutely useless and might be avoided. The number of young men who, as the President had said, presented themselves when not fully prepared, was very numerous, and he thought they might be detected before the examination, so that the examiners might be saved the trouble of looking over papers that were absolutely hopeless. He suggested that some means might be devised of a preliminary  *viva voce*  examination by the tutors or by some other officials, which would rapidly weed out the hopeless candidates.

The resolution was then put and carried unanimously.

The PRESIDENT, on behalf of the Vice-Presidents, the Council, Officers, Examiners, Supervisors and himself, sincerely thanked the members for the hearty vote of thanks which had been accorded to them. He would speak more freely in responding to the motion, knowing as he did that the great burden of the work had fallen, not upon himself, but very largely upon the Honorary Secretaries and the Examiners; and he was sure that, arduous as their work was, they appreciated and were encouraged by the kindly sympathy of the members. As the present was the last meeting at which he should occupy the chair, he desired to thank the members from the bottom of his heart for the kindness with which he had been received at the meetings, and with which his colleagues on the Council had invariably responded to any little efforts he might have made; and for their ready sympathy at all times. It was indeed something to have been the President of the Institute of Actuaries, and although he could not perhaps look back upon his term of office as one during which any very special results had been achieved, he would certainly be filled for the rest of his lifetime with a feeling of great pride and satisfaction at the remembrance of much kindness.

Mr. A. E. MOLYNEUX, in proposing a vote of thanks to the Auditors (Messrs. W. M. Monilaws, G. A. Brown, and Hugh Lugton) for their services

during the past year, said that now the Institute had a membership of over 900, and £9,000 worth of funds, the management of the accounts was a matter of some importance. Although the members knew the accounts were well kept, it was necessary that a certificate of correctness should be given, and they were therefore very much obliged to the auditors for the careful and conscientious work they had done.

Mr. J. H. BARNES seconded the motion, which was carried unanimously.

The PRESIDENT then announced that the meeting stood adjourned to Monday, November 26th.

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### *Additions to the Library.*

The following works have been added to the Library since the publication of the *Journal* for October 1905:

*By whom presented  
(when not purchased).*

Accountants and Auditors, Society of

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“Relative Results of Valuations by Different Methods”, by D. H. Rose.

“A Method of Verifying Valuation Results”, by W. H. Peiler.

“Some Suggestions regarding the Education of Actuarial Students and the future activities of the Society”, by A. Hunter.

“An Accumulation Formula for Policy Values”, by P. C. H. Papps.

“Note on Limit of Risk”, by R. Henderson.

“Mortality among Athletes and other Graduates of Yale University”, by J. M. Gaines and A. Hunter.

“An Equitable Method of keeping the Accounts of ‘Deferred Dividends’”, by R. W. Weeks.

“Note on Intervaluation”, by S. E. Stilwell.

“A New Form of Select Mortality Table”, by M. M. Dawson.

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“Mortality among Graduates of the Yale Divinity School, 1825-72”, by W. M. Strong.

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“Group Valuation and Mortality Register”, by H. Moir.

“Mortality Experience of the Travellers’ Insurance Company according to Age at Death, Cause of Death, and Geographical Distribution”, by H. J. Messenger.

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“An Analysis of the Profit from Endowment Assurances”, by J. Chatham.

“Notes on Summation and Interpolation”, by A. J. C. Fyfe.

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A collection of all the blank forms used at the Head }  
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## FIFTH INTERNATIONAL CONGRESS OF ACTUARIES.

THE Fifth International Congress was held most successfully at Berlin, from September 10th to 15th, and was largely attended by members from twenty-one countries. The following Governments sent official delegates: Germany and Prussia, Argentine Republic, Austria, Belgium, Chili, Denmark, France, Holland, Hungary, Italy, Japan, Mexico, Norway, Russia, Spain, Sweden, Switzerland, and the United States of America. Official delegates also attended from Insurance Institutions in Austria, Belgium, Canada, Denmark, England, France, Holland, Italy, Japan, Scotland, Sweden, and Switzerland. The delegates officially appointed by the Institute of Actuaries were Messrs. Wyatt (President), Woods, Schooling, and Ackland (Vice-Presidents), Andras, Hughes, Manly, and Phelps.

The Reports and Memoirs submitted to the Congress are included in two volumes, of 820 and 748 pages, which were issued to the members prior to the meetings. The reports, which were submitted for discussion, dealt with the following subjects:

- I. Industrial Assurance; especially on the lives of Children.
- II. Methods of calculating and determining extra premiums for hazardous risks.
- III. Mortality Tables for Annuitants.
- IV. Methods of insuring (a) Abstainers; (b) Persons whose occupations connect them with the manufacture or sale of alcoholic beverages.
- V. Insurances on the lives of Women.
- VI. Taxes imposed upon Insurance Companies.
- VII. The limits within which Insurance is possible.

Memoirs (included in the second volume of the Transactions) were submitted upon the following subjects:

- VIII. Methods of conducting Mortality investigations.
- IX. Methods of adjusting or graduating Tables of Mortality.
- X. The progress of teaching of Actuarial Science in Schools and Colleges.
- XI. The progress of Insurance Legislation.
- XII. Aids to Actuarial Calculations.
- XIII. The uniformity of legal requirements, especially as regards Reports to be made to the Insurance Authorities.
- XIV. Notes on the publications presented to the Congress by
  - (a) The Imperial Office for Supervision of Insurance;
  - (b) The Imperial Office for Workmen's Insurances;
  - (c) The Imperial Statistical Office.

The following were the duly elected Officers of the Congress:

*President*—Dr. FERDINAND HAHN.

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# INDEX TO VOL. XL.

---

## A.

Ackland (T. G.). Notes on an Approximate Method of Valuation of Whole-Life Assurances, with allowance for Selection, 42.

——— Remarks on "Spurious Selection" in Amalgamated Mortality Tables, 239.

## ACTUARIAL NOTES—

Interpolation (Two Variables). H. H. Edwards, 289.

——— J. Spencer, 293.

Law of Uniform Seniority. Editorial, 302.

Summation, Approximate. W. P. Elderton, 116.

Actuaries, Fifth International Congress of, 416.

Austin (H. H.). Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 88.

## B.

Bacon (J.). Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 92.

——— Remarks on "Spurious Selection" in Amalgamated Mortality Tables, 234, 304.

Board of Trade. Report of Committee on Bond Investment Companies, 99.

——— Returns, Summary of, for 1905, 313.

Bond Investment Companies. Report of the Departmental Committee on, 99.

British Offices O<sup>[M]</sup> Select Tables. On their application to the Valuation of Whole-Life Policies. G. King, 1.

——— O. F. Diver, 15.

——— T. G. Ackland, 42.

## C.

Canada. Vital Statistics; with particular reference to the Province of Ontario. M. D. Grant, 125.

Catchlove (C. H. L.). Expression of regard from the profession in Australia, 98.

Certification (Actuarial) in connection with Friendly Societies. Historical Memorandum, 212.

Cockburn (H.). Remarks on Reversionary Securities as Investments, 339.

Colenso (F. E.). Remarks on Reversionary Securities as Investments, 337.

Congress, Fifth International Actuarial, 416.

## CORRESPONDENCE:

Determination of Average Ages by methods of Weighting. S. E. Macnaghten, 120.

"Spurious Selection." J. Bacon, 304.

——— W. P. Elderton, 309.

Use of O<sup>[M]</sup> Select Premiums for Valuation purposes. D. C. Fraser, 122.

Coutts (C. R. V.). Reversionary Securities as Investments, 317.

## D.

Dawes (J. A.). Remarks on Registration of Title to Land, 288.

Diver (O. F.). On a property of the  $O^{(M)}$  Select Tables, and its application to the Valuation of Whole-Life Policies, 15.

## E.

Edwards (H. H.). Interpolation by Finite Differences (Two Independent Variables), 289.

Elderton (W. P.). On an alternative type of formula for Approximate Summation, 116.

——— On a form of Spurious Selection which may arise when Mortality Tables are amalgamated, 221. Additional Remarks on the Discussion, 309.

——— Remarks on Variations in Masculinity, 184.

## F.

Faulks (J. E.). Remarks on Registration of Title to Land, 284.

Findlay (Dr. A. W.). Remarks on Registration of Title to Land, 283.

Fraser (D. C.). On the use of  $O^{(M)}$  Select Premiums for Valuation purposes, 122.

French Assurance Law, 1905, 343.

Friendly Societies. Historical Memorandum in connection with Actuarial Certificates, 212.

## G.

Giles (H. Ross). Remarks on Registration of Title to Land, 282.

Gillison (J. B.). Remarks on Registration of Title to Land, 279.

Grant (M. D.). Canadian Vital Statistics; with particular reference to the Province of Ontario, 125, 150.

Green (G.). Remarks on "Spurious Selection" in Amalgamated Mortality Tables, 234.

Grouping of Policies for Valuation by Select Mortality Tables. G. King, 1.

## H.

Hardy (R. P.). Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 87.

Hart (J. R.). Some Aspects of Registration of Title to Land, 246.

Hemming (A. G.). Remarks on Canadian Vital Statistics, 143.

Higham (C. D.). Remarks on Variations in Masculinity, 184.

## I.

## INSTITUTE OF ACTUARIES:

Additions to the Library, 403.

Examinations. Names of Successful Candidates 1906, 395.

——— Papers set April 1906, 375.

——— Revised Rules, 312.

New Members of Council, 400.

Proceedings at the Annual General Meeting 1906, 397.

Report, 390.

Revenue Account and Balance Sheet, for the year ending 31 March 1906, 392-3.

Syllabus of Lectures by Mr. George King, 124.

International Congress of Actuaries. Berlin, 1906, 416.

Interpolation. (Two Independent Variables). H. H. Edwards, 289.

——— J. Spencer, 293.

## K.

King (G.). On the Valuation in Groups of Whole-Life Policies by Select Mortality Tables, 1.

——— Remarks on Canadian Vital Statistics, 147.

## L.

Land Registration. J. R. Hart, 246.

Levine (A.). Demonstration of an extended application of the Law of Uniform Seniority, 302.

——— Remarks on Reversionary Securities as Investments, 334.

——— Tables of Continuous Temporary Annuities,  $OM\ 2\frac{3}{4}$  per-cent, 369.

Lewis (J. N.) and Lewis (Dr. C. J.). The Variations in Masculinity under Different Conditions, 154.

Lidstone (G. J.). Demonstration of an extended application of the Law of Uniform Seniority, 303.

——— Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 83, 84.

——— Remarks on Reversionary Securities as Investments, 340.

Life Assurance Companies of the United Kingdom, 313.

## M.

- Macnaghten (S. E.). On the Determination of Average Ages by methods of Weighting, 120.  
 "Masculinity", the Variations in, under Different Conditions. J. N. and Dr. C. J. Lewis, 154.

## O.

- Ontario. Mortality Tables, 141.

## ORIGINAL TABLES:

- Coefficients, &c., for Interpolation in a Table of Two Independent Variables. H. H. Edwards, 291-2.  
 Continuous Temporary Annuities,  $OM$   $2\frac{3}{4}$  per-cent. A. Levine, 369.  
 Mortality from Census Returns for Ontario. M. D. Grant, 141.  
 Nosological Tables, Canada. M. D. Grant, 137.  
 Probabilities of Marriage and Issue. O. Schjoll, 209.  
 Proportions of Births under Different Conditions. J. N. and Dr. C. J. Lewis, 154.  
 Reserves by  $OM$  Select Table. G. King, 12.  
 Select Valuation Tables. O. F. Diver, 15, 40.  
 ——— T. G. Ackland, 60.

## P.

- Phelps (W. P.). Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 93.  
 Pulley (W. P.). Remarks on Reversionary Securities as Investments, 337.

## R.

- Registration of Title to Land, Some aspects of. J. R. Hart, 246.  
 Reversionary Securities as Investments. C. R. V. Coutts, 317.  
 Rubinstein (J. S.). Remarks on Registration of Title to Land, 276.

## S.

- Schjoll (O.). On the Calculation of the Contributions to be made to an Annuity Fund for Widows and Children, 200.

- Selection. On a form of Spurious Selection. W. P. Elderton, 221.  
 Sharman (W. C.). Remarks on Registration of Title to Land, 273.  
 Spencer (J.). Some Practical Hints on Two-variable Interpolation, 293.  
 Spurious Selection. W. P. Elderton, 221, 309.  
 ——— J. Bacon, 304.  
 Summation (Approximate), On an Alternate Type of Formula for, W. P. Elderton, 116.

## T.

- Thomas (E. C.). Remarks on Methods of Valuation of Whole-Life Assurances, with allowance for Selection, 90.  
 ——— On some Special Features of Widows' and Orphans' Funds, 188.  
 Tilt (R. R.). Remarks on Reversionary Securities as Investments, 335.  
 Todhunter (R.). Remarks on Canadian Vital Statistics, 145.

## U.

- Uniform Seniority, Law of. Demonstrations of its extended application. A. Levine and G. J. Lidstone, 302-3.

## V.

- Valuation of Whole-Life Policies by means of Select Tables. G. King, 1.  
 ——— O. F. Diver, 15.  
 ——— T. G. Ackland, 42.  
 Valuations. On the Use of  $OM$  Select Premiums. D. C. Fraser, 122.  
 Vital Statistics, Canadian. M. D. Grant, 125.

## W.

- Warner (S. G.). Remarks on Methods of Valuation of Whole-Life Assurances, with Allowance for Selection, 91.  
 ——— Remarks on Canadian Vital Statistics, 149.  
 ——— Remarks on "Spurious Selection" in Amalgamated Mortality Tables, 243.  
 ——— Remarks on Reversionary Securities as Investments, 336.

Watson (A. W.). Remarks on Variations in Masculinity, 185.

Watson (J. D.). Remarks on Canadian Vital Statistics, 148.

——— Remarks on Reversionary Securities as Investments, 332.

“Weighting”, Methods of, On the determination of Average Ages by. S. E. Macnaghten, 120.

Whole-Life Assurances, On the Valuation of, by means of Select Mortality Tables. G. King, 1.

——— O. F. Diver, 15.

——— T. G. Aekland, 42.

Widows' and Orphans' Funds, On some Special Features of. E. C. Thomas, 188.

——— On the Calculation of the Contributions, etc. O. Schjoll, 200.

Woods (E.). Remarks on Canadian Vital Statistics, 148.

# Y.

Young (T. E.). Remarks on Variations in Masculinity, 181.

END OF VOL. XL.











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